Notation

Chapter 1

$\mathfrak{A} = \langle A, C; 0, 1\rangle$	computation domain 19
C	code set 19
σ, τ	sequences of elements from A 20
$\mathrm{lh}(\sigma)$	length of σ 20
Θ	computation set 20
$\{a\}^n_\theta$	Θ-computable function with code a 20, 21
$f(\sigma) \simeq z$	the computation tuple $(\hat{f}, \sigma, z) \in \Theta$ 21
φ	functional 21
DC	definition by cases 21
$\mathbf{C}^n$	composition 22
$\mathbf{P}^m_{n,j}$	permutation 22
S^n_m	the s-n-m mapping 22
$f(\sigma)\downarrow$	$f(\sigma)$ is defined 26
$\mathfrak{A} = \langle A, C, N; s, M, K, L\rangle$	computation domain 30
$\langle M, K, L\rangle$	pairing structure 30
$\langle x_1, \ldots, x_n\rangle$	ordered n-tuple 31
$\Gamma_{\mathbf{f}}$	inductive operator 31
$\mathbf{PR}[\mathbf{f}]$	prime computation set in **f** 32
$\lvert a, \sigma, z\rvert_{\mathrm{PR}[\mathbf{f}]}$	length of computation 32
$\Theta \leqslant H$	extension of precomputation theories 34
$\Theta \sim H$	equivalence of theories 34

Chapter 2

$\langle \Theta, <_\Theta\rangle$	computation structure 44
$\mathbf{S}_{(a,\sigma,z)}$	set of subcomputations 44
$\lvert a, \sigma, z\rvert_\Theta$	length of computation 44
$\Gamma_{f,\varphi}$	inductive operator 45
$\mathbf{PR}[f, \varphi]$	prime computation set in f, φ 46
$\lvert a, \sigma, z\rvert_{\mathrm{PR}[f,\varphi]}$	length of computation 46
$<_{\mathrm{PR}[f,\varphi]}$	subcomputation relation 47
$H[\mathbf{f}, \boldsymbol{\varphi}]$	theory generated from H and **f**, $\boldsymbol{\varphi}$ 47
$\langle \Theta, <_\Theta\rangle \leqslant \langle H, <_H\rangle$	extension of computation theories 48
$\langle \Theta, <_\Theta\rangle \sim \langle H, <_H\rangle$	equivalence of theories 48
$\mathbf{E}_B$	functional defining Θ-finiteness 52
WDC	weak definition by cases 59

Chapter 3

$\lvert x\rvert_\Theta$	length of computation $x \in \Theta$ 65
$\mathbf{E}'_S$	functional defining weak Θ-finiteness 66
$\mathbf{Q}$	monotone quantifier 67
$\mathbf{F}^{\#}_{\mathbf{Q}}$	functional associated to quantifier 67
sc*(Θ)	extended section 73
sc(Θ)	section 73
en*(Θ)	extended envelope 73
en(Θ)	envelope 73
$\Gamma\colon 2^A \to 2^A$	inductive operator 79
Γ_α	stage of inductive operator 79
Γ_∞	fixed-point of inductive operator 79
$\lvert\Gamma\rvert$	ordinal of inductive operator 79
IND(**C**)	the **C**-inductive relations 80
$\lvert\mathbf{C}\rvert$	the ordinal of the class **C** 80
HYP(**C**)	the **C**-hyperdefinable relations 80

Chapter 4

$\mathfrak{A} = \langle A, S, \mathbf{S}\rangle$	computation domain on two types 90

$\mathbf{S} = \langle N, s, M, K, L\rangle$ coding scheme 90
$\mathrm{Tp}(S)$ the set ω^S 90
PRF primitive recursive functions 91
PR(L) partial recursive in L 93
$|e, \sigma, x|_{\mathbf{L}}$ length of computation 94
$\kappa_{\mathbf{L}}$ ordinal of computations in L 94
$\langle e, \sigma\rangle\downarrow$ the computation $\{e\}_{\mathbf{L}}(\sigma)$ is defined 94
$\langle e, \sigma\rangle\uparrow$ the computation $\{e\}_{\mathbf{L}}(\sigma)$ is undefined 94
$\mathbf{C}_{\mathbf{L}}$ coded set of L-computations 97

Chapter 5

$(\mathfrak{A}, \preccurlyeq)$ computation domain with prewellordering 111
$|\preccurlyeq|$ ordinal of pwo $\preccurlyeq$ 111
$\mathbf{E}^{\preccurlyeq}$ functional computing the pwo $\preccurlyeq$ 111
$\|\Theta\|$ ordinal of the theory Θ 111
$\Theta^{|w|}$ Θ-computations of length $|w|$ 111
$\mathbf{E}^{1/2}$ functional associated with a selection operator 112
$\mathbf{L}(\mathfrak{A}, \preccurlyeq, \mathbf{R})$ first-order language associated with $\mathfrak{A}, \preccurlyeq, \mathbf{R}$ 113
$\Delta_0(\preccurlyeq, \mathbf{R})$, $\Sigma_1(\preccurlyeq, \mathbf{R})$, $\Pi_1(\preccurlyeq, \mathbf{R})$, $\Delta_1(\preccurlyeq, \mathbf{R})$ classification of relations with respect to the language $\mathbf{L}(\mathfrak{A}, \preccurlyeq, \mathbf{R})$ 113
$\Gamma_\theta(X)$ inductive operator induced by the formula $\theta(\sigma, X)$ 113
Γ_∞ fixed-point of Γ_θ 113
$|\Gamma_\theta|$ ordinal of Γ_θ 113
$\mathrm{PR}[[\preccurlyeq, \mathbf{R}]]$ computation theory generated by R over $(\mathfrak{A}, \preccurlyeq)$ 119
$R(x, \alpha)$, $R_\Theta(x, \alpha)$ "universal" relation in Θ 125, 129
α_Θ, α_F ordinal of Θ, PR[F] 127
$\Psi <_1 \Theta$ extension of Spector theories 127
Θ_τ Θ-computations of length $<\tau$ 129
$m(\mathrm{sc}(\Theta))$ abstract 1-section associated to Θ 135
$\Vdash$ forcing relation 137

Chapter 6

$\lambda x \cdot W^x$ $\preccurlyeq$-enumeration 140
$\lambda a x . W_a^x$ $\preccurlyeq$-parametrization 141
L^β elements of ordinal $<\beta$ in the pwo $\preccurlyeq$ 143
$|\preccurlyeq|^*$ projectum of Θ 143
$|\preccurlyeq|^+$ r.e.-projectum of Θ 144
$\lambda z \cdot K_z$ enumeration of Θ-finite sets 144
$\leqslant_w$ weakly Θ-computable in 144
$\leqslant$ Θ-computable in 144
$\equiv$ equivalence relation derived from $\leqslant$ 145
$\leqslant_d$ Θ-definable in 145
$B^w \prec B^z$ every element of B^w is $\prec$-less than every element of B^z 148
$\leqslant_m$ many-one reducible 149
B' jump of B 149
$\deg(A)$ degree of A 150
$\mathbf{a}$ degree 150
$\Sigma_0, \Pi_0, \Sigma_n, \Pi_n$ definability classification 151
$\lim_\alpha f$ limit of f 151
$\Sigma_2\text{-cf}(\alpha)$ Σ_2-cofinality of α 152
$\Sigma_1\text{-cf}(\beta)$ Σ_1-cofinality of β 160
$\mathfrak{A}_\beta$ the admissible collapse of L_β 163

Chapter 7

$\mathrm{Ord}(X)$ ordinals of Θ-computations in X 168
κ^X the ordinal of Θ-computations in X 168
λ^X ordertype of $\mathrm{Ord}(X)$ 168
$\mathrm{sc}(\Theta)$ Θ-computable subsets of A 171
$\mathrm{sc}(\Theta, \sigma)$ subsets of A Θ-computable in σ 171
$\mathrm{en}(\Theta)$ Θ-semicomputable subsets of A 171
$S\text{-en}(\Theta)$ Θ-semicomputable subsets of S 171
$\Psi <_1 \Theta$ extension of theories on two types 180

Chapter 8

$\{e\}_R(\sigma)$ set-recursive computation relative to R 183
Θ_R the set-recursive computation theory 183
$\|a, \sigma, z\|$ length function in Θ_R 183
$M(A; R)$ set-recursive closure of A relative to R 185
^fA finite subsets of A 185