RECENT ADVANCES IN SAMPLE SURVEY THEORY AND METHODS[1]

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CONTENTS

I. INTRODUCTION

A. Sample surveys and random experiments. The development of statistical theory and methods has, to a large extent, taken place in response to demands for tools to cope with the problem of uncertainty which arises when one is dealing with observations exhibiting "variability", "indeterminism", etc. The research thus initiated has been guided by two basic considerations:

1. The necessity of being able to *measure* the degree of uncertainty; and
2. The desirability of being able to *regulate* the degree of uncertainty.

In certain fields of application, the problem of *measuring* the degree of uncer-

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tainty was tackled by assuming that the variability present in a set of observations could be ascribed to the operation of a chance mechanism in nature. In technical language this approach means that the variability is accounted for by means of a stochastic model. In other fields of application, however, the approach indicated above proved unsatisfactory; the variability could not be explained by sole reference to the operation of a chance mechanism. Such was the case with statistical experiments and sample surveys. The solution of the fundamental problem of measuring the degree of uncertainty in such cases is largely due to Fisher. His was the ingenious idea of randomization: the statistician introduces an element of chance into the selection of the observations, thus *creating* a "random experiment".

The problem of *regulating* the degree of uncertainty is a central one in the theory and methods of statistical inference. On the one hand, desirable properties of procedures for estimation and hypothesis testing have been investigated, and corresponding principles or criteria established; on the other hand, many techniques or designs for collecting and analyzing observations in accordance with some such principle or criterion have been developed. Most of these designs have been constructed for use within a specific field of application. As a consequence, we have today several *branches* of designs, each one with its own terminology: "experiment", "acceptance sampling", "Monte Carlo sampling", "sample survey", etc. From the point of view of the theory and methods used to cope with the problem of uncertainty, these branches of design, when randomization is appropriately introduced, all belong to one and the same class of procedures for statistically planned observation, namely the class of "random experiments". This general *realization* from about a quarter-century back represents by itself an important advance in the history of statistical theory and methods; Cornfield and Tukey (1956) provides the formal link between the "different" theories of the above-mentioned branches.

B. "Sample survey" and "recent" as discussed here. The objective of this paper—to present a review of "recent advances in sample survey theory and methods"—calls for a specification of "*sample survey*" and of "*recent*". "*Sample survey*" as discussed here is that type of random experiment which is the main subject of such textbooks as Cochran (1953), Deming (1950), Hansen, Hurwitz and Madow (1953), Sukhatme (1953), and Yates (1953). In presenting "*recent*" advances, the paper will be restricted to advances published in 1950 or later and not (extensively) dealt with in any of the textbooks just mentioned; for convenience, reference will be made to "before 1950" and "since 1950". Because of the orientation towards *recent* advances, references given must not be interpreted as indicating priority of ideas presented. An effort will be made, however, to review the advances since 1950 against the historical background.

C. References to other review papers. The development of sample survey theory and methods prior to 1950 is thoroughly discussed in Cochran (1947), Seng (1951), Stephan (1948), Sukhatme (1959), Thionet (1959a), and Yates

(1946). The papers by Sukhatme and Thionet should be consulted also for supplementing the present review of the development since 1950.

D. Main trends of development. At first analysis, based on reading an extensive list of references covering the last three decades, the development of sample survey theory and methods since 1950 shows great similarities with the development before 1950. Thus, the majority of contributions which have appeared since 1950 concern the construction of sampling variance models, as did the majority of contributions before 1950. There are, however, indications that important changes are rapidly taking place; the overall picture is likely to be radically changed within the next 5 to 10 years. What is happening may be broadly described as a change of relative emphasis of research. While considerable efforts are still being devoted to the development of sampling variance models, increasing efforts are also being devoted to both the mathematical-statistical foundations of sample survey theory and methods and the development of survey theory and methods which take into account more than the sampling variance, the ultimate goal being to construct practically useful mixed error models.

E. Organization of the review. The organization of the review to be given in this paper will be based on the observation referred to in paragraph D, dealing with the following subjects:

1. Chapter II—Review of the development of the mathematical-statistical foundations.
2. Chapter III—Construction of sampling variance models.
3. Chapter IV—Construction of mixed error models.

II. RECENT ADVANCES—THE MATHEMATICAL-STATISTICAL FOUNDATIONS

A. Distribution theory and analytical tools. The credits for placing sample surveys within the realm of random experiments are largely due to Neyman (1934); this classical paper marks the beginning of the era of what is nowadays referred to as "probability sampling" or the use of "measurable sample designs". The problem of measuring the degree of uncertainty is usually discussed in textbooks on sample survey theory with respect to the mean $\bar{X}$ per element. The use of interval estimation such as "$\bar{x} \pm 3\, s(\bar{x})$" is based on the assumption that the distribution of the unbiased (or only slightly biased) estimator $\bar{x}$ is, to a satisfactory degree of approximation, normal.

The approach just indicated received considerable early empirical support. It had, in addition, some theoretical support. An early reference is Neyman (1934); two later ones are David (1938) and Madow (1948), this latter giving the most far-reaching results published before 1950. The results given in David (1938) and Madow (1948) have recently been generalized. In Motoo (1957) a Lindeberg type condition is shown to be sufficient, when sampling without replacement from a finite population, for the "distribution" of the sample mean to be in the limit normal; reference should also be given to Erdös and Rényi (1959). The same type of condition is shown to be necessary in Hájek (1960). In this connection, mention should also be made of the advances with respect to the analytical tools for deriving higher moments of distributions. The development of polykays in Tukey (1950) and the derivation of moment coefficients of the k-statistics in Wishart (1952) are two examples.

B. Criteria. Regulating the degree of uncertainty calls for the choice of a criterion, that is a measure of efficiency, and for techniques for using it. Neyman (1934) introduced the criterion "minimize the variance, subject to fixed sample size". In Yates and Zacopanay (1935), this criterion was given a more general formulation: "minimize the variance, subject to fixed cost", or vice versa. It is essentially this kind of criterion which governs the design of large scale sample surveys today.

The usual technique for determining the optimum design makes use of Lagrange's multiplier. The variance and the cost are expressed as functions of the "design variables", say Var and C respectively. Thereafter, $F = \text{Var} + \lambda(C - C_0)$, C_0 being the fixed cost, is analyzed for minimum. In Stuart (1954), it is shown how Schwarz' inequality can be used for this same purpose. Further results are given in Hájek (1959a).

Some criticism has been launched on different grounds against the kind of criterion just presented.

1. Thionet (1954) accepts the structure of the criterion mentioned above, that is to minimize one quantity, while keeping another quantity at some prescribed level. He points out, however, that the criterion to be used must reflect the institutional fact that the different components that make up the total cost of a survey may not be completely interchangeable among themselves. Thus, Thionet proposes a criterion of the type "minimize the variance, subject to a set of fixed cost components". This formulation calls for determining the minimum of a function, Thionet (1959*a*),

$$F = \text{Var} + \lambda_a(C_a - C_{a0}) + \lambda_b(C_b - C_{b0}) + \cdots$$

where $a, b, \cdots$ refer to the different cost components.[2]

2. A second line of criticism is concerned with the very structure of the criterion. The use of the criterion discussed above results in a *relative* optimum design, namely the design which is optimum for the chosen level of cost; but it does not help to select the proper level. The development of a statistical decision theory in the 1940's meant an impetus towards the construction of new criteria. Two early examples are given by Blythe, Jr. (1945) and Nordin (1944). A recent contribution is Aggarwal (1959). A somewhat different line is represented by Thionet (1957) and (1958), where an effort is made to have the concept of "lost information" serve as a bridge between the sample survey theory and the theory of information. Thionet conjectures that it will be possible to define "lost information" independently of some specified parameter.

[2] Or rather $F = \text{Var}$; $C_a \leqq C_{a0}\ C_b \leqq C_{b0}$; $\cdots$.

III. RECENT ADVANCES—SAMPLING VARIANCE MODELS

A. Main lines of development. The technical development that has taken place in this area since 1950 represents a natural continuation along lines established before 1950, and especially in the 1930's and the 1940's. Three important lines of development are:

1. The development of new sampling variance models in the nature of general urn-schemes etc.; these models will be referred to here as *basic models.*
2. The development of new techniques for making efficient use of features specific to a certain application; this will be referred to as the development of *resource oriented models.*
3. The development of *simplifying techniques* to be used in applications of fundamentally complicated sample survey designs.

B. Development of basic models. In the 1930's and the 1940's great progress was made in the area of basic models. This progress meant the development of rigorous sampling theory for schemes used long in sample survey practice; it meant improvements of previously developed models; and, finally, it meant the innovation of new models of great efficiency.

Thus, Neyman (1934)[3] formulated and solved the problem of the best allocation of sampling units among strata in stratified sampling. Jessen (1942) demonstrated the efficiency to be achieved from using a "panel", when estimating changes in time. Hansen and Hurwitz (1943) extended the theory of sampling from a finite population to cover more complex designs then previously considered, and, in addition, introduced a scheme for multistage sampling, using probabilities proportional to efficiently determined measures of size for the selection of the primary sampling units. Cochran (1946), Madow and Madow (1944) and Madow (1949) and Yates (1948) developed the theory of systematic sampling.

Much of the development in the field of sampling variance models since 1950 has been in the nature of a direct continuation of previous development; it has brought about changes, improvements and generalizations of already available models. But in this same period, there has also been a fair amount of essentially new development. The review of the development of basic models since 1950 will be made using the following broad scheme of classification:

1. *$\bar{X}$, X and R models:* Models for estimating the overall mean per element, the total or the ratio of two means of a given population.
2. *Time-change models:* Models for estimating the change over time in the overall mean per element, total or ratio of two means of a given population.

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[3] It has been pointed out that Tschuproff published this result already in the early 1920's; in fact, it can be traced back at least to the 1880's. Neyman deserves, however, the credit for realizing the importance to sample survey practice of the result. In this connection, it should be pointed out that the question of priority is often dealt with in a way which tends to "favor" the author of a paper presenting the mathematical-statistical formalization of a problem and (more or less) disregard the statistician, sometimes different, who saw and appreciated the problem, and perhaps provided an intuitive solution to it.

3. *Multiparametric models:* Models for estimating several overall means per element, etc. of a given population.

4. *Models for sub-population studies:* Models for estimating one or more means per element, etc., referring to specific parts (domains of study) of a given population.

5. *Special parameter models:* Models for estimating some other overall parameter than the mean per element, etc.

6. *Multinomial table models:* Models for estimating the frequencies of the RC cells of a table, having R rows and C cells.

7. *Other models:* Models other than those considered in 1–6.

This order of presentation does not imply a ranking with respect to the importance of the seven classes. The same remark applies to the order of presentation of recent advances within each class.

1. *$\bar{X}$, X and R models:*

a. *Simple random sampling.* It has been shown that when sampling with replacement the estimator based on the n_d *distinct* units of the sample has a variance which is at most as large as the variance of the usual estimator based on all n observations. [REFERENCES: Basu (1958); Des Raj and Khamis (1958); Roy and Chakravarti (1960).]

b. *Stratified sampling.* In the early 1950's, the problem of how best to stratify a population into a fixed number, L, of strata was solved in a mathematical sense. Since then, computationally simple methods for approximating the "exact" solution have been developed and adapted to applications. Moreover, some progress has been made as to the determination of the optimum number of strata. [REFERENCES: Cochran (1961); Dalenius (1957); Dalenius and Hodges, Jr. (1957) and (1959); Desabie (1956); Ekman (1959); Stange (1960) and (1961); Strecker (1957); Taga (1953); Zindler (1956).]

c. *Systematic sampling.* The problem of measuring the degree of uncertainty as discussed in paragraph IA may be tackled in two different ways when using systematic sampling. One approach is to use replication: instead of selecting one set of n observations, r independent sets of n/r observations may be selected. Another approach is to account for the role played by the variability by means of a realistic model. The theory of one- and two-dimensional stochastic processes has proved especially useful for the construction of such models in a great many fields of applications. [REFERENCES: Dalenius, Hájek and Zubrzycki (1961); Das (1950); Gautschi (1957); Hájek (1959); Matérn (1960); Whittle (1954); Williams (1956); Zubrzycki (1958).]

d. *Sampling $n > 1$ units with unequal probabilities.* The scheme developed in Hansen and Hurwitz (1943) is characterized by sampling with replacement or the selection of $n_H = 1$ unit from each of $L = n$ strata. This latter device, primarily motivated by efficiency considerations, circumvents a problem specific in sampling without replacement, namely the problem of successive changes in the selection probabilities. One approach to the problem of selecting units without using the device mentioned above is obviously to use sampling with replacement, continuing sampling until n_d distinct units have been selected. A second approach is to sample without replacement and take the computational difficulties as implied above associated with this type of scheme. A third approach, finally, is to use a sampling scheme which selects a sample with probability proportional to the sum of the measure of size of the units making up the sample. Existing sample survey theory permits a rational choice to be made among these three approaches, which may be looked upon as applications of the Rao-Blackwell theorem. Sampling with unequal probabilities is often used for the selection of the primary sampling units in multi-stage sampling. Since 1950, some new schemes for sub-sampling within a sample of primary sampling units have been developed. [REFERENCES: Basu (1954) and (1958); Das (1951); Des Raj (1956a); Des Raj and Khamis (1958); Durbin (1953); Grundy (1954); Hájek (1949) and (1959); Henry (1951); Horvitz and Thompson (1952); Jebe (1952); Lahiri (1951); Midzuno (1952); Murthy (1957); Narain (1951); Roy and Chakravarti (1960); Sen (1955); Sen, Anderson and Finkner (1954); Wilks (1960); Yates and Grundy (1953); Žarković (1960).]

e. *Special techniques for restricted sample selection.* The use of sampling without replacement, stratified sampling, sampling with unequal probabilities, cluster sampling etc. may be looked upon as means of exercising restrictions on the procedure, by which a set of elements is selected to make up *the* sample. More specifically, these procedures aim at a regulation of the probabilities by which different "patterns" of elements will be selected.

In the 1950's, new techniques have been developed for such a *direct* regulation of the "sample pattern"; "controlled selection", "lattice sampling" and "two-way stratified sampling" are three important examples. Moreover, a technique for an *indirect* restriction of the selection procedure has been developed. This technique uses an acceptance inspection approach: a sample is selected, and a known parameter P is estimated. If the estimate P^* is "sufficiently close" to P, the sample is accepted; otherwise, the sample is rejected and the procedure repeated.

It is interesting to observe that many of the techniques for exercising direct or indirect restriction of the solution procedure were adhered to long ago in schemes for purposive selection, that is in schemes which today would be referred to as "non-probability sampling" or "non-measurable designs". [REFERENCES: Bryant, Hartley and Jessen (1960); Dalenius (1961); Goodman and Kish (1950); Koller (1958); Patterson (1954).]

f. *Special estimation procedures.* The unbiased estimator $\bar{x}$ of a mean per element $\bar{X}$ may be looked upon as a special case of the following estimator:

$$\bar{x}_\theta = \bar{x} + \theta(\bar{Y} - \bar{y})$$

where $\bar{y}$ is the unbiased estimator of the known mean $\bar{Y}$; for $\theta = 0$, $\bar{x}_\theta = \bar{x}$. The use of $\theta \neq 0$ corresponds to the use of different *special* estimators. Thus for $\theta = k$, a constant chosen independent of the sample, $\bar{x}_\theta$ is the difference estimator. For $\theta = b$, the estimator of the regression coefficient, $\bar{x}_\theta$ is the regression estimator. For $\theta = r = \bar{x}/\bar{y}$, x_θ is the ratio estimator; as is well known, this ratio estimator is (usually) biased, although the bias is small relative to the variance, if the sample size is large.

Among recent advances mention will be made of the construction of unbiased ratio estimators and also of estimators which make use of more than one auxiliary variable. [REFERENCES: Des Raj (1954); Durbin (1959); Fieller and Hartley (1954); Goodman and Hartley (1958); Hájek (1958a) and (1958b); Hartley and Ross (1954); Lahiri (1951); Mickey (1959); Nanjamma, Murthy and Sethi (1959); Olkin (1958); Pascual (1961); Quenouille (1956); Robson (1957); Žarkovič (1956).]

2. *Time-change models:* Many sample surveys are undertaken in order to estimate parameters of a "time-dependent" population at two or more different times; typical is the estimation of the difference between two successive means, called the "change over time". Efficient design usually calls for use of some kind of a panel technique: the sampling is carried out in such a way that two samples drawn at successive times have some units in common. A considerable amount of theory for the simplest kind of panel system ("one-level rotation") is available in the textbooks mentioned in paragraph IB. In the 1950's this theory has been greatly extended; moreover, theory has been developed for more elaborate panel systems ("two-level rotation"). [REFERENCES: Eckler (1955); Hansen, Hurwitz, Nisselson and Steinberg (1955).]

3. *Multiparametric models:* While most models available in the literature are models for the estimation of a *single* parameter, typically $\bar{X}$, X or R, the real life problem usually calls for the estimation of several parameters. It may happen that a design which is very efficient for the estimation of certain parameters is much less efficient for the estimation of other parameters. To tackle this multiparametric problem calls for the construction of a measure of precision, which takes into account all parameters to be estimated. Among approaches proposed, the use of the generalized variance and a "programming" technique may be mentioned; a review is given in Dalenius (1957). [REFERENCES: Chakravarti (1954); Dalenius (1957); Des Raj (1956b); Ghosh (1958).]

4. *Models for sub-population studies:* A primary purpose of many sample surveys is to estimate parameters referring to specific *parts* of a given population. Such surveys have become known as "analytic surveys", but will be referred to here as "sub-population studies"; for the parts thus studied, U. N. Sub-Commission on Statistical Sampling coined the expression "domains of study". The area of sub-population studies has in the 1950's been the subject of a growing interest. There seems to be, however, a considerable amount of confusion and misunderstanding concerning certain technical aspects which need to be cleared up.

If the domains can be identified in the frame prior to the selection of the sample, the theory of stratified sampling is directly applicable to the design problem: a separate sample is selected from each one of the domains. If the domains cannot be identified in the frame prior to the selection of the sample, standard theory is still applicable; this situation calls for the application of stratified sampling in combination with ratio estimation. While thus, contrary to what is sometimes stated, the problem of constructing models for sub-population studies can be tackled by procedures which were developed already before 1950, there are, nevertheless, opportunities for improvements, as shown very recently. [REFERENCES: Dalenius (1957); Durbin (1958); Hartley (1959) and (1961).]

5. *Special parameter models:* Especially worth mentioning are models for the estimation of a cost of living index I. [REFERENCES: Banerjee (1959); Des Raj (1956–57); McCarthy (1961).]

6. *Multinomial table models:* The development since 1950 has been concerned with two related problems:

a. The choice of an overall measure of precision of a set of sample estimates of the *RC* cell frequencies; and

b. The choice of estimation procedures to be used to insure that marginal totals of a table with estimated cell frequencies agree with known marginal frequencies. [REFERENCES: El-Badry and Stephan (1955); Lahiri and Ganguli (1951); Poti (1955); Thionet (1959b).]

7. *Other models:* Among models not included in the previous discussion, mention will be made only of models for "matching lists by samples": there are k lists of some kind of units, for example k lists of names; the unknown proportion of units appearing on all k lists is to be estimated by means of a sample survey. [REFERENCES: Deming and Glasser (1959); Goodman (1952).]

C. Development of resource oriented models. Some of the sample designs developed in the 1940's—an outstanding example is found in Hansen and Hurwitz (1943)—provide instructive illustrations of successful efforts to achieve gains in efficiency by incorporating into the design one or more of the specific administrative and organizational features of the framework within which the survey was to be carried out. Instructive illustrations of applications made of these developments are found in Mahalanobis (1944), U. S. Bureau of the Census (1947), and also in the previously mentioned textbooks. This development of resource oriented models has continued and been strengthened. For natural reasons—the progress reflects the variations there are between different agencies and different resources for carrying out sample surveys—it is difficult to attempt to present a systematic review. The following examples illustrate the nature of this development:

1. Development of designs which make an efficient use of an existing field-organization.

2. Development of techniques for an efficient revision of a "master sample" of primary sampling units.

3. Development of techniques which make an efficient use of the repetitive nature of a survey.

4. Development of new estimation procedures which exploit the specific capacity of electronic computers.

5. Development of techniques for combined use of "list" and "area" sampling.

[REFERENCES: Daly, Hansen and McPherson (1961); Hansen, Hurwitz, Nisselson and Steinberg (1955); Jabine, Hurley and Hurwitz (1962); Keyfitz (1951); Kish and Hess (1959a); Woodruff (1959).]

D. Simplifying techniques. The highly increased complexity of the sample designs developed in the 1930's and the 1940's has served as an impetus toward the development of techniques simplifying their application. An outstanding early example is provided by the introduction of the ultimate cluster concept in estimating variances for multistage samples, Hansen and Hurwitz (1953), Vol. I. Considerable progress in the development of simplifying techniques has been made since 1950. Thus, techniques have been developed for the selection of a multi-stage sample in a way which takes full advantage of the potentiality of the ultimate cluster concept mentioned above. Moreover, new simple devices for use in variance estimation have been constructed. [REFERENCES: Deming (1956) and (1960); Keyfitz (1957a) and (1957b); Kish and Hess (1959b) and (1959c).]

E. The development since 1950—Selected illustrations. In this paragraph, some of the advances mentioned in paragraph A will be presented in detail. The choice of illustrations has been made subjectively; it does *not* represent any kind of evaluation.

1. *Basic models.*

a. *Methods for determining stratum boundaries.* The theory of stratified sampling was, before 1950, largely concerned with the allocation problem. The starting point of the theory was a population, divided into L strata; the problem of the theory was to determine the number of sampling units to select from each stratum.

Since 1950, considerable work has been done on the problem of *how* to stratify a given population into a specified number L of strata; in addition, some work has been done on the problem of determining the optimum number L of strata.

Using the device of representing the population by a density f, it was shown in Dalenius (1950) that the best points x_H, $H = 1 \cdots L - 1$, of stratification are given by

$$\frac{\sigma_H^2 + (x_H - \mu_H)^2}{\sigma_H} = \frac{\sigma_{H+1}^2 + (x_H - \mu_{H+1})^2}{\sigma_{H+1}}$$

when using minimum variance allocation to estimate the mean. In Dalenius (1957), a review is given of the work using the above mentioned device. The equations given above are troublesome to solve. A number of approximate methods of determining stratum boundaries have, however, been developed; a partial review is given in Dalenius and Hodges (1959). Here, two such approximate methods will be presented.

In Dalenius and Hodges (1957), the following approach is used. Consider the new variable y defined by

$$y = G(x) = \int_{-\infty}^{x} [f(u)]^{\frac{1}{2}}\, du$$

The quantity $K = G(\infty)$ is finite. Then the points x_H^* defined by

$$G(x_H^*) = \frac{H}{L} K$$

are used as (first) approximations to the best points x_H. In Dalenius and Hodges (1959), the use of this approach is illustrated.

In Ekman (1959), it is proposed to use the points x_H satisfying

$$W_H(x_H - x_{H-1}) = \text{const.}$$

where W_H is the relative size of the Hth stratum.

In Cochran (1961), a comparison is made of the performance of four simple approximations, including the ones given above, on actual populations having skew distributions with a long positive tail: experience shows that stratified sampling represents an efficient design for estimating the mean or total of such populations. The comparison showed that the two approximations presented here performed "consistently well".

b. *Sampling $n > 1$ units with unequal probabilities.* Consider a population of N units with their associated probabilities of selection:

$$X_1 \cdots X_I \cdots X_N \qquad P_1 \cdots P_I \cdots P_N$$

where, of course, $\sum P_I = 1$. Recent contributions representing each one of the three approaches mentioned in paragraph B, will be presented here.

(1) The first approach is characterized as "sampling with replacement". This selection procedure results in a set of n observations $X_1 \cdots X_i \cdots X_n$ containing $n_d \leqq n$ *distinct* units. It is shown in Basu (1958) that for the estimation of the total X, the estimator

$$x_d = \frac{1}{n_d} \sum_{i=1}^{n_d} \frac{X_i}{P_i}$$

based on the distinct units, has on the average a smaller variance than the corresponding estimator based on the n observations.

(2) The second approach is characterized as "sampling without replacement". For simplicity, consider the case $n = 2$. Denote the outcome of the first draw by $X_{1=I}$. For the population of the remaining $N - 1$ units, new probabilities of selection P'_K are computed as follows:

$$P'_K = \frac{P_K}{1 - P_{1=I}} \qquad K \neq I.$$

Using these probabilities of selection, a second unit $X_{2=J}$ is selected.

It is now possible to construct several estimators of the total X. Des Raj (1956a) proposes

$$t = \tfrac{1}{2}(t_1 + t_2)$$

where

$$t_1 = \frac{X_{1=I}}{P_{1=I}}$$

and

$$t_2 = X_{2=J} \Big/ \frac{P_{2=J}}{1 - P_{1=I}} + X_{1=I}$$

the first term of t_2 being an estimator of $X - X_{1=I}$, the total remaining after the selection of the first unit. Obviously,

$$t = \frac{1}{2}\left[(1 + P_{1=I})\frac{X_{1=I}}{P_{1=I}} + (1 - P_{1=J})\frac{X_{2=J}}{P_{2=J}}\right].$$

One property worth mentioning is that the variance estimator cannot assume negative values; this is in contradistinction to some other estimators proposed.

The Des Raj estimator presented above is an example of an *ordered* estimator: it reflects the fact that the two units X_I and X_J making up the sample are selected in this specific order. To emphasize this fact, the estimator given above may be written $t(I, J)$. If the outcome had been $X_{1=J}$, $X_{2=I}$, the estimator is

$$t(J, I) = \frac{1}{2}\left[(1 + P_{1=J})\frac{X_{1=J}}{P_{1=J}} + (1 - P_{1=I})\frac{X_{2=I}}{P_{2=I}}\right]$$

It can be shown that for every ordered estimator, there exists a corresponding *unordered* estimator (that is, an estimator which does not take into account the order into which the sampling units are selected) which is more efficient. Murthy (1957), generalizing previous results in Basu (1954), gives such an unordered estimator:

$$t_U = \frac{P(I, J)t(I, J) + P(J, I)t(J, I)}{P(I, J) + P(J, I)}$$

and also gives an unbiased variance estimator which is always nonnegative.

(3) The third approach is as follows: The first unit is selected using the initial probabilities of selection $P_1 \cdots P_I \cdots P_N$ given above. The outcome is denoted by $X_{1=J}$. Thereafter, from the population of the remaining $N - 1$ units, a second unit is selected, with equal probability of selection:

$$P_{2=K} = 1/(N - 1).$$

It is easy to show that by this procedure, described in Hájek (1949) and, independently, in Midzuno (1952), the probability of getting X_J *and* X_K in the sample, irrespective of the order of drawing, is proportionate to $P_J + P_K$. In Lahiri (1951), another selection procedure is presented having the same property. As shown in Dalenius (1961), Lahiri's scheme may be used for the selection of n units with probability of selection proportional to the sum of the sizes of the n

units, subject to a "latin square restriction" of the type considered by Bryant, Hartley and Jessen (1960).

c. *Unbiased ratio estimation.* Let

$$X_1 \cdots X_j \cdots X_n \qquad Y_1 \cdots Y_j \cdots Y_n$$

be a simple random sample from a population of N elements, with unknown mean $\bar{X}$ to be estimated and known mean $\bar{Y}$. The following ratio estimators $\bar{x}'$ and $\bar{x}''$ may be considered:

$$\bar{x}' = (\bar{x}/\bar{y})\bar{Y} = r\bar{Y}$$

and

$$\bar{x}'' = \left(\frac{1}{n}\sum_j \frac{X_j}{Y_j}\right)\bar{Y} = \bar{r}\bar{Y}$$

where $\bar{x}$ and $\bar{y}$ are the usual, unbiased mean-per-element estimators. As pointed out in paragraph B, $\bar{x}'$ is usually biased, although the bias is small relative to the variance for large n. The same holds true of $\bar{x}''$. In recent years, however, procedures for unbiased ratio estimation have been developed. Two different approaches have been used:

(1) The first approach, illustrated here by Hartley and Ross (1954), calls for estimation of a "correction term" to be added to the estimator $\bar{x}''$ of $\bar{X}$:

$$\bar{x}''' = \bar{r}\bar{Y} + \frac{n}{n-1}\,\frac{N-1}{N}\,(\bar{x} - \bar{r}\bar{y}).$$

Other unbiased ratio estimators have been proposed by Goodman and Hartley (1958), Mickey (1959), Robson (1957), and others.

(2) The second approach is to select the sample with probability proportionate to the denominator in the ratio

$$r = \sum X_j / \sum Y_j$$

as proposed by Hájek (1949), Lahiri (1951) and Midzuno (1952) and discussed above.

d. *Estimating several parameters in stratified sampling.* As an illustration of the design conflict mentioned in paragraph B which may result from efforts to apply uniparametric theory to a multiparametric problem, we will consider a case involving stratified sampling. Consider a population of N elements, divided into L strata, with $N_H = W_H N$ elements in the Hth stratum. The means $\bar{X} = \sum_H W_H \bar{X}_H$ and $\bar{Y} = \sum_H W_H \bar{Y}_H$ are to be estimated, by $\bar{x}$ and $\bar{y}$ respectively. It is well known that the size $n_H(X)$ of the sample to be selected from the Hth stratum in order to make the total sample size a minimum, subject to the condition that $\text{Var}\,\bar{x} = \text{Var}_0\,\bar{x}$, will in general not be equal to the size $n_H(Y)$ of the sample to be selected from the Hth stratum in order to make the total sample size a minimum, subject to the condition that $\text{Var}\,\bar{y} = \text{Var}_0\,\bar{y}$. In Dalenius (1957), a review is given of some approaches to the problem of determining the best overall allocation n_H in the multiparametric case. One such approach due to the author will be presented here. This approach is simply that of expressing the design criterion in terms of inequalities.

The total sample size $n = \sum_H n_H$ is minimized subject to a set of inequalities:

$$\text{Var}\,\bar{x} \leqq \text{Var}_0\,\bar{x}, \qquad \text{Var}\,\bar{y} \leqq \text{Var}_0\,\bar{y}.$$

This "programming" approach can be given an illuminating geometrical interpretation. Consider first the uniparametric case, with $L = 2$ strata. The problem to be considered is the best choice of n_H, subject to the condition that $\text{Var}\,\bar{x} = \text{Var}_0\,\bar{x}$. The relation

$$\text{Var}_0\,\bar{x} = W_1^2 \frac{\sigma_1^2(X)}{n_1} + W_2^2 \frac{\sigma_2^2(X)}{n_2}$$

may be represented by a conic in the usual plane (see Figure 1). In the same plane, $n = n_1 + n_2$, or $n_1 = n - n_2$ is a family of straight lines ("budget lines"). One of these lines is a tangent to the conic; the point of tangency corresponds to the point $n_1(X)$, $n_2(X)$ as defined above.

Now consider the multiparametric case, the problem being to determine the best choice of n_H in order to estimate $\bar{X}$ and $\bar{Y}$, subject to the condition that $\text{Var}\,\bar{x} \leqq \text{Var}_0\,\bar{x}$ and $\text{Var}\,\bar{y} \leqq \text{Var}_0\,\bar{y}$ respectively. In Figure 2, $\text{Var}_0\,\bar{x}$ and $\text{Var}_0\,\bar{y}$ are represented by two conics. Clearly, the sample size n_H satisfying the condition given must be looked for in the "hatched" area. More specifically the best n_H is determined by the point where the line $n_1 = n - n_2$ is a "tangent" to the "hatched" area.

2. *Resource oriented models.*

a. *The "surprise-stratum technique"*. Many populations studied by sample surveys are markedly skew: a few large units contribute a substantial proportion of the total X. In order to estimate X, it is often efficient to stratify the popu-

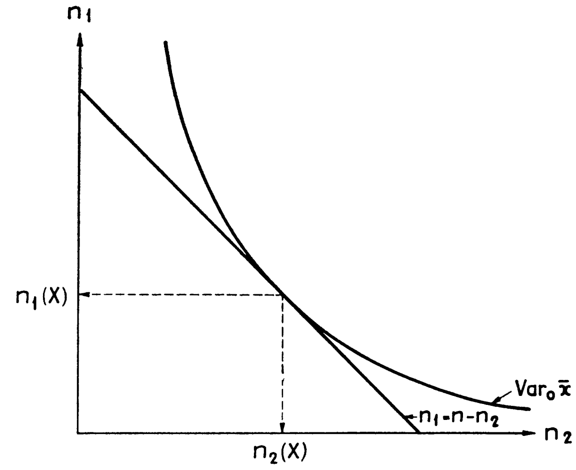

FIG. 1

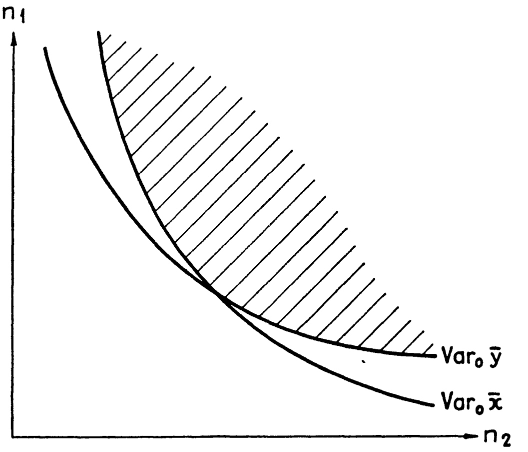

FIG. 2

lation by some measure of size and select relatively more large units than small units. In any given stratum of any sample design, there are likely to be a few very large sampling units which account for a large proportion of the variance from the stratum. This can happen in spite of careful attempts at stratification. These sampling units will be referred to as "surprise units" (they have also been referred to as "large observations", "rare events" and "outliers" known as members of the population). In a one-time survey, such "surprise units" may be dealt with in two ways, neither one of which may be attractive:

(1) The "surprise units" are kept in the sample; this results in a large variance; or

(2) The "surprise units" are rejected; this results in a bias.

In a repetitive survey, it is possible to deal with this problem in a much more satisfactory way provided the sample is rotated from period to period to some degree. The method is referred to here as the "surprise-stratum technique". Woodruff (1959) furnished an illustration of this technique.

The "Retail Trade Survey" carried out by the U. S. Bureau of the Census is a monthly sample survey designed to estimate total receipts. In this survey a large proportion of the universe is represented by an area sample which is rotated from one month to the next. The "surprise-stratum technique" is applied as follows. In month m, those units are identified which are (according to some definition of "large") "surprise units" in *all* previous $m - 1$ months, whether in the current sample or not; all these units (but not those units which are "surprise units" in only some of the previous $m - 1$ months) are surveyed in the mth month; they constitute a sample from the "surprise stratum". The weight of these units (and all like them observed in the current panel) is then divided by m. This drastically reduces their effect on the estimate and the variance of the estimate.

As indicated in Woodruff (1959) and Kish and Hess (1959a) and (1959b), the "surprise-stratum technique" has a wide range of applicability. It may for example be used in connection with area sampling to cope with the problem that some "small" segments selected for canvass in a survey may in fact be very large, due to growth, new construction, etc. It may, of course, also be used for rotating samples which are not area samples.

b. *The impact of computers*. The use of electronic computers for automatic data processing has meant a revolution of the administration and design of large scale statistical surveys. An illustration of the impact on the estimation procedure of a sample survey will be discussed here. The illustration refers to the "Current Population Survey" program carried out by the U. S. Bureau of the Census. The design of this survey is discussed in Hansen, Hurwitz, Nisselson and Steinberg (1955). For the purpose of illustration, it is sufficient to point to two characteristics of the survey design:

(1) The use of multistage sampling for the selection of the sample households;

and

(2) The use of a panel technique: approximately 75% of the households are common from month to month.

Before 1955, a total, for example the number X_m of persons employed in month m, was estimated by a "two-stage ratio estimator":

$$x_m = \sum_a \frac{x_a}{y_a} Y_a$$

the summation being over age-sex-veteran status groups; x_a and y_a are themselves ratio estimators. The computations of the estimates were largely carried out by means of punch card equipment, partly by a process of substitution or elimination of units in the sample. Because of the large amount of work involved, variance estimates were computed on a rather restricted scale.

Since 1955, the computations of the estimates have been carried out by means of Bureau of the Census' Univac system. This has made it possible to make use of a considerably more sophisticated estimator. Thus the substitution and elimination procedure mentioned above has been replaced by the computation of appropriate weights, and use is made of a "composite" estimator of the following type:

$$x_m'' = (1 - K)(x_{m-1}'' + x_{m;\,m-1} - x_{m-1;\,m}) + Kx_m$$

where

x_m'' is the composite estimator of X_m ;

x_{m-1}'' is the composite estimator of X_{m-1} ;

$x_{m;\,m-1}$ is the ratio estimator of X_m based on those households which are in the sample month $m - 1$ and m;

$x_{m-1;\,m}$ is the corresponding ratio estimator of X_{m-1} ; and finally,

K is a weight fulfilling the condition $0 \leqq K \leqq 1$.

The use of this estimator reduces considerably the contributions to the total variance from variation *within* primary sampling units. In spite of the fact that as a consequence of the formula used and also of an increase in the sample size the volume of computations involved is much larger than before, the change has meant a reduction of the processing costs by approximately 50%.

IV. RECENT ADVANCES—MIXED ERROR MODELS

A. Main lines of development. "Non-sampling errors", primarily in the nature of biases, were a source of an increasing concern in the 1930's and in the 1940's. This concern was especially pronounced in two major fields of survey applications: socio-economic surveys and crop-estimation surveys. Palmer (1943), Deming (1944), and Hilgard and Payne (1944) presented illustrations referring to the first-mentioned field, while Mahalanobis (1946) and Sukhatme (1947) contributed illustrations referring to the last-mentioned field.

In the 1940's, two lines of development emerged in the work dealing with non-sampling errors:

1. Development of theory and methods for coping with specific sources of non-sampling errors.

2. Development of a comprehensive theory of non-sampling errors, or in the present terminology, of mixed error models.

B. Development of theory and methods for coping with specific sources of non-sampling errors. Many powerful techniques for coping with specific sources of non-sampling errors were developed in the 1930's and, especially, in the 1940's. Some of these techniques have recently been improved upon.

1. The risk of bias associated with the use of "plots" as sampling units in crop-cutting experiments was thoroughly studied in the 1940's. It was found in some cases that crop-cutters may have a tendency of consistently including borderline plants in the plot, thus introducing an upward bias in the estimate of the yield per unit area. Mahalanobis (1946) and Sukhatme (1947) are two important references; a recent contribution is Masuyama (1954).

2. The opposite type of bias, that is excluding from observation units which do belong to sampling units selected for a survey, has been observed when using segments as sampling units; Manheimer and Hyman (1949) gives an illustration. The combined use of area and list sampling, mentioned in paragraph IIIC, can be applied to cope with this source of error, as demonstrated in Jabine, Hurley and Hurwitz (1962).

3. New ways of tackling the non-response problems were given in Hansen and Hurwitz (1946) and Politz and Simmons (1949). Recent advances are illustrated by Birnbaum and Sirken (1950), Dalenius (1957), Deming (1953), Kish and Hess (1959a) and Simmons (1954).

4. The technique of "interpenetrating samples" was developed in India to cope with the errors introduced by the field-workers; the use of this technique, also referred to as "replication", is described in Mahalanobis (1944) and (1946). Lahiri (1958) illustrates recent applications in India. As will be shown in a forth-

coming publication by the U. S. Bureau of the Census, the technique of interpenetrating samples has been adapted for use in the "evaluation studies" in the 1960 population and housing censuses in the United States.[4]

5. The theory of double sampling was originally developed, in Neyman (1938) and later papers, as a sampling variance model in the sense of paragraph IIIB. In the 1950's, use has been made of this device as an efficient means of coping with certain types of response errors; Robson and King (1952) provides an illustration.

6. A recent, especially note-worthy development is the application of large scale electronic computers to editing of forms. As demonstrated by the U. S. Bureau of the Census, this has meant a reduction of the frequency and size of "editing" errors; it certainly means drastic improvement of the control over editing processes.

C. The development of mixed error models. In the early 1950's, great efforts were devoted to the construction of models which could serve as a basis for measuring and regulating the *overall* error of a survey. Hansen, Hurwitz, Marks and Mauldin (1951), Stock and Hochstim (1951) and Sukhatme (1952) are three important contributions; many of these early results are summarized in Hyman (1954).

A most important result from the early 1950's is a better understanding of the nature of the "interviewer effect." It is often true that each interviewer introduces a systematic error of his own into his observations; that means that there is a positive correlation between the errors introduced by a single interviewer. The presence of such a correlation means that the workloads of the interviewers exercise an influence on the overall error which is analogous to the influence on the sampling error from using homogeneous clusters as sampling units.

The mathematical models developed in the early 1950's to cope with the response errors either did not take this correlation into account or did it in a way which was not entirely satisfactory. The recent paper by Hansen, Hurwitz and Bershad (1961) represents, however, a major step forward towards the construction of statistically satisfactory mixed error models. Characteristic of this paper is that all relevant concepts have been rigorously defined in terms of sampling theory.

D. Evaluation surveys. In this chapter, mention should be made of the use of "evaluation surveys" in connection with censuses and surveys. Excellent illustrations are provided by Eckler and Pritzker (1951), Eckler and Hurwitz (1958) and U. S. Bureau of the Census (1960). In the 1950's and the early 1940's the primary purpose of evaluation surveys was to provide an appraisal of the ac-

[4] It is interesting to note that using interpenetrating samples as part of a sample survey design represents the use of a *combined* (mixed) type of survey design. The usefulness of such designs is by no means limited to "evaluation surveys", as exemplified by Brunk and Federer (1953); they seem to have a considerably greater potentiality than has as yet been appreciated.

curacy of the census or the survey. Since then, a noteworthy change as to the objective of such studies can be observed. Thus, while such studies are still considered important tools for appraising the accuracy of censuses and surveys, a second and equally important objective has emphasized the use of such studies for the improvement of the *efficiency* of censuses and surveys. The development of mean square error models is clearly instrumental to such uses.

V. FINAL CHAPTER

A. Comments on the scope of this review. This review has been restricted to that type of a random experiment which is the main subject of such textbooks as Cochran (1953), as stated in paragraph IB. Among the consequences of this restriction, the following ones deserve special mentioning.

1. The restriction to *random* experiments means that, in general, the entire domain of non-random experiments has been excluded from the review as given in Chapters II–IV. Among recent advances thus not reported mention will be made here of the increased understanding of the "performance" of such a method as quota sampling, as analysed in Moser and Stuart (1953) and Stephan and McCarthy (1958).

2. The further restriction to the *type* of random experiments, which is the subject of the textbooks referred to in paragraph IB, means that the specific development, which has taken place in such areas as statistical quality control, acceptance sampling, Monte Carlo sampling and so forth has been excluded from the review. Terminological variations tend to over-emphasize the differences between different fields of application with respect to sampling procedures used; for example, "sampling with probabilities proportional to measures of size" as discussed in Hansen and Hurwitz (1943) is known as "importance sampling" in textbooks on Monte Carlo sampling. It may, nonetheless, be a worthwhile project to study the procedures used in specific fields of application from the point of view of their wider applicability.

3. A third consequence of the restriction imposed upon this review is that no mention is made of the Bayesian approach to sampling that is being advocated in Raiffa and Schlaifer (1961).

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