PREFACE

The term "structure" in the title is used in the Bourbakist sense. Chapter 0 is devoted to the exposition of a certain notorious failure, or inadequacy, of currently used integration structures, including those presented in Book 5 of the Bourbaki treatise and in the well-known text of P.R. Halmos. In Section G of that chapter, the nature of the integration structures presented here is briefly described. This description is amplified somewhat in the pre-ambles to Chapters 2, 3 and 4.

To indicate the issue involved, I would first repeat what was already said by people learned in these matters and was even recorded, for example by my distinguished friends in their book which is listed among the references as item [9]. Namely, the problem of integration with respect to a vector valued measure which has finite (and σ-additive) variation is trivial. For, we do not in fact integrate with respect to such a vector measure; we integrate with respect to its variation which is a true (positive) measure. The integral with respect to the given vector measure is then a uniquely determined (vector valued) continuous linear functional in the space of functions integrable with respect to the variation.

In contrast, the problem of integration with respect to a vector valued measure having infinite variation seems to be nontrivial even when the range-space is one-dimensional. For, such a vector measure does not generate a continuous linear functional in the space of integrable functions with respect to any positive measure. I should note here, perhaps, that, since the appearance of the work of R.G. Bartle, N. Dunford and J.T. Schwartz, listed among the references as item [2], integration with respect to certain vector measures of possibly infinite variation can be reduced, using duality, to integration with respect to families of scalar valued measures of finite variation. However, this device is surely not available for all measures of infinite variation; for example, it is not available for measures with values in a finite-dimensional space.

From a sufficiently abstract point of view, the integration structures presented here can be seen as instances of a single general structure. That structure is intended to make trivial also the problem of integration 'with respect to measures of infinite variation'. It represents a construction of a complete normed function space - which of course cannot be an L^1-space in general - such that a given vector measure generates a continuous linear functional in it. Indeed, if we do not succeed in making this problem trivial, then, in my view, we do not have a chance to tackle successfully those problems whose solutions for measures of finite variation are so brilliantly exposed by my distinguished friends in the mentioned book.

These remarks indicate, I hope, that I opted for an approach to this problem which is different from the approaches found in the literature. That explains, to some extent, the list of references or, rather, the obvious omissions from it. So, for example, the works of R.H. Cameron and his collaborators are not mentioned although a considerable proportion of my motivation derives from the problems arising in connection with the Feynman integral. Or, the names of R. Henstock and J. Kurzweil do not appear here even though my theme concerns non-absolutely convergent integrals. Similarly, in Chapter 5, I introduce bilinear integrals, but the works of R.G. Bartle and of I. Dobrakov are not referred to. This presents for me a certain difficulty, even embarrassment. It is true that I have not discovered nontrivial relationship, at the technical level, between the results presented here and those results reported in the literature that concern similar themes but were obtained from different perspectives. On the other hand, I am also aware of the fact that I reached the point of view presented here only because I was influenced - possibly and admittedly only indirectly - by the works of the mentioned and of many other authors.

I have still greater difficulties with giving due credit and expressing my gratitude to friends and colleagues who assisted me by their thoughts not available publicly. It is simply impossible for me to trace all such influences, not to speak of their explicit articulation. What is more, in some cases in which I would be able to do so, I do not know the names of the persons who assisted me in this manner. They are, for example, the referees of my journal articles, even, or especially, those which (happily) were not accepted for publication.

However, I am too conscious of the generous help rendered to me by Brian Jefferies, Susumu Okada and Werner Ricker not to mention them by name. I wish I were able to express better my gratitude for their criticism of my numerous attempts at the realization of this project and for helping me to maintain the confidence in its viability.

I am delighted that I am able to put on record my gratitude to Neil Trudinger who created the possibility for me to work on this project at the Centre for Mathematical Analysis and to make the results of my effort available to the public in this form.

In my endeavour to facilitate the reading of the text I was greatly assisted by Dorothy Nash. I would like to thank her for the expert advice about lay-out, for the understanding, even anticipation, of my intentions, for the initiative with which she explored the possibilities of the available equipment for their realization and, generally, for her pleasant cooperation.

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