Preface

The present issue of Advanced Studies in Pure Mathematics is an outgrowth of the international conference entitled "Lie Groups, Geometric Structures, and Differential Equations - One Hundred Years after Sophus Lie - " , which took place, on the occasion of the centennial after the death of Sophus Lie (1842 - 1899), in Kyoto and Nara in December 1999. A rapid version of the proceedings has already appeared as one volume in Lecture Note Series of RIMS Kyoto University (RIMS Kokyuroku 1150, April 2000).

The influence of Sophus Lie on our mathematics today is so immense and so deep that it is almost out of our scope to overview all the aspects developed after Lie. But we wished that the conference would give us an opportunity to contemplate and discuss what Lie pursued, what developments had been achieved after Lie and what would be important for the coming new century, with the emphasis on the links between Lie groups, geometry and differential equations.

In this volume seventeen papers are presented, which reflect the contents, atmosphere and outgrowth of the above conference. Most of the papers are research papers of usual style, whose topics are rather diversified, but in a broader sense related to the developments of Lie's ideas. Subjects treated in this volume include Lie groups, geometry of differential equations, contact transformations, Bäcklund transformations, differential algebras, Cartan geometry, CR-geometry and some topics in differential geometry as well as topics in Mathematical Physics. Each contributed paper standing independently by itself, we will not give here comments on each paper, but mention briefly two survey papers specially prepared for this volume.

The paper, by Morimoto, develops a synthetic study of Lie groups, geometric structures and differential equations from a unified view point of nilpotent geometry, by studying objects on filtered manifolds through their first order approximations, nilpotent graded Lie algebras. Generalization from the abelian to the nilpotent proves to be algebraically natural, geometrically useful, and analytically revealing new phenomena. This survey will give an overview on links between groups, geometric structures and differential equations by trying to make clear the underlying structures common to them three.

The paper, by Yamaguchi and Yatsui, discusses the geometry of higher order differential equations of finite type. Starting from a reductive graded Lie algebra of the first kind and its faithful irreducible

representation, they specify a nice class of differential equations of finite type to which one can associate Cartan connections. Invariants of these Cartan connections being found in the generalized Spencer cohomology groups, they carry out detailed calculus of these cohomology groups. Thus there will be presented concrete models in which Lie groups, geometric structures and differential equations are beautifully woven.

The year when Lie passed away was also marked as the opening of a new epoch 1900 - 1910 in which Élie Cartan achieved his deep works on Pfaffian systems, infinite continuous groups, and differential equations, to make a great progress on what Lie had pursued. The underlying philosophy of the two surveys may be said to stem from this period.

We would like to express our sincere gratitude to all those who have helped us to organize the above-mentioned conference and to prepare and publish this volume.

"Don't follow in the wake of a sage of old,
but try to embark on the quest he pursued."
(Matsuo Basho, Translated by Shigeharu Ando)

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Editors

All papers in this volume have been refereed and are in final form. No version of any of them will be submitted for publication elsewhere.