Perspectives in Mathematical Logic

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Computability in Analysis and Physics

Springer-Verlag

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Ω-Group:
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Preface to the Series

Perspectives in Mathematical Logic

(Edited by the Ω-group for "Mathematische Logik" of the Heidelberger Akademie der Wissenschaften)

On Perspectives. *Mathematical logic arose from a concern with the nature and the limits of rational or mathematical thought, and from a desire to systematise the modes of its expression. The pioneering investigations were diverse and largely autonomous. As time passed, and more particularly in the last two decades, interconnections between different lines of research and links with other branches of mathematics proliferated. The subject is now both rich and varied. It is the aim of the series to provide, as it were, maps of guides to this complex terrain. We shall not aim at encyclopaedic coverage; nor do we wish to prescribe, like Euclid, a definitive version of the elements of the subject. We are not committed to any particular philosophical programme. Nevertheless we have tried by critical discussion to ensure that each book represents a coherent line of thought; and that, by developing certain* themes, *it will be of greater interest than a mere assemblage of results and techniques.*

The books in the series differ in level: some are introductory, some highly specialised. They also differ in scope: some offer a wide view of an area, others present a single line of thought. Each book is, at its own level, reasonably self-contained. Although no book depends on another as prerequisite, we have encouraged authors to fit their book in with other planned volumes, sometimes deliberately seeking coverage of the same material from different points of view. We have tried to attain a reasonable degree of uniformity of notation and arrangement. However, the books in the series are written by individual authors, not by the group. Plans for books are discussed and argued about at length. Later, encouragement is given and revisions suggested. But it is the authors who do the work; if, as we hope, the series proves of value, the credit will be theirs.

History of the Ω-Group. *During 1968 the idea of an integrated series of monographs on mathematical logic was first mooted. Various discussions led to a meeting at Oberwolfach in the spring of 1969. Here the founding members of the group (R.O. Gandy, A. Levy, G.H. Müller, G. Sacks, D.S. Scott)* *discussed the project in earnest and decided to go ahead with it. Professor F.K. Schmidt and Professor Hans Hermes gave us encouragement and support. Later Hans Hermes joined the group. To begin with all was fluid. How ambitious should we be? Should we write the books ourselves? How long would it take? Plans for authorless books were promoted, savaged and scrapped. Gradually there emerged a form and a method. At the end of an infinite discussion we found our name, and that of the series. We established our centre in Heidelberg. We agreed to meet twice a year together with authors, consultants and assistants, generally in Oberwolfach. We soon found the value of collaboration: on the one hand the permanence of the founding group gave coherence to the overall plans; on the other hand the stimulus of new contributors kept the project alive and flexible. Above all, we found how intensive discussion could modify the authors' ideas and our own. Often the battle ended with a detailed plan for a better book which the author was keen to write and which would indeed contribute a perspective.*

Oberwolfach, September 1975

Acknowledgements. *In starting our enterprise we essentially were relying on the personal confidence and understanding of Professor Martin Barner of the Mathematisches Forschungsinstitut Oberwolfach, Dr. Klaus Peters of Springer-Verlag and Dipl.-Ing. Penschuck of the Stiftung Volkswagenwerk. Through the Stiftung Volkswagenwerk we received a generous grant (1970–1973) as an initial help which made our existence as a working group possible.*

Since 1974 the Heidelberger Akademie der Wissenschaften (Mathematisch-Naturwissenschaftliche Klasse) has incorporated our enterprise into its general scientific program. The initiative for this step was taken by the late Professor F.K. Schmidt, and the former President of the Academy, Professor W. Doerr.

Through all the years, the Academy has supported our research project, especially our meetings and the continuous work on the Logic Bibliography, in an outstandingly generous way. We could always rely on their readiness to provide help wherever it was needed.

Assistance in many various respects was provided by Drs. U. Felgner and K. Gloede (till 1975) and Drs. D. Schmidt and H. Zeitler (till 1979). Last but not least, our indefatigable secretary Elfriede Ihrig was and is essential in running our enterprise.

We thank all those concerned.

Heidelberg, September 1982

R.O. Gandy *H. Hermes*
A. Levy *G.H. Müller*
G. Sacks *D.S. Scott*

Authors' Preface

This book is concerned with the computability or noncomputability of standard processes in analysis and physics.

Part I is introductory. It provides the basic prerequisites for reading research papers in computable analysis (at least when the reasoning is classical). The core of the book is contained in Parts II and III. Great care has been taken to motivate the material adequately. In this regard, the introduction to the book and the introductions to the chapters play a major role. We suggest that the reader who wishes to skim begin with the introduction to the book and then read the introductions to Chapters 2, 3, and 4—with a brief glance at the introductions to Chapters 0 and 1.

The book is written for a mixed audience. Although it is intended primarily for logicians and analysts, it should be of interest to physicists and computer scientists—in fact to graduate students in any of these fields. The work is self-contained. Beyond the section on Prerequisites in Logic and Analysis, no further background is necessary. The reasoning used is classical—i.e. in the tradition of classical mathematics. Thus it is not intuitionist or constructivist in the sense of Brouwer or Bishop.

It is a pleasure to thank our mathematical colleagues for stimulating conversations as well as helpful comments and suggestions: Professors Oliver Aberth, John Baldwin, William Craig, Solomon Feferman, Alexander Kechris, Manuel Lerman, Saunders MacLane, Yiannis Moschovakis, Jack Silver, Dana Scott and Pat Suppes. Thanks are also due to Professor Yoshi Oono of the Department of Physics of the University of Illinois in Urbana, who, together with some of his colleagues, went through the manuscript in a seminar and commented extensively on questions of interest to physicists.

The Mathematical Sciences Research Institute at Berkeley provided hospitality and support to one of the authors (M.B.P.) during the fall of 1985.

Kate Houser did a superb job of typing the manuscript.

Finally, we wish to thank Professor Gert Müller and Springer-Verlag for their help above and beyond the call of duty.

We hope that this short monograph will provide an easy entrance into research in this area.

January 1988
Minneapolis, Minnesota

Marian Boykan Pour-El
Ian Richards

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Major Interconnections