Put $C = AB$. Let C' be the transposed matrix of C. We have from (2), (4)–(7)

$$2Tr(A^2B^2) + Tr((AB)^2) = 2Tr(CC') + Tr(C^2) = 0. \tag{8}$$

The left side of (8) is equal to $\sum_{ij=1}^{n} (c_{ij}^2 + c_{ij}c_{ji} + c_{ji}^2)$, which is positive unless all $c_{ij} = 0$ $(i, j = 1, \cdots, n)$. Hence we have $C = AB = 0$, q.e.d.

Corollary 1 follows from Theorem 1 and the theorem of Craig. Corollary 2 results from observing that independence of Q_1 and Q_2 implies (2).

B. Matérn proved, that if A, B are nonnegative, then $AB = 0$ follows from a unique condition $F_{11} = 2Tr(AB) = 0$. If only one of the matrices A, B is assumed to be nonnegative, we have

THEOREM 2. *Let A be nonnegative. Then from two conditions $F_{11} = 0$, $F_{12} = 0$ in (2) follows the relation $AB = 0$.*

PROOF. From (4), (5) follows $Tr(AB^2) = 0$. Since A is nonnegative, we can choose a real symmetric matrix A_0 such that $A = A_0^2$. Put $C_0 = A_0B$. Then we have $Tr(AB^2) = Tr(C_0C_0') = 0$ and from this follows $C_0 = 0$. Hence we have also $AB = A_0C_0 = 0$, q.e.d.

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ERRATA TO "CONTROL CHART FOR LARGEST AND SMALLEST VALUES"

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In the paper cited in the title (*Annals of Math. Stat.*, Vol. 20 (1949), p. 306), there are some numerical errors in Table I. Values of $d_2/2$ and d_4 are given by H. J. Godwin in "Some Low Moments of Order Statistics" in the same issue of the *Annals*. These values are more accurate than those heretofore available. A corrected Table I based on these values is as follows:

n	d_2	d_4	A_2	A_3	A_4	n
2	1.1284	.8256	1.8800	2.6951	3.0411	2
3	1.6926	.7480	1.0233	1.8258	3.0902	3
4	2.0588	.7012	.7286	1.5218	3.1330	4
5	2.3259	.6690	.5768	1.3629	3.1699	5
6	2.5344	.6449	.4832	1.2634	3.2020	6
7	2.7043	.6260	.4193	1.1945	3.2303	7
8	2.8472	.6107	.3725	1.1434	3.2556	8
9	2.9700	.5978	.3367	1.1038	3.2784	9
10	3.0775	.5868	.3083	1.0720	3.2992	10

ABSTRACTS OF PAPERS

(Abstracts of papers presented at the Berkeley meeting of the Institute, August 5, 1950)

1. Sampling from Populations with Overlapping Clusters. Z. W. Birnbaum, University of Washington, Seattle.

In cluster sampling it is usually assumed that the clusters are disjoint. In this paper situations are considered in which this assumption is not fulfilled. Let the population π consist of N individuals "j", having the variates $V[j], j = 1, 2, \cdots, N$, and let K clusters $C[i]$, $i = 1, 2, \cdots, K$, be such that each "j" belongs to at least one cluster. Let $s[j] \geq 1$ be the number of different clusters to which "j" belongs (the multiplicity of "j"). The cluster $C[i]$ contains N_i individuals with the variates $V[i, t]$, $t = 1, 2, \cdots, N_i$; $i = 1, 2, \cdots, K$. In a sampling procedure, let sub-sample sizes $n[i]$ be given for each $C[i]$, and weights $\lambda[i, t]$ for each $V[i, t]$; a random sample of k clusters $C[i_u]$, $u = 1, 2, \cdots, k$ is obtained, then $n[i_u]$ individuals are sampled from $C[i_u]$, and for each of them its variate and its multiplicity are recorded. Necessary and sufficient conditions are derived for $S = \sum_{u=1}^{k} \sum_{v=1}^{n[i_u]} V[i_u, t_v] \lambda[i_u, t_v]$ being an unbiased estimate of $\bar{V} = \frac{1}{N} \sum_{j=1}^{N} V_j$. The variance of S is found, the weights are studied which minimize this variance, and some practically important special cases are derived.

2. A Simple Nonparametric Test of Independence. Nils Blomqvist, University of Stockholm.

Consider a sample of size n from a two-dimensional distribution $F(x, y)$. Let $\tilde{x}$ and $\tilde{y}$ denote the two sample medians and compute the number of individuals, say k, satisfying the inequality $x < \tilde{x}$, $y < \tilde{y}$ (the trivial difficulty arising when n is an odd number can easily be overcome). A test of independence based on k is nonparametric. As a matter of fact one has under the null hypothesis that

$$P(k) = \binom{m}{k}^2 \bigg/ \binom{2m}{m},$$