Towards the C^0 flux conjecture

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In this note, we generalise a result of Lalonde, McDuff and Polterovich concerning the C^0 flux conjecture, thus confirming the conjecture in new cases of symplectic manifolds. We also prove the continuity of the flux homomorphism on the space of smooth symplectic isotopies endowed with the C^0 topology, which implies the C^0 rigidity of Hamiltonian paths, conjectured by Seyfaddini.

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1 Introduction and main results

Let (M, ω) be a closed and connected symplectic manifold. The celebrated Eliashberg–Gromov rigidity theorem (see Eliashberg [4; 5] and Gromov [10]) states that the group $\mathrm{Symp}(M, \omega)$ of symplectomorphisms of M is C^0–closed inside the group $\mathrm{Diff}(M)$ of diffeomorphisms of M. A related natural conjecture (called the C^0 flux conjecture) states the following: The group $\mathrm{Ham}(M, \omega)$ of Hamiltonian diffeomorphisms of M is C^0–closed inside $\mathrm{Symp}_0(M, \omega)$, the connected component of identity in $\mathrm{Symp}(M, \omega)$.

The reader may wonder why it is asked if $\mathrm{Ham}(M, \omega)$ is C^0–closed in $\mathrm{Symp}_0(M, \omega)$ rather than $\mathrm{Symp}(M, \omega)$. The difficulty in addressing the latter question is that, although the Eliashberg–Gromov rigidity theorem tells us that $\mathrm{Symp}(M, \omega)$ is C^0–closed in the group $\mathrm{Diff}(M)$ of diffeomorphisms of M, it is not known if $\mathrm{Symp}_0(M, \omega)$ is C^0–closed in $\mathrm{Symp}(M, \omega)$. To avoid this difficulty the C^0 flux conjecture is usually stated for $\mathrm{Symp}_0(M, \omega)$.

A weak form of the C^0 flux conjecture is the C^1 flux conjecture, which states that $\mathrm{Ham}(M, \omega)$ is C^1–closed in $\mathrm{Symp}_0(M, \omega)$. This statement is equivalent to the flux group $\Gamma \subset H^1(M, \mathbb{R})$ being discrete. Some cases of the C^1 flux conjecture were proven by Banyaga [1], Lalonde, McDuff and Polterovich [12] and McDuff [14]; it was finally confirmed in full generality by Ono [17]. However, the C^0 flux conjecture still remains open in case of a general symplectic manifold. It has been confirmed

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The author would like to point out he also uses the spelling "Buhovski" for his family name.

by Lalonde, McDuff and Polterovich in certain cases [12] (these cases are described below). Also, Humilière and Vichery established more cases of the C^0 flux conjecture (private communication).

A different weak form of the C^0 flux conjecture (the "C^0 rigidity of Hamiltonian paths") was proposed by Seyfaddini [21]: Is it true that on any closed and connected symplectic manifold, the space of smooth Hamiltonian isotopies is C^0–closed in the space of smooth symplectic isotopies? In [21], Seyfaddini showed that a smooth symplectic isotopy which is a C^0 limit of a sequence of smooth Hamiltonian isotopies is itself Hamiltonian, provided that the corresponding sequence of generating Hamiltonians is a Cauchy sequence in the $L^{(1,\infty)}$ topology.

The results of this note are concerned with the C^0 flux conjecture, and with the mentioned conjecture of Seyfaddini (the C^0 rigidity of Hamiltonian paths).

1.1 The C^0 flux conjecture

We denote $H = \mathrm{Ham}(M, \omega) \subseteq G = \mathrm{Symp}_0(M, \omega)$, by $\widetilde{G} = \widetilde{\mathrm{Symp}}_0(M, \omega)$ we denote the universal cover of $G = \mathrm{Symp}_0(M, \omega)$, and by $\widetilde{H} \subseteq \widetilde{G}$ we denote the lift of H to $\widetilde{G}$ (ie the set of those elements of $\widetilde{G}$ whose endpoint belongs to H). Next, by $H_0 \subseteq G$ we denote the C^0 closure of H inside G, and by $\widetilde{H}_0 \subseteq \widetilde{G}$ we denote the lift of H_0 to $\widetilde{G}$. Also, we use the notation $\mathrm{Map}_0(M)$ for the connected component of the identity in the space of all smooth maps $M \to M$.

Let us recall that the flux homomorphism $\mathrm{Flux} \colon \widetilde{G} \to H^1(M, \mathbb{R})$ can be described as follows: given $\widetilde{\phi} \in \widetilde{G}$ its flux is the class in $H^1(M, \mathbb{R})$ which assigns to a smooth loop γ in M the integral of ω over the cylinder $C_\gamma \colon S^1 \times [0, 1] \to M$ defined by $C_\gamma(s, t) = \phi^t(\gamma(s))$, where $\phi^t, t \in [0, 1]$, is a smooth symplectic isotopy representing $\widetilde{\phi}$. This definition is independent of the choice of the isotopy $\{\phi^t\}$.

Denote by $\Gamma \subset H^1(M, \mathbb{R})$ the flux group, ie the image of $\widetilde{H}$ (or, equivalently, of $\pi_1(G)$) under the flux homomorphism, and by $\Gamma_0 \subset H^1(M, \mathbb{R})$ the image of $\widetilde{H}_0$ under the flux homomorphism. It is not hard to see that the C^0 flux conjecture is equivalent to the equality $\Gamma_0 = \Gamma$ (this follows from the well-known fact that for a smooth path $\phi^t, t \in [0, 1]$, of symplectic diffeomorphisms, its endpoint ϕ^1 belongs to H if and only if its flux belongs to Γ). The restriction of the flux homomorphism to $\pi_1(G)$ admits a natural extension to a homomorphism (which we again call flux homomorphism) from $\pi_1(\mathrm{Map}_0(M))$ to $H^1(M, \mathbb{R})$: for any $\mathfrak{a} \in \pi_1(\mathrm{Map}_0(M))$, we consider a (piecewise) smooth loop $f^t, t \in [0, 1]$, in $\mathrm{Map}_0(M)$ representing $\mathfrak{a}$, and then we define the flux of $\mathfrak{a}$ to be the class in $H^1(M, \mathbb{R})$ which assigns to a smooth loop γ in M the integral of ω over the cylinder $C_\gamma \colon S^1 \times [0, 1] \to M$ defined by $C_\gamma(s, t) = f^t(\gamma(s))$. This definition

is independent of the choice of the loop $\{f^t\}$. Following [12], we denote by Γ_{top} the image of $\pi_1(\text{Map}_0(M))$ under the flux homomorphism. Consider the evaluation homomorphism ev: $\pi_1(\text{Map}_0(M)) \to \pi_1(M)$. For any $a \in \pi_1(M)$ we denote by $\Gamma_{\text{top}}^a \subseteq \Gamma$ the image of $\text{ev}^{-1}(a) \subseteq \pi_1(\text{Map}_0(M))$ under the flux homomorphism.

The following result was proved in [12]:

Theorem 1.1 *If M is Lefschetz, then $\Gamma_0 \subseteq \Gamma_{\text{top}}$.*

As a consequence, Lalonde, McDuff and Polterovich conclude:

Corollary 1.2 *Assume that M is Lefschetz and that $\Gamma_{\text{top}} = \Gamma$. Then the C^0 flux conjecture holds for M.*

As an example, one can take M to be a closed and connected Kähler manifold of nonpositive curvature such that its fundamental group has a trivial centre. As another example, one can take the $2n$–dimensional torus with a translation-invariant symplectic structure. See [12] for more details.

Now we turn to our results. Our main result is as follows:

Theorem 1.3 *Let (M, ω) be a closed and connected symplectic manifold. Then $\Gamma_0 \subseteq \Gamma_{\text{top}} + \overline{\Gamma_{\text{top}}^e}$, where $e \in \pi_1(M)$ is the identity, and $\overline{\Gamma_{\text{top}}^e} \subseteq H^1(M, \mathbb{R})$ is the closure of Γ_{top}^e inside $H^1(M, \mathbb{R})$.*

As a result, we obtain the following corollary:

Corollary 1.4 *Let (M, ω) be a closed and connected symplectic manifold such that $\Gamma_{\text{top}} = \Gamma$. Then the C^0 flux conjecture holds for M.*

Indeed, if $\Gamma_{\text{top}} = \Gamma$, then since Γ is closed (Γ being closed is exactly the statement of the C^1 flux conjecture proven in [17]), it follows that $\Gamma_{\text{top}} + \overline{\Gamma_{\text{top}}^e} = \Gamma$ and hence $\Gamma_0 = \Gamma$ by Theorem 1.3.

In particular, the C^0 flux conjecture holds for a closed and connected symplectically aspherical symplectic manifold (M, ω) whose fundamental group $\pi_1(M)$ has a trivial centre. Indeed, if the centre of $\pi_1(M)$ is trivial, we get $\text{ev}(\pi_1(\text{Map}_0(M))) = \{e\}$ and so $\Gamma_{\text{top}} = \Gamma_{\text{top}}^e$, and moreover, since M is symplectically aspherical, we conclude that $\Gamma_{\text{top}}^e = \{0\}$. Therefore $\Gamma_{\text{top}} = \{0\}$, hence $\Gamma_{\text{top}} = \Gamma = \{0\}$.

As another example, we get that the C^0 flux conjecture holds for any product $(M, \omega) = (\mathbb{T}^{2k} \times N, \sigma \oplus \tau)$, where $(\mathbb{T}^{2k}, \sigma)$ is a symplectic torus with a translation invariant

σ, and (N, τ) is a closed and connected symplectically atoroidal symplectic manifold. Indeed, since $\mathbb{T}^{2k}$ is symplectically aspherical, and N is symplectically atoroidal, it follows that for any $a = (b, c) \in \pi_1(M) \cong \pi_1(\mathbb{T}^{2k}) \times \pi_1(N)$ and any $\mathfrak{a} \in \pi_1(\mathrm{Map}_0(M))$ with $\mathrm{ev}(\mathfrak{a}) = a$, the flux of $\mathfrak{a}$ is uniquely determined by b. Moreover, since translations of the torus $\mathbb{T}^{2k}$ generate a large enough subgroup of symplectomorphisms of $(\mathbb{T}^{2k}, \sigma)$, for any $b \in \pi_1(\mathbb{T}^{2k})$ we can find an element $\mathfrak{b} \in \pi_1(G) = \pi_1(\mathrm{Symp}_0(M, \omega))$ such that $\mathrm{ev}(\mathfrak{b}) = (b, 0)$. Therefore we conclude $\Gamma_{\mathrm{top}} = \Gamma$.

Remark 1.5 The reader may ask if there exist examples where we have $\Gamma_{\mathrm{top}} \neq \Gamma$. The following construction is due to Seidel [19; 20]. Let (N, ω_N) be a closed and connected symplectic manifold with $H^1(N, \mathbb{R}) = 0$, let $\psi \colon N \to N$ be a symplectic diffeomorphism which is smoothly isotopic to the identity, but which is not isotopic to the identity via a smooth path of symplectic diffeomorphisms. Look at the symplectic mapping torus $E = E_\psi$ of ψ, which is the total space of the fibration over the two-torus with fibre N and monodromy ψ in one direction, or explicitly,

$$E = \mathbb{R}^2 \times N / \big((p, q, x) \sim (p - 1, q, x) \sim (p, q - 1, \psi(x)) \big),$$
$$\omega_E = dp \wedge dq + \omega_N.$$

Because ψ is smoothly isotopic to the identity, the fibration $E \to \mathbb{T}^2$ is trivial as a smooth one, and it is easy to see that for E we have $\Gamma_{\mathrm{top}} = H^1(E, \mathbb{Z})$. However, it is possible that for E we have $\Gamma \neq H^1(E, \mathbb{Z})$. The closed 1–form dp on E generates the symplectic vector field $\frac{\partial}{\partial q}$, whose time-1 map is $\phi(p, q, x) = (p, q + 1, x) = (p, q, \psi(x))$. If ϕ turns out to be a non-Hamiltonian diffeomorphism, then we get that $[dp] \notin \Gamma$, so in particular $\Gamma \neq \Gamma_{\mathrm{top}}$. One way of detecting this is by looking at the Floer cohomology $HF^*(\psi)$. That is, if we are in a situation when $HF^*(\psi)$ has total rank different from that of $H^*(N)$, then $HF^*(\phi) \cong H^*(\mathbb{T}^2) \otimes HF^*(\psi)$ is not isomorphic to $H^*(E)$, and hence in particular ϕ is a non-Hamiltonian diffeomorphism.

1.2 C^0 rigidity of Hamiltonian paths

Theorem 1.6 *Let (M, ω) be a closed and connected symplectic manifold. Fix a Riemannian metric g on M, which induces a distance function $d \colon M \times M \to \mathbb{R}$, which in turn, induces a distance d between maps $M \to M$: for any $f, h \colon M \to M$ we set $d(f, h) = \sup_{x \in M} d(f(x), h(x))$. Fix a norm $|\cdot|$ on $H^1(M, \mathbb{R})$. Then there exist constants $c = c(M, \omega, g)$, $C = C(M, \omega, g, |\cdot|)$, such that for any smooth path ϕ^t, $t \in [0, 1]$ of symplectomorphisms of M, $\phi^0 = \mathrm{id}_M$, $\phi^1 = \phi$, with $\max_{t \in [0,1]} d(\mathrm{id}_M, \phi^t) < c$, we have $|\mathrm{Flux}(\{\phi^t\})| \leqslant C d(\mathrm{id}_M, \phi)$.*

[Theorem 1.6](#) has a direct corollary:

Corollary 1.7 (1) *On any closed and connected symplectic manifold, the flux homomorphism is continuous with respect to the C^0 distance between smooth paths of symplectomorphisms.*

(2) *C^0 rigidity of Hamiltonian paths: On any closed and connected symplectic manifold, the space of smooth Hamiltonian isotopies of M is C^0–closed in the space of smooth symplectic isotopies of M. This confirms the above mentioned conjecture of Seyfaddini.*

Let us remark that there is another weak version of the C^0 flux conjecture, which is due to Seyfaddini, and is concerned with topological (continuous) Hamiltonian dynamics initially introduced by Oh and Müller [15]: Is it true that any Hamiltonian homeomorphism (in the sense of [15]) that belongs to $\mathrm{Symp}_0(M, \omega)$ is in fact a Hamiltonian diffeomorphism?

Notations Let $A > 0$. We denote by $B(A) \subset \mathbb{R}^2$ the open euclidean disc centred at the origin having area A, ie

$$B(A) = \{z \in \mathbb{R}^2 \mid \pi|z|^2 < A\}.$$

We denote by

$$S(A) = \partial B(A) = \{z \in \mathbb{R}^2 \mid \pi|z|^2 = A\}$$

the euclidean circle centred at the origin enclosing a disc of area A. Also, we use the notation $B'(A) = B(A) \setminus \{0\} \subset \mathbb{R}^2$ for the punctured disc. On T^*S^1 with canonical coordinates (q, p), where $q \in \mathbb{R}/\mathbb{Z}$, $p \in \mathbb{R}$, and with the standard symplectic form $dp \wedge dq$, we use the notation $S^1 \subset T^*S^1$ for the zero section, and we denote

$$W(A) = \{(q, p) \mid |p| < A\} \subset T^*S^1,$$

so that $W(A)$ is a neighbourhood of the zero section in T^*S^1 having area $2A$.

2 Proofs

Consider the evaluation homomorphism ev: $\pi_1(\mathrm{Map}_0(M)) \to \pi_1(M)$. In the next lemma we show that its restriction to $\pi_1(G)$ can be naturally extended to a homomorphism ev′: $\widetilde{H}_0 \to \pi_1(M)$.

Lemma 2.1 *The homomorphism* $\mathrm{ev}|_{\pi_1(G)}$: $\pi_1(G) \to \pi_1(M)$ *admits a natural extension to a homomorphism* ev′: $\widetilde{H}_0 \to \pi_1(M)$.

Proof Before we turn to the proof, let us sketch an easier argument which shows that the evaluation homomorphism ev naturally extends to a homomorphism

$$\mathrm{ev}': \widetilde{H} \to \pi_1(M).$$

Let $\widetilde{\phi} \in \widetilde{H}$ and let $\phi^t, t \in [0,1]$, be a smooth path of symplectomorphisms representing $\widetilde{\phi}$, and denote $\phi = \phi^1$. Since ϕ is a Hamiltonian diffeomorphism, one can choose a smooth Hamiltonian flow $\psi^t, t \in [0,1]$ such that $\psi^1 = \phi$. This defines a loop $f^t, t \in [0,2]$, of symplectomorphisms: first we go forward with the flow ϕ^t, ie $f^t = \phi^t$ for $t \in [0,1]$, and then we go backwards with the flow of ψ^t, that is, $f^t = \psi^{2-t}$ for $t \in [1,2]$. Then $\mathrm{ev}'(\widetilde{\phi})$ is by definition the evaluation $\mathrm{ev}(\{f^t\})$. To show the independence of the choice of the flow $\psi^t, t \in [0,1]$, it is enough to show that for any two such flows ψ_1^t and ψ_2^t, the evaluation homomorphism ev vanishes on the Hamiltonian loop obtained by first going forward with the flow of ψ_1^t and then going backwards with the flow ψ_2^t. But it follows from the Arnold's conjecture [3; 6; 7; 8; 9; 11; 13; 16; 18], that on any Hamiltonian loop, the evaluation homomorphism vanishes. Our main message here is that this argument can be generalised to further extend ev′ to a homomorphism

$$\mathrm{ev}': \widetilde{H}_0 \to \pi_1(M).$$

Let us explain our construction of ev′: $\widetilde{H}_0 \to \pi_1(M)$. Fix a Riemannian metric g on M. Let $\widetilde{\phi} \in \widetilde{H}_0$ and let $\phi^t, t \in [0,1]$, be a smooth path of symplectomorphisms representing $\widetilde{\phi}$, and denote $\phi = \phi^1$. Consider a Hamiltonian diffeomorphism $\psi \in H$ such that ψ is sufficiently C^0–close to ϕ (it is possible to find such a Hamiltonian diffeomorphism, since by our assumption ϕ lies in the C^0 closure of H inside G). Let $\psi^t, t \in [0,1]$, be a smooth Hamiltonian isotopy of M, such that $\psi^1 = \psi$. Define a continuous loop $f^t, t \in [0,3]$, in $\mathrm{Map}_0(M)$, such that $f^t = \phi^t$ for $t \in [0,1]$, such that for any $x \in M$, the path $f^t(x), t \in [1,2]$ is the shortest g–geodesic connecting $\phi(x)$ and $\psi(x)$, and such that $f^t = \psi^{3-t}$ for $t \in [2,3]$. We now define $\mathrm{ev}'(\phi)$ to be the value of the evaluation map ev at the loop $f^t, t \in [0,3]$.

Let us show that the definition does not depend on the choice of ψ and of the path ψ^t, $t \in [0,1]$. Let $\psi_1^t, \psi_2^t, t \in [0,1]$, be two smooth Hamiltonian isotopies of M, such

that $\psi_1 = \psi_1^1$ and $\psi_2 = \psi_2^1$ are sufficiently C^0–close to ϕ. Define the corresponding loops f_1^t, f_2^t, $t \in [0, 3]$, as above. Define the loop h_1^t, $t \in [0, 6]$, in $\mathrm{Map}_0(M)$ by

$$h_1^t = \begin{cases} f_1^{3-t} & t \in [0, 3], \\ f_2^{t-3} & t \in [3, 6]. \end{cases}$$

It is enough to show that the value of ev at the loop h_1^t, $t \in [0, 6]$, equals $e \in \pi_1(M)$. Clearly, the loop h_1^t, $t \in [0, 6]$, is homotopic to the loop h_2^t, $t \in [0, 4]$, where $h_2^t = \psi_1^t$ for $t \in [0, 1]$, where for any $x \in M$ the path $h_2^t(x)$, $t \in [1, 2]$, is the shortest g–geodesic between $\psi_1(x)$ and $\phi(x)$, and the path $h_2^t(x)$, $t \in [2, 3]$, is the shortest g–geodesic between $\phi(x)$ and $\psi_2(x)$, and finally $h_2^t = \psi_2^{4-t}$ for $t \in [3, 4]$. Also, since ψ_1 and ψ_2 are C^0–close to ϕ, it follows that the loop h_2^t, $t \in [0, 4]$ is homotopic to the loop h_3^t, $t \in [0, 3]$, where $h_3^t = \psi_1^t$ for $t \in [0, 1]$, where for any $x \in M$ the path $h_3^t(x)$, $t \in [1, 2]$ is the shortest g–geodesic between $\psi_1(x)$ and $\psi_2(x)$, and where $h_3^t = \psi_2^{3-t}$ for $t \in [2, 3]$. It is enough to show that the value of the evaluation map ev at h_3^t, $t \in [0, 3]$, equals $e \in \pi_1(M)$. Now pick some increasing bijective smooth function $\nu: [0, 1] \to [0, 1]$, such that the derivatives of ν of all orders vanish at $1 \in [0, 1]$, and look at the smooth Hamiltonian flow h^t, $t \in [0, 2]$, defined by $h^t = \psi_1^{\nu(t)}$ for $t \in [0, 1]$ and $h^t = \psi_2^{\nu(2-t)} \circ (\psi_2)^{-1} \circ \psi_1$ for $t \in [1, 2]$. Then by the solution of the Arnold's conjecture [3; 6; 7; 8; 9; 11; 13; 16; 18],[1] the time-2 map h^2 of the Hamiltonian flow h^t, $t \in [0, 2]$, has a fixed point $p \in M$, such that its trajectory under the flow is a contractible loop. Hence we get $\psi_1(p) = \psi_2(p)$, and as a result, the loop $t \mapsto h_3^t(p)$, $t \in [0, 3]$, is contractible. Therefore the value of the evaluation map ev at the loop h_3^t, $t \in [0, 3]$, equals $e \in \pi_1(M)$.

Finally, it is easy to see the independence of ev' of the choice of metric g, and that ev' is a homomorphism. $\qquad\square$

The main step in the proof of Theorem 1.3 is the following proposition:

Proposition 2.2 *Let (M, ω) be a closed and connected symplectic manifold, and let $\widetilde{\phi} \in \widetilde{H}_0$. Then $\mathrm{Flux}(\widetilde{\phi}) \in \Gamma_{\mathrm{top}}^a$, where $a = \mathrm{ev}'(\widetilde{\phi}) \in \pi_1(M)$.*

Proof Choose a Riemannian metric g on M. Denote by $\phi \in G$ the endpoint of $\widetilde{\phi}$, and let ϕ^t, $t \in [0, 1]$, be a smooth symplectic isotopy of M representing $\widetilde{\phi}$, such that $\phi^0 = \mathrm{id}_M$, $\phi^1 = \phi$. Choose a smooth embedded loop $\gamma: [0, 1] \to M$, and define the loop $\alpha = \phi \circ \gamma$. There exists a neighbourhood U of $\alpha([0, 1])$ which is

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[1]Strictly speaking, except for the case of a semi-positive symplectic manifold, the existing proofs of the Arnold's conjecture depend on virtual cycle techniques, which are not yet accepted by all the experts.

symplectomorphic to the product of a neighbourhood of the zero section in T^*S^1 and $n-1$ small 2–dimensional discs, ie there exists $\epsilon > 0$, such that for

$$W(\epsilon) = \{(q, p) \in T^*S^1 \mid |p| < \epsilon\} \subset T^*S^1$$

(here $S^1 \cong \mathbb{R}/\mathbb{Z}$, so that the symplectic area of $W(\epsilon)$ in T^*S^1 is 2ϵ) and for the standard 2–dimensional disc $B(\epsilon) \subset \mathbb{R}^2$ of area ϵ centred at the origin, we have a symplectic embedding

$$\iota \colon W(\epsilon) \times B(\epsilon)^{\times n-1} \to M,$$

such that $S^1 \times \{0\} \times \cdots \times \{0\}$ is mapped onto $\alpha([0, 1])$, where $S^1 \subset T^*S^1$ is the zero section. We set $U = \iota(W(\epsilon) \times B(\epsilon)^{\times n-1})$.

Now let $\psi \in H$ be sufficiently C^0–close to ϕ, and let ψ^t, $t \in [0, 1]$, be a smooth Hamiltonian isotopy on M such that $\psi^1 = \psi$. Define, as in the proof of Lemma 2.1, a continuous loop f^t, $t \in [0, 3]$, in $\mathrm{Map}_0(M)$, such that $f^t = \phi^t$ for $t \in [0, 1]$, such that for any $x \in M$, the path $f^t(x)$, $t \in [1, 2]$, is the shortest g–geodesic connecting $\phi(x)$ and $\psi(x)$, and such that $f^t = \psi^{3-t}$ for $t \in [2, 3]$. Then since ψ is sufficiently C^0–close to ϕ, Lemma 2.1 tells us that the value of ev at the loop f^t, $t \in [0, 3]$, equals $\mathrm{ev}'(\widetilde{\phi})$. Define smooth cylinders $w, u, v \colon [0, 1] \times [0, 1] \to M$ by

$$w(s, t) = f^t(\gamma(s)) = \phi^t(\gamma(s)),$$
$$u(s, t) = f^{t+1}(\gamma(s)),$$
$$v(s, t) = f^{t+2}(\gamma(s)) = \psi^{1-t}(\gamma(s)),$$

for $(s, t) \in [0, 1] \times [0, 1]$. The loop $\beta := \psi \circ \gamma$ is C^0–close to the loop $\alpha = \phi \circ \gamma$, hence the image $\beta([0, 1])$ lies inside U, and moreover the image $u([0, 1] \times [0, 1])$ lies inside U. The union of the images of w, u, v is a torus, which is the isotopy of the loop γ via the path f^t, $t \in [0, 3]$. We therefore have the equality $\omega(w) + \omega(u) + \omega(v) = \mathrm{Flux}(f^t)(\gamma)$. We have $\omega(w) = \mathrm{Flux}(\phi^t)(\gamma)$, and $\omega(v) = 0$ since the isotopy ψ^t, $t \in [0, 1]$, is Hamiltonian. Thus we get $\mathrm{Flux}(f^t)(\gamma) - \mathrm{Flux}(\phi^t)(\gamma) = \omega(u)$. Hence it is enough to show that for any initially chosen loop $\gamma \colon [0, 1] \to M$ as above, the symplectic area $\omega(u)$ is arbitrarily small, provided that ψ is sufficiently C^0–close to ϕ.

We show this by a contradiction. Assume the contrary, ie that there exists some $\epsilon' > 0$, such that one can find ψ arbitrarily C^0–close to ϕ for which we have $|\omega(u)| \geqslant \epsilon'$. Now pick $\psi \in H$ which is sufficiently C^0–close to ϕ and such that $|\omega(u)| \geqslant \epsilon'$.

Let us first describe the idea of the rest of the proof. The restriction of the symplectic form ω to the neighbourhood U of α is exact. Consider a slightly smaller tubular neighbourhood U' of α, which is compactly contained in U. Since ψ is arbitrarily C^0–close to ϕ, the map $\psi \circ \phi^{-1}$ sends U' inside U. Picking a 1–form which is a

primitive of ω on U, we obtain that the actions of α and $\beta = \psi \circ \phi^{-1} \circ \alpha$ differ by at least ϵ'. Hence on U', the difference between our primitive 1–form and its pullback by the map $\psi \circ \phi^{-1}$ is represented by a closed 1–form whose cohomology class in $H^1(U', \mathbb{R})$ is not small. Then we consider a certain Lagrangian torus inside U', and as a result of the previous observation, we conclude that the cohomology classes of the restrictions of our primitive 1–form to this torus and to the image of the torus by $\psi \circ \phi^{-1}$, are not close one to the other, while on the other hand, these tori are C^0–close since ψ is arbitrarily C^0–close to ϕ. Finally, we show that such situation is impossible, using one of the ways which are described below.

Now let us explain the details. First, without loss of generality we may assume that $\epsilon' < \epsilon$. Recall that we have a symplectic embedding

$$\iota: W(\epsilon) \times B(\epsilon)^{\times n-1} \to M,$$

such that $S^1 \times \{0\} \times \cdots \times \{0\}$ is mapped onto $\alpha([0, 1])$, and we have

$$U = \iota\big(W(\epsilon) \times B(\epsilon)^{\times n-1}\big).$$

Put $\delta = \epsilon'/2$, and consider the Lagrangian torus

$$L = S^1 \times S(\delta) \times \cdots \times S(\delta) \subset W(\epsilon') \times B(\epsilon')^{\times n-1} \subset W(\epsilon) \times B(\epsilon)^{\times n-1},$$

where $S(\delta) = \{z \in \mathbb{R}^2 \mid \pi|z|^2 = \delta\}$ is the circle on $\mathbb{R}^2$ centred at the origin enclosing a disc of area δ. Since ψ is sufficiently C^0–close to ϕ, it follows that $\psi \circ \phi^{-1}$ is sufficiently C^0–close to id_M, and in particular

$$\psi \circ \phi^{-1}\big(\iota\big(W(\epsilon') \times B(\epsilon')^{\times n-1}\big)\big) \subset U = \iota\big(W(\epsilon) \times B(\epsilon)^{\times n-1}\big),$$

$$\psi \circ \phi^{-1}(\iota(L)) \subset \iota\big(W(\epsilon'/4) \times B'(\epsilon)^{\times n-1}\big)$$

(recall that $B'(\epsilon) = B(\epsilon) \setminus \{0\} \subset \mathbb{R}^2$ is the open punctured euclidean disc centred at the origin having area ϵ). Now, if we denote

$$\widetilde{L} := \iota^{-1}\big(\psi \circ \phi^{-1}(\iota(L))\big) \subset W(\epsilon'/4) \times B'(\epsilon)^{\times n-1},$$

then $\widetilde{L}$ is a Lagrangian torus which is C^0–close to L, and $\pi_1(\widetilde{L})$ is generated by the loops $\widetilde{\beta}_1, \ldots, \widetilde{\beta}_n$, which are the push-forwards of the loops $\beta_1, \ldots, \beta_n$ on $L = S^1 \times S(\delta) \times \cdots \times S(\delta)$, such that the homotopy classes of $\beta_1, \ldots, \beta_n$ in $\pi_1(L)$ correspond to the factors $S^1, S(\delta), \ldots, S(\delta)$. Consider the 1–form

$$\lambda = p_0 dq_0 + \tfrac{1}{2}(x_1 dy_1 - y_1 dx_1) + \cdots + \tfrac{1}{2}(x_{n-1} dy_{n-1} - y_{n-1} dx_{n-1})$$
$$= p_0 dq_0 + \tfrac{1}{2}r_1^2 d\theta_1 + \cdots + \tfrac{1}{2}r_{n-1}^2 d\theta_{n-1}$$

on $W(\epsilon) \times B(\epsilon)^{\times n-1}$, where

$$x_i = r_i \cos \theta_i, \quad y_i = r_i \sin \theta_i$$

for $i = 1, 2, \ldots, n-1$. Then

$$d\lambda = dp_0 \wedge dq_0 + dx_1 \wedge dy_1 + \cdots + dx_{n-1} \wedge dy_{n-1}$$

is the standard symplectic form on $W(\epsilon) \times B(\epsilon)^{\times n-1}$. The map $\iota^{-1} \circ \psi \circ \phi^{-1} \circ \iota$ is well-defined on $W(\epsilon') \times B(\epsilon')^{\times n-1}$, since we have

$$\psi \circ \phi^{-1}\big(\iota\big(W(\epsilon') \times B(\epsilon')^{\times n-1}\big)\big) \subset U = \iota\big(W(\epsilon) \times B(\epsilon)^{\times n-1}\big).$$

Moreover, the map $\iota^{-1} \circ \psi \circ \phi^{-1} \circ \iota$ is symplectic, hence the 1–form

$$(\iota^{-1} \circ \psi \circ \phi^{-1} \circ \iota)^* \lambda - \lambda$$

on $W(\epsilon') \times B(\epsilon')^{\times n-1}$ is closed, and its evaluation at the loop

$$S^1 \times \{0\} \times \cdots \times \{0\} \subset W(\epsilon') \times B(\epsilon')^{\times n-1}$$

equals the symplectic area $\omega(u)$. Hence at the level of cohomology we have

$$[(\iota^{-1} \circ \psi \circ \phi^{-1} \circ \iota)^* \lambda - \lambda] = \omega(u)[dq]$$

on $W(\epsilon') \times B(\epsilon')^{\times n-1}$. In particular, we have

$$\lambda(\widetilde{\beta}_1) = (\iota^{-1} \circ \psi \circ \phi^{-1} \circ \iota)^* \lambda(\beta_1) = \omega(u) + \lambda(\beta_1) = \omega(u),$$

and for $2 \leqslant j \leqslant n$ we have

$$\lambda(\widetilde{\beta}_j) = (\iota^{-1} \circ \psi \circ \phi^{-1} \circ \iota)^* \lambda(\beta_j) = \lambda(\beta_j) = \delta.$$

Now let us present two possible ways of finishing the proof via arriving to a contradiction. The second way is easier and was suggested by Seyfaddini.

First way　Consider the 1–form

$$\lambda' = \tfrac{1}{2}(x_0 dy_0 - y_0 dx_0) + \tfrac{1}{2}(x_1 dy_1 - y_1 dx_1) + \cdots + \tfrac{1}{2}(x_{n-1} dy_{n-1} - y_{n-1} dx_{n-1})$$
$$= \tfrac{1}{2} r_0^2 d\theta_0 + \tfrac{1}{2} r_1^2 d\theta_1 + \cdots + \tfrac{1}{2} r_{n-1}^2 d\theta_{n-1}$$

on $B(\epsilon'/2) \times B'(\epsilon)^{\times n-1}$ endowed with the coordinates $(x_0, y_0, \ldots, x_{n-1}, y_{n-1})$, where

$$x_i = r_i \cos \theta_i, \quad y_i = r_i \sin \theta_i$$

for $i = 0, 1, \ldots, n-1$. We have that

$$d\lambda' = dx_0 \wedge dy_0 + dx_1 \wedge dy_1 + \cdots + dx_{n-1} \wedge dy_{n-1}$$

is the standard symplectic form on $B(\epsilon'/2) \times B'(\epsilon)^{\times n-1}$. Consider the embedding

$$\iota': W(\epsilon'/4) \times B'(\epsilon)^{\times n-1} \hookrightarrow B(\epsilon'/2) \times B'(\epsilon)^{\times n-1},$$

given by

$$\iota'(q_0, p_0, x_1, y_1, \ldots, x_{n-1}, y_{n-1}) = (r_0, \theta_0, x_1, y_1, \ldots, x_{n-1}, y_{n-1}),$$
$$\pi r_0^2 = p_0 + \epsilon'/4,$$
$$\theta_0 = 2\pi q_0.$$

Then we have

$$(\iota')^* \lambda' = \lambda + \tfrac{1}{4}\epsilon' dq_0.$$

Hence for the Lagrangian torus

$$\hat{L} = \iota'(\tilde{L}) \subset B(\epsilon'/2) \times B'(\epsilon)^{\times n-1} \subset \mathbb{R}^2 \times B'(\epsilon)^{\times n-1}$$

and the loops

$$\hat{\beta}_j := \iota' \circ \tilde{\beta}_j, \quad j = 1, \ldots, n,$$

generating $\pi_1(\hat{L})$, we have

$$\lambda'(\hat{\beta}_1) = \omega(u) + \epsilon'/4.$$

Therefore, if we consider $\hat{L}$ as a Lagrangian inside $\mathbb{R}^2 \times B'(\epsilon)^{\times n-1}$ endowed with the standard symplectic form, then it follows that the symplectic area of any disc in $\mathbb{R}^2 \times B'(\epsilon)^{\times n-1}$ with boundary on $\hat{L}$ is an integer multiple of $\omega(u) + \epsilon'/4$, and hence its absolute value is greater than or equal to $|\omega(u)| - \epsilon'/4$ and

$$|\omega(u)| - \epsilon'/4 \geqslant \epsilon' - \epsilon'/4 = 3\epsilon'/4 > \epsilon'/2.$$

By the Chekanov theorem [2], the displacement energy $e(\hat{L})$ of $\hat{L}$ inside $\mathbb{R}^2 \times B'(\epsilon)^{\times n-1}$ is greater than or equal to the minimal area of a non-constant holomorphic disc with boundary on $\hat{L}$. Hence we conclude that $e(\hat{L}) > \epsilon'/2$. But on the other hand, since $\hat{L} \subset B(\epsilon'/2) \times B'(\epsilon)^{\times n-1}$, one can clearly displace $\hat{L}$ with a smooth Hamiltonian isotopy of energy $\epsilon'/2$. Contradiction.

Second way Consider the Liouville form

$$\lambda' = p_0 dq_0 + p_1 dq_1 + \cdots + p_{n-1} dq_{n-1}$$

on $T^*\mathbb{T}^n$. Let

$$\iota': W(\epsilon'/4) \times B'(\epsilon)^{\times n-1} \to T^*\mathbb{T}^n$$

be the symplectic embedding given by

$$\iota'(q_0, p_0, r_1, \theta_1, \ldots, r_{n-1}, \theta_{n-1}) = (q_0, p_0, q_1, p_1, \ldots, q_{n-1}, p_{n-1}),$$
$$\pi r_i^2 - \delta = p_i,$$
$$\theta_i = 2\pi q_i,$$

for $i = 1, 2, \ldots, n-1$. The image of ι' lies inside

$$W(\epsilon'/4) \times (T^* S^1)^{\times n-1} = W(\epsilon'/4) \times T^* \mathbb{T}^{n-1}.$$

We have

$$(\iota')^* \lambda' = \lambda - \frac{\delta}{2\pi} d\theta_1 - \cdots - \frac{\delta}{2\pi} d\theta_{n-1}.$$

Hence for the Lagrangian torus

$$\widehat{L} = \iota'(\widetilde{L}) \subset W(\epsilon'/4) \times T^* \mathbb{T}^{n-1} \subset T^* \mathbb{T}^n$$

and the loops

$$\widehat{\beta}_j := \iota' \circ \widetilde{\beta}_j, \quad j = 1, \ldots, n,$$

generating $\pi_1(\widehat{L})$, we have $\lambda'(\widehat{\beta}_1) = \omega(u)$, and $\lambda'(\widehat{\beta}_j) = 0$, $2 \leqslant j \leqslant n$. Now consider the symplectic shift

$$\Phi \colon T^* \mathbb{T}^n \to T^* \mathbb{T}^n$$

given by

$$\Phi(q_0, p_0, q_1, p_1, \ldots, q_{n-1}, p_{n-1}) = (q_0, p_0 - \omega(u), q_1, p_1, \ldots, q_{n-1}, p_{n-1}).$$

Then we see that the shifted Lagrangian $\Phi(\widehat{L}) \subset T^* \mathbb{T}^n$ is exact, and moreover it does not intersect the zero section, since

$$\widehat{L} \subset W(\epsilon'/4) \times T^* \mathbb{T}^{n-1},$$
$$|\omega(u)| \geqslant \epsilon' > \epsilon'/4.$$

However, by the theorem of Gromov [10, Section 2.3. B_4''], a closed exact Lagrangian submanifold of a cotangent bundle must intersect the zero section. Contradiction. □

Now, Theorem 1.3 is a straightforward consequence of Proposition 2.2:

Proof of Theorem 1.3 By Proposition 2.2, for any $\widetilde{\phi} \in \widetilde{H}_0$ we have

$$\mathrm{Flux}(\widetilde{\phi}) \in \overline{\Gamma_{\mathrm{top}}^a} = \Gamma_{\mathrm{top}}^a + \overline{\Gamma_{\mathrm{top}}^e} \subseteq \Gamma_{\mathrm{top}} + \overline{\Gamma_{\mathrm{top}}^e},$$

where $a = \mathrm{ev}'(\widetilde{\phi}) \in \pi_1(M)$. Hence $\Gamma_0 \subseteq \Gamma_{\mathrm{top}} + \overline{\Gamma_{\mathrm{top}}^e}$. □

Now we turn to the proof of Theorem 1.6.

Proof of Theorem 1.6 It is clearly enough to prove that for any smooth embedded loop $\gamma\colon [0,1] \to M$, there exist constants

$$c = c(M, \omega, g, \gamma), \quad C = C(M, \omega, g, \gamma),$$

such that for any smooth path ϕ^t, $t \in [0,1]$ of symplectomorphisms of M, $\phi^0 = \mathrm{id}_M$, $\phi^1 = \phi$, with $\max_{t \in [0,1]} d(\mathrm{id}_M, \phi^t) < c$, we have

$$|\mathrm{Flux}(\{\phi^t\})(\gamma)| \leqslant C d(\mathrm{id}_M, \phi).$$

Now fix a smooth embedded loop $\gamma\colon [0,1] \to M$. Then a neighbourhood of $\gamma([0,1])$ in M is standard, and hence for some $\epsilon > 0$ one can find a symplectic embedding

$$\iota\colon W(\epsilon) \times B(\epsilon)^{\times n-1} \to M$$

such that

$$\iota(S^1 \times \{0\} \times \cdots \times \{0\}) = \gamma([0,1]).$$

We set

$$c_1 = d\big(\iota\big(W(\epsilon/2) \times S(\epsilon/3)^{\times n-1}\big), M \setminus \iota\big(W(\epsilon) \times B'(\epsilon)^{\times n-1}\big)\big)$$

(recall that the notation $B'(\epsilon) = B(\epsilon) \setminus \{0\} \subset \mathbb{R}^2$ stands for the punctured disc). Now let ϕ^t, $t \in [0,1]$ be a smooth path of symplectomorphisms of M, $\phi^0 = \mathrm{id}_M$, $\phi^1 = \phi$, with $\max_{t \in [0,1]} d(\mathrm{id}_M, \phi^t) < c_1$. Define the "flux function"

$$\kappa\colon [0,1] \to \mathbb{R}, \quad \kappa(t) = \mathrm{Flux}\big(\{\phi^s\}_{s \in [0,t]}\big)(\gamma).$$

We assume that $\kappa(1) = \mathrm{Flux}(\{\phi^t\})(\gamma) \neq 0$. If $\max_{t \in [0,1]} |\kappa(t)| > \epsilon/3$, then we define $T \in [0,1]$ to be minimal such that $|\kappa(T)| = \epsilon/3$, otherwise we set $T = 1$. We have $|\kappa(t)| \leqslant \epsilon/3$ for all $t \in [0,T]$. Now, on $\iota(W(\epsilon) \times B(\epsilon)^{\times n-1})$, consider the time-dependent vector field

$$X_\kappa^t = \kappa'(t) \frac{\partial}{\partial p_0},$$

where $(q_0, p_0, x_1, y_1, \ldots, x_{n-1}, y_{n-1})$ are the standard coordinates on $W(\epsilon) \times B(\epsilon)^{\times n-1}$. Denote by X^t the time-dependent symplectic vector field of the flow ϕ^t. The difference

$$Y^t = X^t - X_\kappa^t, \quad t \in [0,T],$$

is a time-dependent Hamiltonian vector field on $\iota(W(\epsilon) \times B(\epsilon)^{\times n-1})$. With help of a cut-off, we can find a time-dependent Hamiltonian vector field Z^t, $t \in [0,T]$ on M, such that

$$Y^t(x) = Z^t(x)$$

for any $x \in \iota(W(\epsilon/2) \times B(\epsilon/2)^{\times n-1})$ and $t \in [0, T]$. Now look at the time-dependent symplectic vector field

$$\widetilde{X}^t = X^t - Z^t, \quad t \in [0, T],$$

on M, and denote by $\widetilde{\phi}^t$, $t \in [0, T]$, its symplectic flow on M. Then

$$\psi^t := (\phi^t)^{-1} \circ \widetilde{\phi}^t, \quad t \in [0, T],$$

is a Hamiltonian flow on M since it has zero flux at all times. Consider the Lagrangian torus

$$L = \iota(S^1 \times S(\epsilon/3) \times \cdots \times S(\epsilon/3)) \subset \iota(W(\epsilon) \times B(\epsilon)^{\times n-1}) \subset M.$$

We have that $\widetilde{X}^t(x) = X_\kappa^t(x)$ for any $x \in \iota(W(\epsilon/2) \times B(\epsilon/2)^{\times n-1})$ and $t \in [0, T]$, and hence

$$\widetilde{\phi}^t\big(\iota(q_0, p_0, x_1, y_1, \ldots, x_{n-1}, y_{n-1})\big) = \iota(q_0, p_0 + \kappa(t), x_1, y_1, \ldots, x_{n-1}, y_{n-1})$$

whenever

$$\iota(q_0, p_0, x_1, y_1, \ldots, x_{n-1}, y_{n-1}) \in L \quad \text{and} \quad t \in [0, T],$$

and so for any $t \in [0, T]$, $\widetilde{\phi}^t(L)$ is obtained from L by shifting it by $\kappa(t)$ in the "p_0 direction". Clearly there exists a constant $c_2 = c_2(M, \omega, g, \gamma, \iota)$, such that the distance $d(\widetilde{\phi}^T(L), L)$ is greater than or equal to $c_2|\kappa(T)|$. Also note that since

$$\max_{t \in [0,1]} d(\mathrm{id}_M, \phi^t) < c_1,$$

we get that

$$\psi^t(L) = (\phi^t)^{-1} \circ \widetilde{\phi}^t(L) \subset \iota\big(W(\epsilon) \times B'(\epsilon)^{\times n-1}\big)$$

for each $t \in [0, T]$. Now, L cannot be Hamiltonianly displaced *inside* $\iota(W(\epsilon) \times B'(\epsilon)^{\times n-1})$, and so we must have

$$\psi^T(L) \cap L = (\phi^T)^{-1} \circ \widetilde{\phi}^T(L) \cap L \neq \varnothing.$$

Since in addition we have $d(\widetilde{\phi}^T(L), L) \geqslant c_2|\kappa(T)|$, we conclude that $d(\mathrm{id}_M, \phi^T) \geqslant c_2|\kappa(T)|$.

We have shown that for any smooth path ϕ^t, $t \in [0, 1]$ of symplectomorphisms of M, $\phi^0 = \mathrm{id}_M$, $\phi^1 = \phi$, with $\max_{t \in [0,1]} d(\mathrm{id}_M, \phi^t) < c_1$, we must have

$$d(\mathrm{id}_M, \phi^T) \geqslant c_2|\kappa(T)|.$$

Thus, if we set $c = \min(c_1, c_2\epsilon/3)$ and $C = 1/c_2$, then for any smooth path ϕ^t, $t \in [0, 1]$, of symplectomorphisms of M, $\phi^0 = \mathrm{id}_M$, $\phi^1 = \phi$, with $\max_{t \in [0,1]} d(\mathrm{id}_M, \phi^t) < c$,

we have

$$c_2\epsilon/3 \geqslant c > d(\mathrm{id}_M, \phi^T) \geqslant c_2|\kappa(T)|,$$

hence $|\kappa(T)| \neq \epsilon/3$, which means that $T = 1$, and we therefore get

$$|\mathrm{Flux}(\{\phi^t\})(\gamma)| = |\kappa(1)| = |\kappa(T)| \leqslant Cd(\mathrm{id}_M, \phi^T) = Cd(\mathrm{id}_M, \phi). \qquad \square$$

References

[1] **A Banyaga**, *Sur la structure du groupe des difféomorphismes qui préservent une forme symplectique*, Comment. Math. Helv. 53 (1978) 174–227   MR490874

[2] **Y V Chekanov**, *Lagrangian intersections, symplectic energy, and areas of holomorphic curves*, Duke Math. J. 95 (1998) 213–226   MR1646550

[3] **C C Conley**, **E Zehnder**, *The Birkhoff–Lewis fixed point theorem and a conjecture of V. I. Arnol′d*, Invent. Math. 73 (1983) 33–49   MR707347

[4] **Y M Eliashberg**, *Rigidity of symplectic and contact structures*, preprint (1981)

[5] **Y M Eliashberg**, *A theorem on the structure of wave fronts and its application in symplectic topology*, Funktsional. Anal. i Prilozhen. 21 (1987) 65–72, 96   MR911776

[6] **A Floer**, *Morse theory for Lagrangian intersections*, J. Differential Geom. 28 (1988) 513–547   MR965228

[7] **A Floer**, *Symplectic fixed points and holomorphic spheres*, Comm. Math. Phys. 120 (1989) 575–611   MR987770

[8] **K Fukaya**, **K Ono**, *Arnold conjecture and Gromov–Witten invariant*, Topology 38 (1999) 933–1048   MR1688434

[9] **K Fukaya**, **K Ono**, *Floer homology and Gromov–Witten invariant over $\mathbb{Z}$ of general symplectic manifolds—summary*, from: "Taniguchi Conference on Mathematics Nara '98", (M Maruyama, T Sunada, editors), Adv. Stud. Pure Math. 31, Math. Soc. Japan, Tokyo (2001) 75–91   MR1865088

[10] **M Gromov**, *Pseudoholomorphic curves in symplectic manifolds*, Invent. Math. 82 (1985) 307–347   MR809718

[11] **H Hofer**, **D A Salamon**, *Floer homology and Novikov rings*, from: "The Floer memorial volume", (H Hofer, C H Taubes, A Weinstein, E Zehnder, editors), Progr. Math. 133, Birkhäuser, Basel (1995) 483–524   MR1362838

[12] **F Lalonde**, **D McDuff**, **L Polterovich**, *On the flux conjectures*, from: "Geometry, topology, and dynamics", (F Lalonde, editor), CRM Proc. Lecture Notes 15, Amer. Math. Soc., Providence, RI (1998) 69–85   MR1619124

[13] **G Liu**, **G Tian**, *Floer homology and Arnold conjecture*, J. Differential Geom. 49 (1998) 1–74   MR1642105

[14] **D McDuff**, *Symplectic diffeomorphisms and the flux homomorphism*, Invent. Math. 77 (1984) 353–366 MR752824

[15] **Y-G Oh**, **S Müller**, *The group of Hamiltonian homeomorphisms and C^0–symplectic topology*, J. Symplectic Geom. 5 (2007) 167–219 MR2377251

[16] **K Ono**, *On the Arnol′d conjecture for weakly monotone symplectic manifolds*, Invent. Math. 119 (1995) 519–537 MR1317649

[17] **K Ono**, *Floer–Novikov cohomology and the flux conjecture*, Geom. Funct. Anal. 16 (2006) 981–1020 MR2276532

[18] **Y Ruan**, *Virtual neighborhoods and pseudo-holomorphic curves*, from: "Proceedings of 6th Gökova Geometry–Topology Conference", (S Akbulut, T Ander, R J Stern, editors), volume 23 (1999) 161–231 MR1701645

[19] **P Seidel**, *Abstract analogues of flux as symplectic invariants* arXiv:1108.0394

[20] **P Seidel**, *Lectures on categorical dynamics and symplectic topology*, lecture notes Available at http://www-math.mit.edu/~seidel/937/lecture-notes.pdf

[21] **S Seyfaddini**, *A note on C^0 rigidity of Hamiltonian isotopies*, J. Symplectic Geom. 11 (2013) 489–496 MR3100803

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