11. $(abc' + ab'c + a'b'c = 1) = (b = c')\ (c' < a);$

12. $(abc' + ab'c + a'b'c' = 1) = (bc = 0)\ (a = b + c);$

13. $(abc' + a'b'c + a'b'c' = 1) = (ac = 0)\ (a = b);$

14. $(ab'c + a'b'c + a'b'c' = 1) = (b = 0)\ (a < c).$

The last two causes, as we know, are the equality (1) itself and the absurdity $(1 = 0)$. It is evident that the cause independent of a is the 8th $(b = 0)\ (c = 1)$, and the cause independent of c is the 10th $(a = 0)\ (b = 0)$. There is no cause, properly speaking, independent of b. The most "natural" cause, the one which may be at once divined simply by the exercise of common sense, is the 12th:

$$(bc = 0)\ (a = b + c).$$

But other causes are just as possible; for instance the 9th $(b = 0)\ (a = c)$, the 7th $(c = 0)\ (a = b)$, or the 13th $(ac = 0)\ (a = b)$.

We see that this method furnishes the complete enumeration of all possible cases. In particular, it comprises, among the *forms* of an equality, the solutions deducible therefrom with respect to such and such an "unknown quantity", and, among the *consequences* of an equality, the resultants of the elimination of such and such a term.

48. The Geometrical Diagrams of Venn.—Poretsky's method may be looked upon as the perfection of the methods of Stanley Jevons and Venn.

Conversely, it finds in them a geometrical and mechanical illustration, for Venn's method is translated in geometrical diagrams which represent all the constituents, so that, in order to obtain the result, we need only strike out (by shading) those which are made to vanish by the data of the problem. For instance, the universe of three terms a, b, c, represented by the unbounded plane, is divided by three simple closed contours into eight regions which represent the eight constituents (Fig. 1).

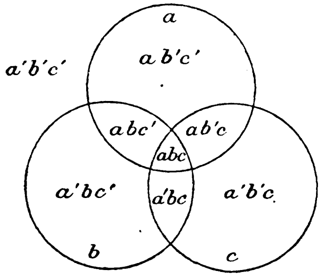

Fig. 1.

To represent geometrically the data of VENN's problem we must strike out the regions abc, $ab'c'$, $a'bc$ and $a'bc'$; there will then remain the regions abc', $ab'c$, $a'b'c$, and $a'b'c'$ which will constitute the universe *relative to the problem*, being what PORETSKY calls his *logical whole* (Fig. 2). Then

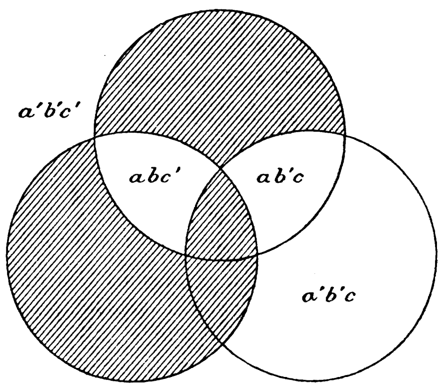

Fig. 2.

every class will be contained in this universe, which will give for each class the expression resulting from the data of the problem. Thus, simply by inspecting the diagram, we see that the region bc does not exist (being struck out); that the region b is reduced to abc' (hence to ab); that all a is b or c, and so on.

This diagrammatic method has, however, serious inconveniences as a method for solving logical problems. It does not show how the data are exhibited by canceling certain constituents, nor does it show how to combine the remaining constituents so as to obtain the consequences sought. In short, it serves only to exhibit one single step in the argument, namely the equation of the problem; it dispenses neither with the previous steps, *i. e.*, "throwing of the problem into an equation" and the transformation of the premises, nor with the subsequent steps, *i. e.*, the combinations that lead to the various consequences. Hence it is of very little use, inasmuch as the constituents can be represented by algebraic symbols quite as well as by plane regions, and are much easier to deal with in this form.

49. **The Logical Machine of Jevons.**—In order to make his diagrams more tractable, VENN proposed a mechanical device by which the plane regions to be struck out could be lowered and caused to disappear. But JEVONS invented a more complete mechanism, a sort of *logical piano*. The keyboard of this instrument was composed of keys indicating the various simple terms (a, b, c, d), their negatives, and the signs $+$ and $=$. Another part of the instrument consisted of a panel with movable tablets on which were written all the combinations of simple terms and their negatives; that is, all the constituents of the universe of discourse. Instead of writing out the equalities which represent the premises, they are "played" on a keyboard like that of a typewriter. The result is that the constituents which vanish because of the premises disappear from the panel. When all the premises have been "played", the panel shows only those constituents whose sum is equal to 1, that is, forms the universe with respect to the problem, its logical whole. This mechanical method has the advantage over VENN's geometrical method of performing automatically the "throwing into an equation", although the premises must first be expressed in the form of equalities; but it throws no more light than the geometrical method on the operations to be per-