TABLE OF CONTENTS

1. INTRODUCTION

- 1.1 A short history of variational principles.
- 1.2 The concept of geodesics.
- 1.3 Definition and some elementary properties of harmonic maps.
- 1.4 Mathematical problems arising from the concept of harmonic maps.
- 1.5 Some examples of harmonic maps.
- 1.6 Some applications of harmonic maps.
- 1.7 Composition properties of harmonic maps.
- GEOMETRIC PRELIMINARIES Almost linear functions, approximate fundamental solutions, and representation formulae. Harmonic coordinates.
 - 2.1 Outline of the chapter.
 - 2.2 Jacobi field estimates.
 - 2.3 Applications to geodesic constructions.
 - 2.4 Convexity of geodesic balls.
 - 2.5 The distance as a function of two variables.
 - 2.6 Almost linear functions.
 - 2.7 Approximate fundamental solutions and representation formulae.
 - 2.8 Regularity properties of coordinates. Harmonic coordinates.
- 3. THE HEAT FLOW METHOD Existence, regularity, and uniqueness results for a nonpositively curved image.
 - 3.1 Approaches to the existence and regularity question.
 - 3.2 Short time existence.
 - 3.3 Estimates for the energy density of the heat flow.
 - 3.4 The stability lemma of Hartman.
 - 3.5 A bound for the time derivative.
 - 3.6 Global existence and convergence to a harmonic map (Theorem of Eells-Sampson).
 - 3.7 Estimates in the elliptic case.
 - 3.8 The uniqueness results of Hartman.
 - 3.9 The Dirichlet problem.
 - 3.10 An open question.

- REGULARITY OF WEAKLY HARMONIC MAPS. Regularity, existence, and uniqueness of solutions of the Dirichlet problem, if the image is contained in a convex ball.
 - 4.1 The concept of weak solutions.
 - 4.2 A lemma of Giaquinta-Giusti-Hildebrandt.
 - 4.3 Choice of a test function.
 - 4.4 An iteration argument. Continuity of weak solutions.
 - 4.5 Hölder continuity of weak solutions.
 - 4.6 Applications to the Bernstein problem.
 - 4.7 Estimates at the boundary.
 - 4.8 C¹-estimates.
 - 4.9 Higher estimates.
 - 4.10 The existence theorem of Hildebrandt-Kaul-Widman.
 - 4.11 The uniqueness theorem of Jager-Kaul.

5. HARMONIC MAPS BETWEEN SURFACES

- 5.1 Nonexistence results.
- 5.2 Some lemmata.
- 5.3 The existence theorem of Lemaireand Sacks-Uhlenbeck.
- 5.4 The Dirichlet problem, if the image is homeomorphic to S^2 . Two solutions for nonconstant boundary values.
- 5.5 Conformal diffeomorphisms of spheres. The Riemann mapping theorem.
- 5.6 Existence of harmonic diffeomorphisms, if the image is contained in a convex ball.
- 5.7 Existence of harmonic diffeomorphisms between closed surfaces.
- 5.8 Some remarks.