Perspectives in Mathematical Logic

Peter G. Hinman

Recursion-Theoretic Hierarchies

Springer-Verlag
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Ω-Group:
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G. E. Sacks D. S. Scott

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For *M* and *m*

who make life fun

Preface to the Series

On Perspectives. *Mathematical logic arose from a concern with the nature and the limits of rational or mathematical thought, and from a desire to systematise the modes of its expression. The pioneering investigations were diverse and largely autonomous. As time passed, and more particularly in the last two decades, interconnections between different lines of research and links with other branches of mathematics proliferated. The subject is now both rich and varied. It is the aim of the series to provide, as it were, maps or guides to this complex terrain. We shall not aim at encyclopaedic coverage; nor do we wish to prescribe, like Euclid, a definitive version of the elements of the subject. We are not committed to any particular philosophical programme. Nevertheless we have tried by critical discussion to ensure that each book represents a coherent line of thought; and that, by developing certain themes, it will be of greater interest than a mere assemblage of results and techniques.*

The books in the series differ in level: some are introductory some highly specialised. They also differ in scope: some offer a wide view of an area, others present a single line of thought. Each book is, at its own level, reasonably self-contained. Although no book depends on another as prerequisite, we have encouraged authors to fit their book in with other planned volumes, sometimes deliberately seeking coverage of the same material from different points of view. We have tried to attain a reasonable degree of uniformity of notation and arrangement. However, the books in the series are written by individual authors, not by the group. Plans for books are discussed and argued about at length. Later, encouragement is given and revisions suggested. But it is the authors who do the work; if, as we hope, the series proves of value, the credit will be theirs.

History of the Ω-Group. *During 1968 the idea of an integrated series of monographs on mathematical logic was first mooted. Various discussions led to a meeting at Oberwolfach in the spring of 1969. Here the founding members of the group (R. O. Gandy, A. Levy, G. H. Müller, G. Sacks, D. S. Scott) discussed the project in earnest and decided to go ahead with it. Professor F. K. Schmidt and Professor Hans Hermes gave us encouragement and support. Later Hans Hermes joined the group. To begin with all was fluid. How ambitious should we be? Should* *we write the books ourselves? How long would it take? Plans for authorless books were promoted, savaged and scrapped. Gradually there emerged a form and a method. At the end of an infinite discussion we found our name, and that of the series. We established our centre in Heidelberg. We agreed to meet twice a year together with authors, consultants and assistants, generally in Oberwolfach. We soon found the value of collaboration: on the one hand the permanence of the founding group gave coherence to the over-all plans; on the other hand the stimulus of new contributors kept the project alive and flexible. Above all, we found how intensive discussion could modify the authors' ideas and our own. Often the battle ended with a detailed plan for a better book which the author was keen to write and which would indeed contribute a perspective.*

Acknowledgements. *The confidence and support of Professor Martin Barner of the Mathematisches Forschungsinstitut at Oberwolfach and of Dr. Klaus Peters of Springer-Verlag made possible the first meeting and the preparation of a provisional plan. Encouraged by the Deutsche Forschungsgemeinschaft and the Heidelberger Akademie der Wissenschaften we submitted this plan to the Stiftung Volkswagenwerk where Dipl. Ing. Penschuck vetted our proposal; after careful investigation he became our adviser and advocate. We thank the Stiftung Volkswagenwerk for a generous grant (1970–73) which made our existence and our meetings possible.*

Since 1974 the work of the group has been supported by funds from the Heidelberg Academy; this was made possible by a special grant from the Kultusministerium von Baden-Württemberg (where Regierungsdirektor R. Goll was our counsellor). The success of the negotiations for this was largely due to the enthusiastic support of the former President of the Academy, Professor Wilhelm Doerr. We thank all those concerned.

Finally we thank the Oberwolfach Institute, which provides just the right atmosphere for our meetings, Drs. Ulrich Felgner and Klaus Gloede for all their help, and our indefatigable secretary Elfriede Ihrig.

Oberwolfach
September 1975

R. O. Gandy	*H. Hermes*
A. Levy	*G. H. Müller*
G. Sacks	*D. S. Scott*

Author's Preface

At a recent meeting of logicians, one speaker complained — mainly, but perhaps not wholly, in jest — that logic is tightly controlled by a small group of people (the cabal) who exercise careful control over the release of new ideas to the general public (especially students) and indeed suppress some material completely. The situation is surely not so grim as this, but any potential reader of this book must have felt at some time that there is at least a minor conspiracy to keep new ideas inaccessible until the "insiders" have worked them over thoroughly. In particular he might well feel this way about the whole subject of Generalized Recursion Theory, which developed in the second half of the 1960s. The basic definitions and results on recursion involving functionals of higher type appeared in the monumental but extremely difficult paper Kleene [1959] and [1963]. Gandy [1967] gave another presentation *ab initio*, but the planned part II of this paper, as well as several other major advances in the subject, never appeared in print. For the theory of recursion on ordinals, the situation was even worse. Much of the basic material had appeared only in the *abstracts* Kripke [1964, 1964a], and although certain parts of the theory had been worked out in papers such as Kreisel-Sacks [1965] and Sacks [1967], there was no reasonably complete account of the basic facts of the subject in print.

When I first contemplated doing something about this situation in the spring of 1971, I planned to write a short monograph on recursion relative to type-2 functionals with enough background on ordinary Recursion Theory to show how the theories fit together. Before I had done much about it, however, the invitation of the Ω-Group to write a volume for this series stimulated me to think in more ambitious terms and my plan expanded gradually to include functionals of types 3 and higher, ordinal recursion, and a more thorough presentation of the material on definability (Chapters III–V). The constant encouragement of the Ω-Group, collective and individual, was essential to the completion of the task.

The original plan arose from a course I gave at the University of Michigan in the Fall Term of 1970. Thanks to Jens-Erik Fenstad and the University of Oslo I had the opportunity to lecture on much of the material during the academic year 1971–1972. Other occasions to lecture on parts of the material were provided by the University of Michigan in 1972–73 and the Winter Term of 1975, the Warsaw

Logic Semester in May, 1973, and the Michigan–Ohio Logic Seminar. The majority of the actual writing was done in the summers of 1973–75 under grants from the National Science Foundation.

Of my many teachers, formal and informal, who have personally helped me to form my conception of this subject, I want especially to mention John Addison, Jens-Erik Fenstad, Robin Gandy, Yiannis Moschovakis, and Joe Shoenfield. Andreas Blass read much of the first draft and made many helpful comments. Mm. Bocuse and Haberlin provided inspiring models of excellence. The boldness of the section and subsection headings in the first third of the book is due to the careful work of Monica Scott and her brown crayon. Barbara Perkel did a superb job of typing. Finally, the person to whom the reader should be most grateful is Anne Zalc. In reading carefully the entire final draft she caught hundreds of errors, serious and minor. More importantly, she was an unrelenting enemy of that peculiar brand of obfuscation which results from an author's implicit assumption that the reader has perfectly understood and remembered every detail of what has preceded any given point. Without her the book would be a denser jungle.

January 30, 1978 Peter G. Hinman
Ann Arbor

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