

Proceedings of the
**CENTRE FOR
MATHEMATICAL ANALYSIS**
AUSTRALIAN NATIONAL UNIVERSITY

Volume 8, 1984

**Miniconference on
NONLINEAR ANALYSIS
(Canberra, July 5-7, 1984)**

Edited by
Neil S. Trudinger and Graham Williams

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NEIL S. TRUDINGER AND GRAHAM H. WILLIAMS

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These proceedings are dedicated to Dr. Jim Michael on the occasion of his retirement from the University of Adelaide.

On December 31, 1983 Dr. J.H. Michael retired from his academic duties at the University of Adelaide. He has had a great deal of influence on the course of Mathematics, and in particular Analysis, in Australia for quite some time. Not only has he contributed significant works of his own but has also supervised several graduate students who have gone on to successful mathematical careers. Four of these former students (A.U. Kennington, L.M. Simon, J. van der Hoek and G.H. Williams) also presented talks at this miniconference and many of the other speakers have had contact with Michael's work.

Jim Michael completed his Ph.D. at the University of Adelaide in 1956 under the supervision of G. Szekeres on a topic concerning Cauchy's Integral Theorem and its applications. Prior to this he had attained his Bachelors degree which he partly completed while soldiering in the Australian Army. (In fact one examination was undertaken on a troop ship fending off bouts of seasickness!). He then spent some time at the Universities of Manchester and Glasgow before returning to a lecturing position at Adelaide in 1958 where he remained (as Lecturer, Senior Lecturer, Reader, Professor and Reader) apart from study tours until his retirement. In addition to the work on Cauchy's Integral theorem he also made significant contributions to problems in the invariance of domains, approximation of general surfaces by Lipschitz graphs and more recently in the field of partial differential equations. He was elected as a Fellow of the Australian Academy of Science in 1973.

Jim Michael's work, while perhaps not large in volume, has always been very thorough and in several cases has presented new ideas which have turned out to be very significant in the later development of the theory. These include in particular his study of Lipschitz approximations of variational integrals [11], his fundamental paper with Leon Simon on Sobolev inequalities on submanifolds [20] and his innovative approach to elliptic equations through interior estimates [15]. Although he has retired from academic duties he has not retired from research as can be seen from the paper presented by him at this conference.

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20. Jointly with L.M. Simon
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Jointly with William P. Ziemer

21. A Lusin type approximation of Sobolev functions by smooth functions - To appear.
22. Interior regularity for solutions to obstacle problems - Submitted for publication.

PREFACE

This volume contains the proceedings of a miniconference on nonlinear analysis held at the Australian National University in July, 1984, at the Centre for Mathematical Analysis. It is divided into two parts. The first consists of reports of expository lectures. The second is devoted to research reports communicated at the conference.

We gratefully acknowledge the support of the contributors to this volume as well as the excellent typing assistance of Marilyn, Dorothy and Norma. We would especially also like to thank Jim Michael for his participation and his agreement to have these proceedings dedicated to him.

Neil S. Trudinger, Graham H. Williams
(Editors)

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