Perspectives in Mathematical Logic

Jens E. Fenstad

General Recursion Theory

An Axiomatic Approach

Springer-Verlag
Berlin Heidelberg New York

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Ω-Group:
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Preface to the Series

On Perspectives. *Mathematical logic arose from a concern with the nature and the limits of rational or mathematical thought, and from a desire to systematise the modes of its expression. The pioneering investigations were diverse and largely autonomous. As time passed, and more particularly in the last two decades, interconnections between different lines of research and links with other branches of mathematics proliferated. The subject is now both rich and varied. It is the aim of the series to provide, as it were, maps or guides to this complex terrain. We shall not aim at encyclopaedic coverage; nor do we wish to prescribe, like Euclid, a definitive version of the elements of the subject. We are not committed to any particular philosophical programme. Nevertheless we have tried by critical discussion to ensure that each book represents a coherent line of thought; and that, by developing certain* themes, *it will be of greater interest than a mere assemblage of results and techniques.*

The books in the series differ in level: some are introductory some highly specialised. They also differ in scope: some offer a wide view of an area, others present a single line of thought. Each book is, at its own level, reasonably self-contained. Although no book depends on another as prerequisite, we have encouraged authors to fit their book in with other planned volumes, sometimes deliberately seeking coverage of the same material from different points of view. We have tried to attain a reasonable degree of uniformity of notation and arrangement. However, the books in the series are written by individual authors, not by the group. Plans for books are discussed and argued about at length. Later, encouragement is given and revisions suggested. But it is the authors who do the work; if, as we hope, the series proves of value, the credit will be theirs.

History of the Ω-Group. *During 1968 the idea of an integrated series of monographs on mathematical logic was first mooted. Various discussions led to a meeting at Oberwolfach in the spring of 1969. Here the founding members of the group (R. O. Gandy, A. Levy, G. H. Müller, G. E. Sacks, D. S. Scott) discussed the project in earnest and decided to go ahead with it. Professor F. K. Schmidt and Professor Hans Hermes gave us encouragement and support. Later Hans Hermes joined the group. To begin with all was fluid. How ambitious should we be? Should we write the books ourselves? How long would it take? Plans for authorless books were promoted, savaged and scrapped. Gradually there emerged a form and a method. At the end of* *an infinite discussion we found our name, and that of the series. We established our centre in Heidelberg. We agreed to meet twice a year together with authors, consultants and assistants, generally in Oberwolfach. We soon found the value of collaboration: on the one hand the permanence of the founding group gave coherence to the over-all plans; on the other hand the stimulus of new contributors kept the project alive and flexible. Above all, we found how intensive discussion could modify the authors' ideas and our own. Often the battle ended with a detailed plan for a better book which the author was keen to write and which would indeed contribute a perspective.*

Acknowledgements. *The confidence and support of Professor Martin Barner of the Mathematisches Forschungsinstitut at Oberwolfach and of Dr. Klaus Peters of Springer-Verlag made possible the first meeting and the preparation of a provisional plan. Encouraged by the Deutsche Forschungsgemeinschaft and the Heidelberger Akademie der Wissenschaften we submitted this plan to the Stiftung Volkswagenwerk where Dipl. Ing. Penschuck vetted our proposal; after careful investigation he became our adviser and advocate. We thank the Stiftung Volkswagenwerk for a generous grant (1970–73) which made our existence and our meetings possible.*

Since 1974 the work of the group has been supported by funds from the Heidelberg Academy; this was made possible by a special grant from the Kultusministerium von Baden-Württemberg (where Regierungsdirektor R. Goll was our counsellor). The success of the negotiations for this was largely due to the enthusiastic support of the former President of the Academy, Professor Wilhelm Doerr. We thank all those concerned.

Finally we thank the Oberwolfach Institute, which provides just the right atmosphere for our meetings, Drs. Ulrich Felgner and Klaus Glöde for all their help, and our indefatigable secretary Elfriede Ihrig.

Oberwolfach
September 1975

R. O. Gandy *H. Hermes*
A. Levy *G. H. Müller*
G. E. Sacks *D. S. Scott*

Author's Preface

This book has developed over a number of years. The aim has been to give a unified and coherent account of the many and various parts of general recursion theory.

I have not worked alone. The Recursion Theory Seminar in Oslo has for a number of years been a meeting place for an active group of younger people. Their work and enthusiasm have been an important part of the present project. I am happy to acknowledge my debts to Johan Moldestad, Dag Normann, Viggo Stoltenberg-Hansen and John Tucker. It has been a great joy for me to work together with them.

But there are other debts to acknowledge. The Oslo group learned much of their recursion theory from Peter Hinman who spent the year 1971–72 in Oslo and Peter Aczel who was here for part of the year 1973. Robin Gandy, Yiannis Moschovakis and Gerald Sacks have also in many ways helped to shape our ideas about general recursion theory.

I appreciate the invitation from the Ω-group to write this book for their series. In particular, I would like to thank Gert Müller for his friendship and helpfulness.

Finally, I would like to thank Randi Møller for her expert typing of the manuscript, David Kierstead for his valuable help with the proof-reading, and the printers and publishers for the excellent production of this volume.

Advice to the reader. The book is in principle self-contained, but we expect that any reader would have had some previous exposure to recursion theory, e.g. Rogers [136]. The books by Barwise [11], Hinman [61] and Moschovakis [115, 118] supplement our account, often from a different perspective.

The exposition is organized around a main core which is reasonably complete. This is the general theory of computations. But the main core flowers into a number of side branches intended to show how computation theories connect with and unify other parts of general recursion theory. Here we are less complete and proofs may sometimes be more in the nature of a comment on the basic ideas involved. In the main body of the text and in an Epilogue we have tried to point beyond to open problems and areas of further research.

Oslo, October 1979 Jens Erik Fenstad

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