

$v = \frac{x' + x''}{2}$. When we integrate this joint density function with respect to u , we obtain the density function of $v = \frac{x' + x''}{2}$ as given by

$$(16) \quad p(v) = 6\sqrt{2}\phi[\sqrt{2}(v - \theta)] \left[1 + G\left(\frac{\sqrt{2}(v - \theta)}{\sqrt{11}}\right) - 2 \int_0^\infty \phi(x) G\left(\frac{3x}{\sqrt{2}} + v - \theta\right) dx \right].$$

The mean and the variance of the distribution of v are given by θ and $\frac{1}{2} + \frac{\sqrt{3}}{4\pi}$ respectively.

It may be remarked that if there is a suspicion that one of the extreme observations in a sample of three does not belong to the normal population under consideration, then the median of the sample is a better estimate than the average of the two closest. The efficiency of the latter compared to that of the former is about 70%, for the variance of the median in this case is given by $1 + \frac{\sqrt{3}}{\pi}$ compared to $\frac{1}{2} + \frac{\sqrt{3}}{4\pi}$ of v , the average of the two closest. The efficiency is here defined as the ratio of the variances for the two estimates.

ERRATA

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The author regrets the following inconsequential, but very disturbing, slips in his paper "On the Kolmogorov-Smirnov limit theorems for empirical distributions" (*Annals of Math. Stat.*, Vol. 19 (1948), pp. 177-189):

(1) In equation (1.4) on p. 178, the exponent $-\nu^2 z^2$ should be replaced by $-2\nu^2 z^2$. The same copying error occurs in the description of Smirnov's table on p. 279. The proof is correct as it stands.

(2) In the formulation of the *continuity-theorem* on p. 180 it is claimed that $u_k \rightarrow f(t)$ whereas in reality the continuity theorem permits only the conclusion that

$$(*) \quad \delta \sum_{r=1}^k u_r \rightarrow \int_0^t f(x) dx.$$

This slip in formulation in no way affects the proofs since only (*) is used. (The assertion that the step functions $\{\xi_k\}$ converge pointwise is not based on a

second application of the continuity theorem, but on the obvious fact that(*) implies

$$\delta \sum_{r=1}^k q_r u_r \rightarrow \int_0^1 q(x)f(x) dx,$$

where the step function $\{q_r\}$ converges uniformly to a continuous monotonic $q(x)$.

The following corrections apply to the paper, "On the normal approximation to the binomial distribution" (*Annals of Math. Stat.*, Vol. 16, (1945), pp. 319-329).

(1) Equation (27) gives two variants of an estimate for the error ρ . The second should simply restate the first one in terms of the variable x ; in other words, the expression $(p^3 + q^3)$ in the second line of (27) should be replaced by $p^3(1 - px/\sigma)^{-3} + q^3(1 + qx/\sigma)^3$.

(2) The estimate $\rho < \sigma^{-6}/300$ given in (28) is not valid over the entire range for which it is claimed. However, the further theory depends only on the fact that $\rho = O(\sigma^{-4})$, and the estimate $\rho < \sigma^{-6}/30$ is both correct and sufficient for our purposes. (Actually, no changes whatever are required in the proofs, since (28) is used explicitly only for a range where it is correct as stated).

(3) On p. 324 it is stated that under the conditions of the main theorem (p. 325) $k \geq 4$, $n - k \geq 4$, whereas in reality the value 3 can occur in extreme cases. Fortunately, the assertion is not used anywhere in the proof, and the error ρ is negligible in all cases.

Accordingly, no changes are required either in the formulation or the proof of the theorems. I am indebted to Dr. W. Hoeffding for calling my attention to the slips.

(4) The first minus sign in footnote 5 should be an equality sign and the second minus in (70) a plus.

ABSTRACTS OF PAPERS

(Abstracts of papers presented at the Chapel Hill meeting of the Institute, March 17-18, 1950)

1. A Method of Estimating the Parameters of an Autoregressive Time Series. S. G. GHURYE, University of North Carolina.

The general autoregressive process of the second order is defined by the equations

$$\begin{aligned} x_t &= X_t + \eta_t, \\ X_t + \alpha_1 X_{t-1} + \alpha_2 X_{t-2} &= \epsilon_t, \end{aligned}$$

where x_t is the value actually observed at time t , X_t the corresponding theoretical value, ϵ_t the disturbance and η_t the superposed variation. The estimates of α_1 , α_2 given by Yule's method are biased and inconsistent if η_t is not identically zero, the permanent bias being a function of the unknown variance of η_t . The present paper proposes a method of estimation