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Existence of global smooth solutions to the Cauchy problem of bipolar Navier–Stokes–Maxwell system

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Abstract.

This work is concerned with smooth periodic solutions for the compressible Navier–Stokes equations coupled with the Maxwell equations through the Lorentz force. The existence and uniqueness of the global smooth solution is established by using energy method.

§1. Introduction

In this paper, we are interested in the Navier–Stokes–Maxwell system

$$\begin{cases} \partial_t n_\mu + \nabla \cdot (n_\mu u_\mu) = 0, \\ \partial_t u_\mu + u_\mu \cdot \nabla u_\mu + \dfrac{\nabla p(n_\mu)}{n_\mu} = q_\mu (E + u_\mu \times B) + \dfrac{\nu_\mu \triangle u_\mu}{n_\mu}, \\ \partial_t E - \nabla \times B = n_e u_e - n_i u_i, \\ \partial_t B + \nabla \times E = 0, \\ \nabla \cdot E = n_i - n_e, \quad \nabla \cdot B = 0. \end{cases} \tag{1.1}$$

Where, $\mu = e, i$, $n_e = n_e(t,x) > 0$, $n_i = n_i(t,x) > 0$, $u_e = u_e(t,x) \in \mathbb{R}^3$, $u_i = u_i(t,x) \in \mathbb{R}^3$, $E = E(t,x) \in \mathbb{R}^3$, $B = B(t,x) \in \mathbb{R}^3$, for $t > 0$, $x \in \mathbb{T} = (\frac{\mathbb{R}}{2\pi})^3$, denoting the electron density, ion density, electron velocity, ion velocity, electric field and magnetic field, respectively. The electrons of charge $q_e = -1$ and a single species of ions of charge $q_i = 1$ is considered. p depending only on n_μ denotes the pressure function with the usual assumption that p is smooth in the argument and $p' > 0$. $\nu_\mu > 0$ is a constant denoting viscosity coefficient. Throughout this

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paper, we set $\nu_\mu = 1$ without loss of generality. The Initial data is given as

$$[n_\mu, u_\mu, E, B]\mid_{t=0} = [n_{\mu 0}, u_{\mu 0}, E, B], \quad x \in \mathbb{T}, \tag{1.2}$$

with the compatible condition

$$\nabla \cdot E_0 = n_{i0} - n_{e0}, \quad \nabla \cdot B_0 = 0, \quad x \in \mathbb{T}. \tag{1.3}$$

The study of the unipolar Navier–Stokes–Maxwell system is due to Duan [2]. In related models, such as Euler–Maxwell equations, some problems have been widely analyzed by many authors. The first mathematical study of the unipolar Euler–Maxwell system is due to Chen et al [1]. Peng and Wang [4] established convergence of the compressible Euler–Maxwell system to the incompressible Euler system for well-prepared smooth initial data. Ueda et al [6] established the existence and uniqueness of global solutions with small amplitude to the unipolar Euler–Maxwell system in three space dimensions.

In this paper, we investigate the existence and uniqueness of global solution to the Cauchy problem (1.1)–(1.3).

§2. Preliminaries and main results

Let us introduce some notations for the use later. $a \sim b$ means $\lambda a \leq b \leq \frac{1}{\lambda}$ for a generic constant $0 < \lambda < 1$. For any integer $m \geq 0$, we use H^m, $\dot{H}^m$ to denote the usual Sobolev space $H^m(\mathbb{T})$ and the corresponding m-order homogeneous Sobolev space, respectively. Set $L^2 = H^0$. We use $\langle \cdot, \cdot \rangle$ to denote the inner product over the Hilbert space $L^2(\mathbb{T})$. For a multi-index $\alpha = [\alpha_1, \alpha_2, \alpha_3]$, we set $\partial_j = \partial_{x_j}$ $(j = 1, 2, 3)$. The main result is stated as follows.

Theorem 2.1. *Let $N \geq 4$ and* (1.3) *hold. There exit $\delta_0 > 0$ sufficiently small and $C > 0$ such that if*

$$\|[n_{\mu 0} - 1, u_{\mu 0}, E_0, B_0]\|_N \leq \delta_0,$$

then, the Cauchy problem (1.1)–(1.2) *has a unique global solution $[n_\mu(t,x)$, $u_\mu(t,x)$, $E(t,x)$, $B(t,x)]$ with*

$$[n_\mu - 1, u_\mu, E, B] \in C([0,\infty); H^N(\mathbb{T})),$$

$$n_\mu - 1 \in L^2((0,+\infty); H^N(T)), \quad \nabla u_\mu \in L^2((0,+\infty); H^N(T)),$$

$$\nabla E \in L^2((0,+\infty); H^{N-2}(T)), \quad \nabla^2 B \in L^2((0,+\infty); H^{N-3}(T)),$$

and

$$\|[n_\mu - 1, u_\mu, E, B]\|_N^2 + \int_0^t \big(\|n_\mu(s) - 1\|_N^2 + \|\nabla u_\mu(s)\|_N^2 + \|\nabla E(s)\|_{N-2}^2 + \|\nabla^2 B(s)\|_{N-3}^2\big) ds \le C\|[n_{\mu 0} - 1, u_{\mu 0}, E_0, B_0]\|_N^2$$

for any $t \geq 0$.

Next, we introduce the reformulated version of (1.1). Set

$$\rho_\mu = n_\mu - 1, \quad v_\mu = \frac{1}{\gamma} u_\mu, \quad \widetilde{E} = \frac{1}{\gamma} E, \quad \widetilde{B} = \frac{1}{\gamma} B, \tag{2.1}$$

with $\gamma = \sqrt{p'(1)} > 0$. Then, $V := [\rho_\mu, v_\mu, \widetilde{E}, \widetilde{B}]$ satisfies

$$\begin{cases} \partial_t \rho_\mu + \gamma \nabla \cdot v_\mu = h_{1\mu}, \\ \partial_t v_\mu + \gamma \nabla \rho_\mu - q_\mu \widetilde{E} - \Delta v_\mu = h_{2\mu}, \\ \partial_t \widetilde{E} - \nabla \times \widetilde{B} + v_i - v_e = h_3, \\ \partial_t \widetilde{B} + \nabla \times \widetilde{E} = 0, \\ \nabla \cdot \widetilde{E} = \dfrac{1}{\gamma}(\rho_i - \rho_e), \quad \nabla \cdot \widetilde{B} = 0, \quad t > 0, \ x \in \mathbb{T}, \end{cases} \tag{2.2}$$

with initial data

$$V|_{t=0} = V_0 := [\rho_{\mu 0}, v_{\mu 0}, \widetilde{E}_0, \widetilde{B}_0], \ x \in \mathbb{T}. \tag{2.3}$$

Here, $V_0 = [\rho_{\mu 0}, v_{\mu 0}, \widetilde{E}_0, \widetilde{B}_0]$ satisfies

$$\nabla \cdot \widetilde{E}_0 = \frac{1}{\gamma}(\rho_{i0} - \rho_{e0}), \quad \nabla \cdot \widetilde{B}_0 = 0, \quad x \in \mathbb{T}. \tag{2.4}$$

Where,

$$h_{1\mu} = -\gamma \nabla \cdot (\rho_\mu v_\mu), \tag{2.5}$$

$$\begin{aligned} h_{2\mu} = & - \gamma v_\mu \cdot \nabla v_\mu - \frac{1}{\gamma}\left(\frac{\nabla p(1+\rho_\mu)}{1+\rho_\mu} - p'(1)\nabla \rho_\mu\right) + q_\mu \gamma v_\mu \times \widetilde{B} \\ & + \left(\frac{1}{1+\rho_\mu} - 1\right) \Delta v_\mu, \end{aligned} \tag{2.6}$$

$$h_3 = (\rho_e v_e - \rho_i v_i). \tag{2.7}$$

In the following, we set the integer $N \geq 4$. Besides, for $V = [\rho_\mu, v_\mu, \widetilde{E}, \widetilde{B}]$, we define the full instant energy functional $\mathcal{E}_N(V(t))$ and the corresponding dissipation rates $\mathcal{D}_N(V(t))$ by

$$\mathcal{E}_N(V(t)) \sim \left\| [\rho_\mu, v_\mu, \widetilde{E}, \widetilde{B}] \right\|_N^2 \tag{2.8}$$

and

$$\mathcal{D}_N(V(t)) \sim \|\rho_\mu\|_N^2 + \|\nabla v_\mu\|_N^2 + \left\|\nabla \widetilde{E}\right\|_{N-2}^2 + \left\|\nabla^2 \widetilde{B}\right\|_{N-3}^2. \tag{2.9}$$

Then, concerning the reformulated Cauchy problem (2.2)-(2.3), one obtain the following global existence result.

Proposition 2.1. *Assume that* $V_0 = [\rho_{\mu 0}, v_{\mu 0}, \widetilde{E}_0, \widetilde{B}_0]$ *satisfies* (2.4). *Then, there exist* $\mathcal{E}_N(\cdot)$ *and* $\mathcal{D}_N(\cdot)$ *such that the following holds true. If* $\mathcal{E}_N(V_0)$ *is small enough, the Cauchy problem* (2.2)–(2.3) *has a unique global nonzero solution* $V = [\rho_\mu, v_\mu, \widetilde{E}, \widetilde{B}]$ *satisfying*

$$V \in C([0, \infty); H^N(\mathbb{T})) \tag{2.10}$$

and

$$\mathcal{E}_N(V(t)) + \lambda \int_0^t \mathcal{D}_N(V(s))ds \leq \mathcal{E}_N(V_0) \tag{2.11}$$

for any $t \geq 0$.

Lastly, it is easy to see that Theorem 2.1 follows from Proposition 2.1. Thus, the rest of this paper is to prove the stated above Proposition.

§3. A priori estimates and the proof of global existence

3.1. A priori estimates

Theorem 3.1. *Assume* $N \geq 4$ *and* $V = [\rho_\mu, v_\mu, \widetilde{E}, \widetilde{B}] \in C([0,T); H^N(\mathbb{T}))$ *is smooth for* $T > 0$ *with*

$$\sup_{0 \leq t \leq T} \|\rho_\mu(t)\|_N \leq 1, \tag{3.1}$$

and suppose that V *solves the system* (2.2) *for* $t \in (0,T)$. *Then, there exist* $\mathcal{E}_N(\cdot)$ *and* $\mathcal{D}_N(\cdot)$ *in the form of* (2.8) *and* (2.9) *such that*

$$\frac{d}{dt}\mathcal{E}_N(V(t)) + \lambda \mathcal{D}_N(V(t)) \leq C\mathcal{E}_N(V(t))^{\frac{1}{2}} \mathcal{D}_N(V(t)) \tag{3.2}$$

for any $0 \leq t \leq T$.

Proof. It is divided by five steps as follows.
Step 1 It holds that

$$\frac{1}{2}\frac{d}{dt}\|V\|_N^2 + \|\nabla v_\mu\|_N^2 \leq C\|V\|_N \|[\rho_\mu, \nabla v_\mu]\|_N^2 . \tag{3.3}$$

In fact, from the first two equations of (2.2), energy estimates on $\partial^\alpha \rho_\mu$ and $\partial^\alpha v_\mu$ for $|\alpha| \leq N$ give

$$\begin{aligned} &\frac{1}{2}\frac{d}{dt}\left\|[\rho_\mu, v_\mu, \widetilde{E}, \widetilde{B}]\right\|_N^2 + \|\nabla v_\mu\|_N^2 \\ &= \sum_{|\alpha|\leq N}\Big(\langle \partial^\alpha h_{1\mu}, \partial^\alpha \rho_\mu\rangle + \langle \partial^\alpha h_{2\mu}, \partial^\alpha v_\mu\rangle + \left\langle \partial^\alpha h_{1\mu}, \partial^\alpha \widetilde{E}\right\rangle\Big). \end{aligned} \tag{3.4}$$

Notice

$$h_{1\mu} \sim \nabla\cdot(\rho_\mu v_\mu),\ h_3 \sim \rho_e v_e - \rho_i v_i, \tag{3.5}$$

$$h_{2\mu} \sim v_\mu \cdot \nabla v_\mu + \rho_\mu \nabla \rho_\mu + v_\mu \times \widetilde{B} + \rho_\mu \Delta v_\mu. \tag{3.6}$$

Then for the second term on the right hand side of (3.4), (3.6) implies that

$$\begin{aligned} &\langle \partial^\alpha h_{2\mu}, \partial^\alpha v_\mu\rangle \\ &\leq C\,|\langle \partial^\alpha(v_\mu\cdot\nabla v_\mu), \partial^\alpha v_\mu\rangle| + C\,|\langle \partial^\alpha(\rho_\mu\cdot\nabla\rho_\mu), \partial^\alpha v_\mu\rangle| \\ &\quad + C|\langle \partial^\alpha(v_\mu\times\widetilde{B}), \partial^\alpha v_\mu\rangle| + C\,|\langle \partial^\alpha(\rho_\mu\Delta v_\mu), \partial^\alpha v_\mu\rangle|\,. \end{aligned} \tag{3.7}$$

Let I_j denote the j−th ($j = 1, 2, 3, 4$) term on the right hand side of (3.7). When $|\alpha| = 0$, one has

$$I_1 \leq C\|v_\mu\|_{L^6}\|\nabla v_\mu\|\,\|v_\mu\|_{L^3} \leq C\|v_\mu\|_N \|\nabla v_\mu\|_N^2, \tag{3.8}$$

$$I_2 \leq C\|v_\mu\|_{L^\infty}\|\nabla\rho_\mu\|\,\|\rho_\mu\| \leq C\|v_\mu\|_N\|\rho_\mu\|_N^2, \tag{3.9}$$

$$I_3 = C\left|\langle v_\mu\times\widetilde{B}, v_\mu\rangle\right| = 0 \tag{3.10}$$

and

$$I_4 \leq C\,\|v_\mu\|_{L^\infty}\|\rho_\mu\|\,\|\Delta v_\mu\| \leq C\,\|v_\mu\|_N\left(\|\rho_\mu\|_N^2 + \|\nabla v_\mu\|_N^2\right). \tag{3.11}$$

When $|\alpha| \geq 1$, one has

$$I_1 \leq I_{11} + \sum_{\beta<\alpha} C_\beta^\alpha I_{12},\ I_2 \leq I_{21} + \sum_{\beta<\alpha} C_\beta^\alpha I_{22}, \tag{3.12}$$

$$I_3 \le I_{31} + \sum_{\beta<\alpha} C_\beta^\alpha I_{32}, \ I_4 \le I_{41} + \sum_{\beta<\alpha} C_\beta^\alpha I_{42}. \tag{3.13}$$

Where,

$$I_{11} = C\left|\langle v_\mu \partial^\alpha \nabla v_\mu, \partial^\alpha v_\mu\rangle\right| \le C\|v_\mu\|_N \|\nabla v_\mu\|_N^2, \tag{3.14}$$

$$I_{12} = C\left|\left\langle \partial^{\alpha-\beta} v_\mu \partial^\beta \nabla v_\mu, \partial^\alpha v_\mu\right\rangle\right| \le C\|v_\mu\|_N \|\nabla v_\mu\|_N^2, \tag{3.15}$$

$$\begin{aligned} I_{21} &= C\left|\langle \rho_\mu \partial^\alpha \nabla \rho_\mu, \partial^\alpha v_\mu\rangle\right| \\ &\le C\|v_\mu\|_N \|\rho_\mu\|_N^2 + C\|\rho_\mu\|_N \left(\|\rho_\mu\|_N^2 + \|\nabla v_\mu\|_N^2\right), \end{aligned} \tag{3.16}$$

$$I_{22} = C\left|\left\langle \partial^{\alpha-\beta} \rho_\mu \partial^\beta \nabla \rho_\mu, \partial^\alpha v_\mu\right\rangle\right| \le C\|v_\mu\|_N \|\rho_\mu\|_N^2, \tag{3.17}$$

$$I_{31} = C\left|\left\langle v_\mu \times \partial^\alpha \widetilde{B}, \partial^\alpha v_\mu\right\rangle\right| \le C\left\|\widetilde{B}\right\|_N \|\nabla v_\mu\|_N^2, \tag{3.18}$$

$$I_{32} = C\left|\left\langle \partial^{\alpha-\beta} v_\mu \times \partial^\beta \widetilde{B}, \partial^\alpha v_\mu\right\rangle\right| \le C\left\|\widetilde{B}\right\|_N \|\nabla v_\mu\|_N^2, \tag{3.19}$$

$$I_{41} = C\left|\langle \rho_\mu \partial^\alpha \Delta v_\mu, \partial^\alpha v_\mu\rangle\right| \le C\|\rho_\mu\|_N \|\nabla v_\mu\|_N^2, \tag{3.20}$$

$$I_{42} = C\left|\left\langle \partial^{\alpha-\beta} \rho_\mu \partial^\beta \Delta v_\mu, \partial^\alpha v_\mu\right\rangle\right| \le C\|\rho_\mu\|_N \|\nabla v_\mu\|_N^2. \tag{3.21}$$

Thus, (3.7)–(3.21) give that

$$\langle \partial^\alpha h_{2\mu}, \partial^\alpha v_\mu\rangle \le C\left\|[\rho_\mu, v_\mu, \widetilde{B}]\right\|_N \|[\rho_\mu, \nabla v_\mu]\|_N^2. \tag{3.22}$$

From (3.5) similar argument give that

$$\langle \partial^\alpha h_{1\mu}, \partial^\alpha \rho_\mu\rangle \le C\|[\rho_\mu, v_\mu]\|_N \|[\rho_\mu, \nabla v_\mu]\|_N^2, \tag{3.23}$$

$$\left\langle \partial^\alpha h_3, \partial^\alpha \widetilde{E}\right\rangle \le C\left\|\widetilde{E}\right\|_N \|[\rho_\mu, \nabla v_\mu]\|_N^2. \tag{3.24}$$

Hence, (3.4) together with (3.22)–(3.24) yield (3.3).
Step 2 It holds that

$$\begin{aligned} &\frac{d}{dt} \sum_{|\alpha|\le N-1} \left(\langle \partial^\alpha v_e, \partial^\alpha \nabla \rho_e)\rangle + \langle \partial^\alpha v_i, \partial^\alpha \nabla \rho_i\rangle\right) + \lambda \|\rho_\mu\|_N^2 \\ &\le C\|\nabla v_\mu\|_N^2 + C\left\|[\rho_\mu, v_\mu, \widetilde{B}]\right\|_N \|[\rho_\mu, \nabla v_\mu]\|_N^2. \end{aligned} \tag{3.25}$$

In fact, Let $|\alpha| \leq N-1$. Applying ∂^α to the second equation of (2.2), multiplying it by $\partial^\alpha(\gamma\nabla\rho_\mu)$, taking integration in x and then using integration by parts and also the final equation of (2.2) gives

$$\begin{aligned} &\frac{d}{dt}\sum_{\mu=e,i}\langle\partial^\alpha(\gamma\nabla\rho_\mu),\partial^\alpha v_\mu\rangle+\lambda\left(\|\partial^\alpha[\rho_e,\rho_i]\|^2+\|\partial^\alpha\nabla[\rho_e,\rho_i]\|^2\right)\\ &\leq\|[\nabla v_e,\nabla v_i]\|_N^2+\langle\partial^\alpha(\gamma\nabla h_{1e}),\partial^\alpha v_e\rangle+\langle\partial^\alpha(\gamma\nabla h_{1i}),\partial^\alpha v_i\rangle\\ &\quad+\langle\partial^\alpha h_{2e},\partial^\alpha(\gamma\nabla\rho_e)\rangle+\langle\partial^\alpha h_{2i},\partial^\alpha(\gamma\nabla\rho_i)\rangle\ , \end{aligned}\tag{3.26}$$

where, Poincare inequality and Cauchy–Schwarz inequality were also used used. Similar argument as estimate for $\langle\partial^\alpha h_{2\mu},\partial^\alpha v_\mu\rangle$ in step 1, we get (3.25).

Step 3 It holds that

$$\begin{aligned} &\frac{d}{dt}\sum_{|\alpha|\leq N-2}\left\langle\partial^\alpha\nabla\times\widetilde{E},\partial^\alpha\nabla\times(v_e-v_i)\right\rangle+\lambda\left\|\nabla\widetilde{E}\right\|_{N-2}^2\\ &\qquad\leq C\left\|[\widetilde{E},\widetilde{B}]\right\|_N\left(\|[\rho_\mu,\nabla v_\mu]\|_N^2+\left\|\nabla\widetilde{E}\right\|_{N-2}^2\right).\\ &\qquad\quad+\varepsilon\left\|\nabla^2\widetilde{B}\right\|_{N-3}^2+C_\varepsilon\|\nabla v_\mu\|_N^2 \end{aligned}\tag{3.27}$$

In fact, for $|\alpha| \leq N-2$, applying ∂^α to the second equation of (2.2), multiplying it by $\partial^\alpha\nabla\times\widetilde{E}$, taking integration in x and then using integration by parts and also the third equation of (2.2) gives (3.27). Where we also used

$$\left\|\partial^\alpha\partial_i\widetilde{E}\right\|=\left\|\partial_i\Delta^{-1}\nabla\times(\nabla\times\partial^\alpha\widetilde{E})\right\|\leq C\left\|\nabla\times\partial^\alpha\widetilde{E}\right\|\tag{3.28}$$

for each $1 \leq i \leq 3$, due to the fact that $\partial_i\Delta^{-1}\nabla$ is bounded from L^p to L^p for $1 < p < \infty$; see [5].

Step 4 It holds that

$$\begin{aligned} &\frac{d}{dt}\sum_{1\leq|\alpha|\leq N-2}\left\langle\partial^\alpha(-\nabla\times\widetilde{B}),\partial^\alpha\widetilde{E}\right\rangle+\lambda\left\|\nabla^2\widetilde{B}\right\|_{N-3}^2\\ &\leq C\left\|\nabla\widetilde{E}\right\|_{N-2}^2+C\|\nabla v_\mu\|_N^2+C\left\|\widetilde{B}\right\|_N\|[\rho_\mu,\nabla v_\mu]\|_N^2\,. \end{aligned}\tag{3.29}$$

In fact, for $1 \leq |\alpha| \leq N-2$, applying ∂^α to the third equation of (2.2), multiplying it by $\partial^\alpha(-\nabla\times\widetilde{B})$, taking integration in x and then using the fourth equation of (2.2) gives (3.29) by further using Cauchy–Schwarz inequality and taking summation over $|\alpha| \leq N-2$, where we also used (3.28) for $\widetilde{B}$.

Step 5 Now following four steps above, we are ready to prove (3.2). In fact, let us define

$$
\begin{aligned}
\mathcal{E}_N(V(t)) = {} & \left\|[\rho_\mu, v_\mu, \widetilde{E}, \widetilde{B}]\right\|_N^2 \\
& + \mathcal{K}_1 \sum_{|\alpha|\le N-1} \left(\langle \partial^\alpha(\gamma\nabla\rho_e), \partial^\alpha\nabla v_e\rangle + \langle \partial^\alpha(\gamma\nabla\rho_i), \partial^\alpha\nabla v_i\rangle\right) \\
& + \mathcal{K}_1 \sum_{|\alpha|\le N-2} \left\langle \partial^\alpha\nabla\times\widetilde{E}, \partial^\alpha\nabla\times(v_e - v_i)\right\rangle \\
& + \mathcal{K}_2 \sum_{1\le|\alpha|\le N-2} \left\langle \partial^\alpha(-\nabla\times\widetilde{B}), \partial^\alpha\widetilde{E}\right\rangle
\end{aligned}
\tag{3.30}
$$

for constants $0 < \mathcal{K}_2 \ll \mathcal{K}_1 \ll 1$ to be determined. Notice that as soon as $0 < \mathcal{K}_i \ll 1$ is sufficiently small for $i = 1, 2$, then $\mathcal{E}_N(V(t)) \sim ||V||_N^2$ holds true. Moreover, by letting $0 < \mathcal{K}_2 \ll \mathcal{K}_1 \ll 1$ be sufficiently small with $\varepsilon\mathcal{K}_1 < \mathcal{K}_2$, the sum of (3.3), (3.25)$\times\mathcal{K}_1$, (3.27)$\times\mathcal{K}_1$ and (3.29)$\times\mathcal{K}_2$ gives that there exists $\lambda > 0$, $C > 0$ such that (3.2) also holds true with $\mathcal{D}_N(\cdot)$ defined in (2.9). This completes the proof of the Theorem 3.1. Q.E.D.

3.2. The proof of global existence.

Proof of Proposition 2.1. The global existence of smooth solutions follows from the standard argument by using the local existence result, the a priori estimate (3.2) given in Theorem 3.1 and the continuous extension argument, see [3]. This completes the proof of the Proposition 2.1. Q.E.D.

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