Note that the last four causes are based on the inclusion

$$0 < 1.$$

The last two causes (15. and 16.) are obtained either by adding *all* the missing constituents or by not adding any. In the first case, the sum of all the constituents being equal to 1, we find

15. $$1 = 0,$$

that is, absurdity, and this confirms the paradoxical proposition that the false (the absurd) implies any proposition (is its cause). In the second case, we obtain simply the given equality, which thus appears as one of its own causes (by the principle of identity):

16. $$ab' + bc' = 0.$$

If we disregard these two extreme causes, the number of causes properly so called will be

$$2^{2^n - m} - 2.$$

46. Forms of Consequences and Causes.—We can apply the law of forms to the consequences and causes of a given equality so as to obtain all the forms possible to each of them. Since any equality is equivalent to one of the two forms

$$N = O, \qquad N' = 1,$$

each of its consequences has the form[1]

$$NX = 0, \qquad \text{or } N' + X' = 1,$$

and each of its causes has the form

$$N + X = 0, \qquad \text{or } N'X' = 1.$$

[1] In § 44 we said that a consequence is obtained by taking a part of the constituents of the first member N, and not by multiplying it by a term X; but it is easily seen that this amounts to the same thing. For, suppose that X (like N) be developed with respect to the n terms of discourse. It will be composed of a certain number of constituents. To perform the multiplication of N by X, it is sufficient to multiply all their constituents each by each. Now, the product of two identical constituents is equal to each of them, and the product of two different constituents is 0. Hence the product of N by X becomes reduced to the sum of the constituents common to N and X, which is, of course, contained in N. So, to multiply N by an arbitrary term is tantamount to taking a part of its constituents (or all, or none).

In fact, we have the following formal implications:

$$(N + X = 0) < (N = 0) < (NX = 0),$$
$$(N'X' = 1) < (N' = 1) = (N' + X' = 1).$$

Applying the law of forms, the formula of the consequences becomes

$$U = (N' + X')\,U + NXU',$$

and the formula of the causes

$$U = N'X'U + (N + X)\,U';$$

or, more generally, since X and X' are indeterminate terms, and consequently are not necessarily the negatives of each other, the formula of the consequences will be

$$U = (N' + X)\,U + NYU',$$

and the formula of the causes

$$U = N'XU + (N + Y)\,U'.$$

The first denotes that U is contained in $(N' + X)$ and contains NY; which indeed results, *a fortiori*, from the hypothesis that U is contained in N' and contains N.

The second formula denotes that U is contained in $N'X$ and contains $N' + Y$ whence results, *a fortiori*, that U is contained in N' and contains N.

We can express this rule verbally if we agree to call every class contained in another a *sub-class*, and every class that contains another a *super-class*. We then say: To obtain all the consequences of an equality (put in the form $U = N'U + NU'$), it is sufficient to substitute for its logical whole N' all its super-classes, and, for its logical zero N, all its sub-classes. Conversely, to obtain all the causes of the same equality, it is sufficient to substitute for its logical whole all its sub-classes, and for its logical zero, all its super-classes.

47. Example: Venn's Problem.—*The members of the administrative council of a financial society are either bondholders or shareholders, but not both. Now, all the bond-*