universe of discourse is divided with reference to the n given terms. In the same way we shall call the factors of the development of 0 the *maxima* of discourse, because they are the largest classes that can be determined in the universe of discourse by means of the n given terms.

22. Properties of the Constituents.—The constituents or minima of discourse possess two properties characteristic of contradictory terms (of which they are a generalization); they are *mutually exclusive*, *i. e.*, the product of any two of them is 0; and they are *collectively exhaustive*, *i. e.*, the sum of all "exhausts" the universe of discourse. The latter property is evident from the preceding formulas. The other results from the fact that any two constituents differ at least in the "sign" of one of the terms which serve as factors, *i. e.*, one contains this term as a factor and the other the negative of this term. This is enough, as we know, to ensure that their product be null.

The maxima of discourse possess analogous and correlative properties; their combined product is equal to 0, as we have seen; and the sum of any two of them is equal to 1, inasmuch as they differ in the sign of at least one of the terms which enter into them as summands.

For the sake of simplicity, we shall confine ourselves, with Boole and Schröder, to the study of the constituents or minima of discourse, *i. e.*, the developments of 1. We shall leave to the reader the task of finding and demonstrating the corresponding theorems which concern the maxima of discourse or the developments of 0.

23. Logical Functions.—We shall call a *logical function* any term whose expression is complex, that is, formed of letters which denote simple terms together with the signs of the three logical operations.[1]

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[1] In this algebra the logical function is analogous to the *integral function* of ordinary algebra, except that it has no powers beyond the first.

A logical function may be considered as a function of all the terms of discourse, or only of some of them which may be regarded as unknown or variable and which in this case are denoted by the letters x, y, z. We shall represent a function of the variables or unknown quantities, x, y, z, by the symbol $f(x, y, z)$ or by other analogous symbols, as in ordinary algebra. Once for all, a logical function may be considered as a function of any term of the universe of discourse, whether or not the term appears in the explicit expression of the function.

24. **The Law of Development.**—This being established, we shall proceed to develop a function $f(x)$ with respect to x. Suppose the problem solved, and let

$$ax + bx'$$

be the development sought. By hypothesis we have the equality

$$f(x) = ax + bx'$$

for all possible values of x. Make $x = 1$ and consequently $x' = 0$. We have

$$f(1) = a.$$

Then put $x = 0$ and $x' = 1$; we have

$$f(0) = b.$$

These two equalities determine the coefficients a and b of the development which may then be written as follows:

$$f(x) = f(1)x + f(0)x',$$

in which $f(1)$, $f(0)$ represent the value of the function $f(x)$ when we let $x = 1$ and $x = 0$ respectively.

Corollary.—Multiplying both members of the preceding equalities by x and x' in turn, we have the following pairs of equalities (MacColl):

$$xf(x) = ax \qquad x'f(x) = bx'$$
$$xf(x) = xf(1), \quad x'f(x) = x'f(0).$$

Now let a function of two (or more) variables be developed