PREFACE

These notes originated from a series of lectures I delivered at the Centre for Mathematical Analysis at Canberra. The purpose of the lectures was to introduce mathematicians familiar with the basic notions and results of linear elliptic partial differential equations and Riemannian geometry to the subject of harmonic mappings. I selected some topics to the presentation of which I felt I could contribute something, while on the other hand it was possible to provide complete and detailed proofs of them during these lectures. Thus, these notes are not meant to cover all that is known about harmonic maps, but nevertheless I believe that they give a good account of many of the interesting aspects of the subject and a fair idea of the variety of techniques used in the field.

After some introductory material in chapter 1, we present useful geometric constructions in chapter 2. In particular, we introduce almost linear functions on Riemannian manifolds and prove some properties of approximate fundamental solutions and harmonic coordinates. Most of this material originated from [JK1]. In chapter 3, we present the heat flow method to obtain existence, regularity, and uniqueness properties of harmonic maps into nonpositively curved manifolds. This covers the basic results of Eells-Sampson [ES]. Our approach also uses some ideas as presented by Hartman [Ht], von Wahl [vW], and Jost [J4].

In chapter 4, we prove the existence (due to Hildebrandt-Kaul-Widman [HKW3]) and uniqueness (due to Jäger-Kaul [JäK2]) of harmonic maps with image contained in a strictly convex ball, which solve a Dirichlet problem. The a-priori estimates based on the work of Hildebrandt-Widman [HW2] will be simplified by using the results of chapter 2. In chapter 5, we finally are concerned with harmonic maps between surfaces. We prove the existence result of Lemaire ([L1], [L2]) and Sacks-Uhlenbeck [SkU], as well as a result of Jost [J7] and Brézis-Coron [BC2] yielding the existence of two homotopically distinct solutions for nonconstant Dirichlet boundary data in S^2. We then turn to the question of the existence of harmonic diffeomorphisms, proving the results of Jost [J3] and Jost-Schoen [JS]. They are based on the deep estimates of E. Heinz [Hz5] for the Jacobian of univalent harmonic maps from below. These estimates, however, will not be proved in the present notes. We refer to [JK1] instead. Moreover, we show how a simple variational procedure can produce conformal diffeomorphisms between spheres as well as a version of the Riemann mapping theorem. For more details on harmonic mappings between surfaces, we refer to the author's notes [J8].

I thank Stefan Hildebrandt for his continuous advice and support over many years and for making the financial support of the SFB 72 at the University of Bonn available to me, and Richard Schoen for a fruitful collaboration and interesting discussions about harmonic maps between surfaces. To Hermann Karcher I owe many insights into the geometric aspects of the theory, and what I learned from him or what evolved during our collaboration is not only represented in chapter 2, but also penetrates chapter 4, and I regret that we did not find the opportunity to work out these notes together.

Moreover, I am grateful to Leon Simon for inviting me to Canberra and to the colleagues who attended my lectures for their interest and their stimulating queries and comments and to the Centre for Mathematical Analysis for its support of my work. Finally, I thank Dorothy Nash and Norma Chin for typing these notes with great care and patience.