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**Special Program on
Inverse Problems**

Edited by R.S. Anderssen
and G.N. Newsam

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PREFACE

In the second half of 1986, the Centre for Mathematical Analysis hosted a special study of inverse problems as one of its major activities for that year. As well as Australian researchers, a number of leading international experts, with wide and varied experience with inverse problems, were invited to participate. They included :

Dr. R. Barakat Harvard University	Phase Retrieval	19/10/86 - 01/12/86
Dr.R. Davies University College of Wales, Aberystwyth	Numerical Differentiation and Convolution	01/09/86 - 03/10/86
Prof. H.W. Engl University of Linz	Theory and Application of Regularization Methods	01/08/86 - 19/08/86
Prof. O. Hald University of California	Inverse Eigenvalue Problems	05/09/86 - 30/10/86
Prof. K-H Hoffman University of Augsburg	Optimal Control, Asymptotic Regularization, and parameter identification for elliptic pde	06/11/86 - 06/12/86
Prof. J. McLaughlin Rensselaer Polytechnic Institute	Inverse Eigenvalue Problems	14/09/86 - 05/10/86
Prof. M. Overton New York University	Inverse Eigenvalue Problems	27/10/86 - 10/06/87
Prof. T. Seidman University of Maryland	Theory and Application of Regularization Methods	01/11/86 - 30/11/86
Prof. C. Vogel University of Montana	Theory and Application of Regularization Methods	15/11/86 - 24/12/86
Prof. M. Vogelius University of Maryland	Determination of an Unknown Conductivity from Boundary Measurements	06/07/86 - 20/08/86

For the study, attention was focussed on specific topics which related in one way or another to previous Australian research and potential future interest. The hope was that such concentration would assist with maximizing the success of the research undertaken. As a direct result of the interaction and collaboration thereby fostered, considerable progress was made with developing new methodologies for various classes of inverse problems such as methods based on the weak formulation for the aquifer transmissivity identification problem (Research Report R01-87); as phase retrieval in two dimensions (Research Report R15-86); as asymptotic regularization techniques for parameter identification (Research Report - in preparation); and as hyperbolic approximations for a Cauchy problem for the heat equation (Research Report R44-86). In addition, a number of new and exciting results were discovered. They included a variational method for impedance computed tomography (Research Report R50-86); a proof that the distance between nodal points uniquely determine the density of a vibrating string (Research Report - in preparation); optimal parameter choice for general regularization methods (Research Report R35-86).

In addition to the above mentioned reports, some of the work done in the Centre on inverse problems during this period has been collected together as a series of papers for this Proceedings. These papers highlight in various ways the specific topics chosen as the points of focus; namely numerical differentiation and convolution, theory and application of regularization methods, and inverse eigenvalue problems.

As a prototype for a wide class of inverse problems, including fractional differentiation and first kind integral equations, numerical

differentiation has and continues to be studied in great detail. In fact, it is often used to characterize and compare the degree of ill-posedness in a wider variety of situations. Dr. Davies paper examines different measures which have been used to optimize the numerical differentiation of data using regularization techniques, as well as considers practical questions related to implementation.

A popular computational approach to the solution of inverse problems is not to first stabilize the original problem, but to stabilize the discretization of the original problem derived from the application of some approximation methodology. As a direct consequence, the analysis and stabilization of algebraic inverse problems is important in the construction of suitable algorithms. The papers by Professor Eldén and Dr. Newsam examine this aspect in some detail. Professor Eldén focused attention on algorithms for the computation of functionals defined on the solution of a discrete ill-posed problem; while Dr. Newsam has given a detailed analysis of the asymptotic distribution of the eigenvalues of discretizations of compact operators, since such operators are often used as the prototype problem for inverse problems and because the distribution of such eigenvalues plays such a key role in assessing the numerical performance of algebraic problems.

The most popular form of stabilization for inverse problems, which have a natural operator equation setting, is Tikhorov regularization. The paper by Professor Engl and Professor Groetsch reviews some recent advances in the theoretical examination of such methods; while the paper by Professors Jonca and Vogel considers the application of regularization

methods to the practical problem of determining magnetic relief from aeromagnetic survey data.

The three remaining papers examine a number of independent aspects connected with the solution of inverse eigenvalue problems. Since such inverse problems do not fall naturally into the standard operator equation setting mentioned above, their analysis represents an independent facet in the analysis of inverse problems, especially since inverse eigenvalue problems model important practical situations. Professors Hald and McLaughlin derive uniqueness results as well as an algorithm and bounds for such problems when information about the nodal positions of the eigenfunctions are known instead of the eigenvalues.

Just as for operator equations, the stabilization of an algebraic discretization of an inverse eigenvalue problem leads naturally to an examination of inverse algebraic eigenvalue problems. Recent research related to this subject is reviewed in Professor Overton's paper, which also examines the extremal eigenvalue problem. Finally, the paper by Dr. Paine shows how regularization procedures can be used to recover piecewise constant Sturm-Liouville potentials. Now, however, one works not with an algebraic discretization of the Sturm-Liouville problem, but a Sturm-Liouville problem with piecewise constant co-efficients which are approximations to the coefficients of the original Sturm-Liouville problem.

The editors wish to acknowledge with sincere thanks the cooperation and support of the mathematicians directly involved with the special activity on inverse problems; of the Director of the Centre, Professor Neil Trudinger; and of the secretaries Marilyn Gray, Dorothy Nash and Jillian Smith, who not only assisted with the preparation of the Proceedings, but also with the background organizational details of the special activity.