# THE CONCEPT OF MATTER IN DESCARTES AND LEIBNIZ

It has been the recent fashion to accuse Descartes, and hence all Baroque thinkers, of a simple and disastrous dualism, made famous by Whitehead's phrase "the bifurcation of nature." In the *Lowell Lectures*, Whitehead indeed admitted that rarely had the world ever seen such an assembly of distinguished minds as that period produced, which gives one pause to consider whether they were really as naive as some have pretended. Certainly this dualism was no more primitive than Aristotle's physics of contraries with its crude reduction of Eudoxus' mathematical theory of celestial motions, its four qualities and its arbitrary distinction of substantial and accidental forms the former of which perpetuate themselves in everlasting cycles without shadow of turning. Nor was Aristotle's notion of the Cosmos with its natural place and motion of heavy bodies to its center, the earth, calculated to enrich our experience in spite of Mr. Koyré's attempt to make it seem interesting.[1] In fact, Bishop Tempier of Paris judged it so dreary as to be unfit as a representation of the creation of the Almighty Christian God and banned it from the University of Paris with the most fortunate consequences for the science of physics in the fourteenth century.[2]

But even among the Baroque thinkers, there are, of course, several traditions: (1) that of Kepler and, perhaps, Bruno which passes on to Leibniz and Newton; (2) that of Galileo; and (3) that of Descartes, Huyghens, and Malebranche which passes also through Leibniz. Of these, of course, that of Descartes is the most susceptible to the criticism of Whitehead, but we hope to show that the Cartesian concept of matter and the resulting physics are a fruitful and necessary moment in the dialectic of Baroque theories whose consequences are not yet exhausted. The greatest mind in physical theory and the most important was perhaps Kepler, the least susceptible to the attacks mentioned above, yet unknown to Descartes except for his treatises on light, as he was to Galileo, by design, perhaps, more than by inadvertence. And the stone which Descartes and Galileo rejected will become the chief cornerstone of classical mechanics, although it is with their unconscious help.

The obvious intent of the Cartesian *cogito* is to convert the world to a structure of thought, where thought is equated to awareness.[3] For thought, for Descartes, consists of all sensations as well as clear and distinct ideas, imaginations, and volitions. This all embracing world of thought is divided into active thought, consisting of clear and distinct ideas, and of passive thought consisting of sensations which are not representations of theoretical truths but of useful reactions, that is, of my body's relation to other bodies for pure purposes of bodily survival.[4] Hence the Cartesian intent is even more radical: it is to convert the world to active thought, the thought of clear and distinct ideas, for only in this way is it open to our conquest, not

1 A. Koyré, *Etudes Galiléennes*, I, Paris, 1939; pp. 11-17.
2 P. Duhem, *Le Système du Monde*, Tomes VI-VIII, Paris.
3 *Principia Philosophiae*, I, *Oeuvres de Descartes*, Adam et Tannery, T. VIII, p. 17. This will be quoted as AT hereafter. There are many places here where the same thing is stressed, as well as in the other major works.
4 *Principia*, I, 48, AT VIII, p. 23.

by abstraction but by construction.[5] It is not abstraction which rules the new methods, but construction; and the essence of construction is active thought, in the new sense of essence, that which is implied in its very denial. For, if I doubt all, I know I doubt; if I deny I think, my denial is a thought.[6] This is the essence of thought. And again the essence of that which is opposed to thought is that without which it cannot be thought, extension.

Such a world will indeed appear to the average man a dream world, and Descartes knew this, for he consciously created a dream world, using constantly its images. Passíng over the great dreams of 1619,[7] what are the *Mêditations* but a sequence of dreams, and what is the world of *Le Monde* but a dream of clear and distinct ideas where the ordinary world of common sense does not intrude?[8] The very power of the Cartesian method is in the radicalism of this dream world of the thinking subject, as we see it in the *Regulae* with its purely literary companion, the *Discours de la Mêthode*.

But to disturb this dream world of thought there appear within it two strange obstacles which must be exorcised if the conversion of the ordinary world is to be complete. The first is the idea of perfection which appears necessarily with the imperfection of the thinking subject,[9] and the second is the idea of the world outside of this world, the thought of the world which is not thought, the world of body.[10] The former is an idea in the realm of active thought as beyond it; and the latter in the realm of passive thought as beyond it, neither depending on the will of the thinking subject.—It is well known how the first is absorbed into the dream as the thinking which guarantees all thinking; it is not the purpose of this paper to consider it further.

It is the second idea which is the main concern of the Cartesian method as it is of all Baroque thinking. This is the true paradox which tries the mettle of great men, to reduce that which does not think and which announces itself to the thinking subject as not thought, to reduce this to the power of thought. For such a world, the words "pure potentiality" are anathema and are banished forthwith.[11] The building of a rational physics which can reduce all nature to the domination of thought, this is the main task of Descartes as evidenced in all his letters and in all his works from the beginning to the end of his life.

A thinker, no matter how sublime his manner and no matter how transcendental his method, is always conditioned by the tools that the historical order of his succession furnishes him with. In this regard, Kepler allowed himself more freedom, using analogies from many sources, as he sought to unfold the archetype of God in the visible world, an archetype to be expressed essentially in mathematical form where one was free to consider the relation between bodies in terms of their mutual distances without the reduction to a simple set of axioms and laws immediately clear to the thinking subject. But it was precisely Descartes' goal to find the clear and distinct ideas which would give him control of this world opposed to thought, and his geometry was the model for this. This geometry was mainly concerned on the one hand with the isomorphism between the algebraic representation of the

[5] *Regulae ad Directionem Ingenii*, *Regula* XII, AT X, pp. 418-419.
[6] *Regula* XII, AT X, p. 421.
[7] *Olympica, Cogitationes Privatae*, AT X, pp. 181-183, p. 216.
[8] *Le Monde*, AT XI, p. 31.
[9] *Mêditation Troisième*, AT IX, pp. 32-33.
[10] *Mêditation Troisième*, AT IX, pp. 27-28.
[11] *Regula* XII, AT X, p. 421.

rational operations of addition, subtraction, multiplication, division, involution and evolution with the lines of geometry by which any vestiges of passive thinking inherent in the geometrical imagination could be reduced to the active ideas of algebra,[12] and, on the other hand, with the use of this method in solving the problem of anaclastic curves.[13] Both of these great concerns of Descartes' geometry are embodied in the two great problems of Descartes' physics and dominate it completely: namely, the problem of the percussion of perfectly hard bodies, and the problem of the refraction of light.

To consider the general concept of matter in Descartes without considering its embodiment in these two problems, would be to act irresponsibly as an armchair philosopher, and to indulge in verbalisms of a kind which Descartes violently eschewed. The Cartesian intent is to formulate a method which will reduce the confused ideas of our experience to the clear and distinct ideas of our active thought. This can only be understood in the doing, and the method can only be seen as it develops with the problems it seeks to solve. And we shall see that what has so often been called a simple and obviously false notion of matter was powerful and fruitful at the moment of its definition. In all its shocking simplicity, what was already latent in the *Regulae* is made explicit in *Le Monde*[14] and rigorously so in the *Principia:* matter is three dimensional extension as represented by Euclidean geometry with the modes of motion and rest, and figure,[15] which are indivisibles of thought, and where the real figures of extension depend upon the motions of its parts as opposed to the figures, independent of motion, which are imagined by the mind. It is to this, and finally to the corresponding algebra, that all sensations, that part of passive thinking which refers to a so-called external world which does not have the property of thinking, the vulgar exterior world of sensed body, must be referred. This leads to a paradoxical position: that which is really most proper to sensation exists only in thought (passive thought), while that which in sensation refers beyond thought as existing

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12 *La Géométrie*, AT VI, p. 36 and following. *Regulae* XVI-XVIII. Fermat and others also used algebra for their geometry.

13 *La Géométrie*, AT VI, p. 413: C'est pourquoi je croirai avoir mis ici tout ce qui est requis pour les éléments des lignes courbes, lorsque j'aurai généralement donné la façon de tirer les lignes droites qui tombent à angles droits sur tels de leurs points qu'on voudra choisir. Et j'ose dire que c'est ceci le problème le plus utile et le plus général, non seulement que je sache, mais meme que j'aie jamais désiré de savoir en Géométrie.

p. 424: "Au reste, afin que vous sachiez que la considération des lignes courbes, ici proposée, n'est pas sans usage, et qu'elles ont diverses propriétés qui ne cèdent en rien a celles des sections coniques, je veux encore ajouter ici l'explication de certaines ovales que vous verrez être très utiles pour la théorie de la catoptrique et de la dioptrique."

14 For a discussion of the date of the composition of *Le Monde*, see Le Mouy, *Le dévêlopment de la physique cartésienne 1646-1712*, Paris 1934, pp. 5-8. It was, of course, only published after Descartes' death, but was composed probably from 1629 to 1633.

15 *Regula* XII AT X, p. 418: "Quamobrem hic de rebus non agentes, nisi quantum ab intellectu percipiuntur, illas tantum simplices vocamus, quarum cognitio tam percipua est et distincta, ut in plures magis distincte cognitas mente dividi non possunt: tales figura, extensio, motus, etc.; reliquas autem omnes quodam modo compositas ex his esse concipimus." See also pp. 419-420, 426.

*Le Monde*, AT XI, p. 33: *Principia*, I, 53, AT VIII, p. 25.

outside it has its clear and distinct ideas in active thought. Thus *Principia*, I, 71 reads:[16]

> ". . . and our mind would experience those states which we call the sensations of tastes, smells, sounds, heat, cold, light, colors and similar things which represent nothing placed outside thought [*cogitationem*]. And at the same times also it would perceive magnitudes, figures, motions and such which showed themselves not as sensations, but as certain things [*res*], or modes of things, existing outside of thought although it did not yet notice this difference between them."[17]

Let us compare this with the free expression of the earlier *Le Monde*

> "But since we are taking the liberty of feigning this matter to our fancy, let us attribute to it, if you please, a nature in which there is nothing at all which anyone cannot know as perfectly as possible. And to this effect, let us suppose expressly that it does not have the form of earth, nor of fire, nor of air, nor of any other particular thing as wood, or stone, or metal, nor again the qualities of being hot or cold, dry or wet, light or heavy, or to have any taste, or smell, or sound, or color, or light or any like thing, in the nature of which it can be said that there is something which is not evidently known by everybody.
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> "And, on the other hand, do not let us think that it is this prime matter of the philosophers which has been so well robbed of all its forms and qualities that nothing remains which can be clearly understood. But let us conceive it as true body, perfectly solid, which fills equally all the lengths, widths, and depths of this great space in the midst of which we have poised our thought."[18]

There could not be a clearer and more conscious statement of the adventurous course on which Descartes had decided to embark in respect to the physical world.

Earlier or possibly at the same time, the *Regulae* manifest the very same intent. "For the knowing of things only two things are to be considered, namely we who know, and the things to be known."[19] Rules 12 and 13, and especially Rule 14, everywhere develop this conversion of real body to extension as clearly seen by the imagination and understood by the intellect.

> "And from these it is easily concluded that it will be quite enough if we transfer those things which we understand to be said of magnitudes in the genus, to the species of magnitude which of all things is painted most easily and distinctly in our imagination; that this is the real extension of body abstracted from all else, as well as that it is figured, follows from things said in Rule 12, where we conceived the *phantasia* itself, with ideas existing in it, to be nothing but the true real body, extended and figured."[20]

It is not to be thought this radical reduction of qualities to extension and its modes of motion and figure is made by Descartes without any consideration of a deeper significance. If for him there is on the one hand an extreme

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16 The translations into English are my own.
17 AT VIII, p. 35.
18 AT XI, p. 33.
19 AT X, p. 411.
20 AT X, p. 441.

form of intuition, of identity of mind and object, there is, on the other, a recognition of the act of symbolizing as a fundamental fact of knowing. In the beginning of *Le Monde*, this fact is stated in its most acute form.

> "You well know that words, having no resemblance to the things that they signify, are no less able to make us conceive them, and often without our noticing the sound of the words nor their syllables; so that it can happen that, after having heard a discourse, we cannot say in what language it was pronounced. But if words, which only signify by the institution of men, can make us conceive things they have no resemblance to, why cannot Nature have established a certain sign to give us the feeling of light although this sign is nothing in itself which resembles this feeling."[21]

Thus the sensations of light can be explained in terms of extension, figure, and motion; the ones are the symbols of the others. This notion of symbolization is carried further in the whole conception of this treatise. For leaving the ordinary world of the senses, and abandoning any attempt to understand it by abstraction, the author asks us to enter a new Bruno-like world constructed in terms of what he calls clear and distinct ideas: -

> "For a moment, let your thought leave this world to gaze upon an entirely new one, which I shall bring about in its presence, in imaginary spaces. Philosophers tell us that these spaces are infinite; and they must be believed, since they made them themselves."[22]

This act of symbolization is further discussed in a famous passage of the *Dioptrique* where the relation of the sensation of color to the corresponding motions in the realm of clear and distinct ideas is compared to the knowledge a blind man has of the things around him in terms of the reactions of the end of a stick which he holds in his hand.[23] And the strange arbitrariness of the power of symbolizing is finally placed by Descartes at the very center of knowing in the letter to Mersenne of July 22, 1641, where he says: -

> "And finally I hold that all these [ideas] which include no affirmation or negation, are *innatae* in us; for the sense organs bring us nothing which is like the idea which awakes in us on their occasion, and this idea must have been in us before."[24]

In a sense, it might be said this is only the fulfillment of the vision of the young Descartes in his *Olympica* of 1619 when he spoke almost in parables: -

> "As the imagination uses figures for conceiving bodies, so the intellect uses certain sensible bodies such as wind and light to figure spiritual things: whence by philosophizing in a higher way, we can carry the mind by cognition to a sublime height. It may seem strange what profound statements are in the writings of poets rather than in those of the philosophers. The reason is that poets have written with enthusiasm and with the force of imagination. There are in us the seeds of science, as [sparks] in a stone, which are brought forth by the philosophers through reason, but by poets are struck forth through the imagination and light up the more."[25]

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21 AT XI, p. 4.
22 AT XI, pp. 31-32.
23 AT VI, pp. 84-85.
24 AT III, p. 418.
25 AT X, pp. 217-218.

The inspiration of Descartes will have come full circle from the vague dream of the *Olympica* of 1619 to the clearly articulated dream of the French edition of the *Principia* in 1647. We must see how this development, we have so briefly indicated in general, took place in particular.

We have seen so far that the world of body is the infinite world of extension, having three dimensions, figure, and motion and rest, these last being the modes of dimension, clear and distinct ideas for all, in terms of which all the ideas concerning body must be explained. In Rule 14 of the *Regulae* Descartes defines dimension in a more general way than he will later in *Le Monde* and the *Principia.*

> "By dimension we understand nothing other than the mode and reason according to which some subject is considered to be measurable; so that not only length, width, and depth are the dimensions of body, but also gravity is a dimension according to which subjects are weighed, speed is a dimension of motion and so on indefinitely. For the division into several equal parts whether it be real or only intellectual, is properly a dimension according to which we number things. . ."[26]

Another account of this general notion of dimension is given by Beeckman in his *Journal* for 1628 at about the same time.[27]

> "But in particular he conceives the cube through three dimensions, as also others do; but he conceives it four-dimensional as if from the simple cube which is considered as wooden, there should be made a stone cube, for thus a dimension is added for the whole; and, if another dimension is to be added, he considers an iron cube, then gold etc., which is not only done for gravity, but also in colors and all other qualities."[28]

It is difficult to assess how profound was the intent of this general definition of dimension. In any case, it does not reappear in any definite way in *Le Monde* and the *Principia* where specifically gravity is reduced to the extension of three dimensions and motion.

It follows, of course, that the Cartesian matter is a plenum and that there are no permanent atoms. For body is infinitely divisible as in the Euclidean continuum. Although raised on Clavius, an excellent commentator of Euclid, Descartes could not be subject to the deeper training of Barrow, the first modern commentator to understand the more sophisticated properties of this continuum as presented in Euclid, Book V. In a further dream, Leibniz will begin the dissolution of Descartes' clear and distinct extension with a vision which was to find its rigorous fulfillment in nineteenth century mathematics.

Figure and motion and rest are qualities of this matter in *Le Monde*,[29] in the *Principia* they are called modes of matter or extension.[30] The only motion which is clear and distinct, of course, is local motion, and the local motion of a body is defined as "the translation of one part of matter or of one body, from the neighborhood of those bodies which touch it immediately and

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26 AT X, pp. 447-448.

27 We shall hear more of Beeckman later. He and Descartes met in Holland in 1618, and we shall see what a profound influence he had on Descartes' thinking.

28 AT X, p. 334.

29 *Le Monde*, AT XI, p. 37.

30 *Principia*, I, 53, AT VIII, pp. 25, 26-27.

are considered as at rest, to the neighborhood of others."[31] This "considered as at rest" will naturally lead to considerable trouble.

That motion and rest should thus become clear and distinct ideas, that is, two of the most important "seeds of science" of the *Olympica* or of the "*innatae*" of the letter to Mersenne, has far reaching consequences for physics.[32] Motion and rest are now essential properties of bodies, known immediately by everyone, needing no reduction to other and clearer principles. Indeed motion has been used by geometers to deduce a line from a point and a plane from a line, and the Aristolelian definition of motion is incomprehensible.[33] Motion and rest are taken as distinct qualities, a distinction which promises an uneasy future. In this way, Descartes departs from the doctrine of Kepler, for whom rest only belongs naturally to a body as he says in the introduction to the *Astronomia Nova:* - "Every corporeal substance, in so far as it is corporeal, naturally rests in any place where it is placed alone beyond the sphere of influence of a like body."[34] This idea of motion leads directly to the law of inertia stated for the first time in its clearest form by Descartes.

For in *Le Monde*, Descartes immediately sets out three general laws of motion which one might call the conservation laws:[35]

I. That every part of matter continues always in the same state as long as its contact with others does not force it to change.

II. When one body hits another, it can give it no movement which it does not lose itself nor receive from it except what the other loses itself.

III. Since the straight line is the simplest curve and constant speed the simplest property of motion, therefore the first law is further interpreted in a third law, to mean that every body continues in a straight line at constant speed unless it is interfered with.

To explain this third law, he uses as an argument the case of the stone revolved in a circle by a sling which, as soon as it escapes from the sling, moves off in a straight line tangent to its curve and constantly shows by the pull it exerts its tendency to so move.

The second law will take on a more precise form in the *Principia* and so we proceed to give these laws as they appear there: -

I. "Each thing, in so far as it is simple and undivided, remains as far as it is in itself, always in the same state and never is changed except by external causes. . . If it rest, we do not believe it will ever begin to move unless it is impelled to it by some cause. Nor is there any greater reason, if it move, why we should think it would ever stop that motion spontaneously unimpeded by anything else."[36]

II. "No part of matter, considered separately, ever tends to move so that it pursue any curved line, but only straight lines."[37]

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31 *Principia* II, 15, AT VIII, p. 53.
32 See Koyré, *Etudes Galiléennes* III, *Galilée et la loi d'inertie*, pp. 158-181.
33 *Le Monde*, AT XI, p. 39.
34 *Gesammelte Werke*, Band III, p. 25; the edition of von Dyck and Caspar.
35 AT XI, pp. 38-46.
36 AT VIII, p. 62.
37 AT VIII, p. 63.

III. "Whenever a body which moves meets another, if it have less force [*vis*] to pursue a straight line than the other for resisting it, then it is deflected in the other direction, and, in retaining its movement [speed], loses only its determination; but, if it have a greater force, then it moves the other body with it, and loses just as much of its motion as it gives the other."[38]

In the next paragraph which purports to prove this law, we have, stated explicitly, the distinction between direction and speed, so that speeds in opposite directions are not opposed to each other, but only motion and rest, and that the quantity of motion remains fixed in the transferring of motion from one body to another according to God's immutability. Hence we can summarize these three laws in this way, anticipating further developments in the text which we shall later bring out: -

(1) the law of inertia:-every body moves forever in a straight line at constant speed in the same sense, unless it is interfered with;

and

(2) the quantity of motion in the world remains the same, that is, the sum of the volumes, each multiplied by the absolute value of its speed, remains constant where the volumes are those of absolutely hard bodies, bodies whose parts are at rest relative to each other.

Given the fact that these laws are to operate in an infinite plenum, the gap between principle and verification would seem almost impassable.

In the *Principia*, there follow the famous seven laws of the percussion of hard bodies which are supposedly derived from these general laws. But before we attempt to state them, we must go far back indeed in the development of Cartesian thought to find out where we are. For this is the end of a long argument where there appear such peculiar notions as hard and soft, and such peculiar expressions as "conserving motion" and "determination of motion." To understand we must turn backwards and then forwards, using very restricted examples of Cartesian research and analysis, and their future development in the hands of great successors. For just as it is idle to try to understand a term by itself, so it is impossible to understand a doctrine in a certain moment of its statement when it has been elaborated in terms of carefully circumscribed problems whose history is long and arduous.

We now turn to the two problems of Descartes we have already mentioned: (1) the refraction of light; (2) the percussion of perfectly hard bodies. And (3) we shall touch briefly on the fall of heavy bodies.

In 1618, as we have already mentioned, the young Descartes met the Dutch physicist Beeckman in Holland. By 1614, Beeckman had the doctrine that motion is a state inherent in bodies (not only in the celestial bodies as had been held by everyone): "Everything, once moved, never rests except for some external impediment; and the weaker the impediment, the longer the one in motion moves."[39] As we shall see later, by 1618, he had enunciated the law of conservation of momentum for the percussion of perfectly soft bodies. It is certainly from here that Descartes got many of his ideas although Beeckman held for the vacuum and read not only Kepler's treatises

38 AT VIII, p. 65.
39 *Correspondence du Père Marin Mersenne*, T. II, Paris, 1945; p. 236.

on light as did Descartes, but also the *Astronomia Nova* and the *Epitome* with the appropriate enthusiasm,[40] while Descartes shows no sign of ever having known them.

It is probably through the influence of Beeckman that Descartes read the treatises on light of Kepler, and Milhaud goes so far as to believe that the famous phrase of Descartes "on Nov. 11, 1620, I began to understand the foundation of a marvelous find" refers to his reading of Kepler's treatises in Prague.[41] In any case, it is certain that the appearance of the law of sines for the refraction of light in the *Dioptrique*, proved from the principles of motion, has its roots in this first meeting with Beeckman and with the reading of Kepler. We know that Descartes already had this law in 1628, for Beeckman mentions the fact in his *Journal* for 1628-29.[42] And this is independent of Snell who died in 1626 without publishing his treatise. To understand the treatment in the *Dioptrique* of Descartes, we begin with the exposition of certain matters from the *Dioptrice* of Kepler.

For our purposes we need repeat only the following axioms from that work, which we paraphrase.

1. The angles of refraction in crystal are, up to 30°, sensibly proportional to the angles of inclination.
2. The angle of refraction in crystal, up to this limit, is very approximately a third part of the angle of inclination.
3. The maximum angle of refraction in crystal is about 48°.

It must be remembered that for Kepler the angle of refraction is the difference between the angle of incidence and what is now called the angle of refraction. It is then obvious that in general the angles of incidence and of refraction are not proportional. Kepler then proceeds to show that, if parallel rays meet a spherical crystal surface inside it, then, for angles less than 30°, the refracted rays outside in the air will converge to a point at a distance equal to the diameter of the spherical surface.[43]

And, finally, from this, by a qualitative argument, Kepler shows that, if the rays are to converge for angles greater than 30° and the shape of the surface is conical, then it must be an hyperbola.[44]

It is strange, as others have already remarked, that, from the data here given, Kepler did not find the law of sines, for just such juggling with trigonometric functions is performed in several analogous situations in the *Astronomia nova*.[45] However this may be, Descartes in a letter to Ferrier in November, 1629,[46] explains how one finds the ratio of refraction for a ray passing through a prism of glass, assuming the law of sines, and he speaks here also of a hyperbolic glass. And again in a letter to Golius, in February,

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40 Beeckman to Mersenne, April 30, 1630, *Corresp. du P. Merseene*, T. II, p. 456, where he obviously remembers the discussion of Kepler in *Astronomia Nova*, III, 36. The note of de Waard on p. 469 forgets Kepler, *Epitome*, IV, 2, 3; and also *Ad Vitellionem Paralip.*, I, Prop. IX, where the inverse-square law of illumination is stated and argued rigorously. For Beeckman's knowledge of the *Epitome*, see p. 290.

41 Gaston Milhaud, *Descartes savant*, ch. 4, Paris, 1921.

42 AT X, pp. 335-337.

43 *Gesammelte Werke*, B. IV, p. 357.

44 *Ibid.*, pp. 371-372.

45 *Ibid.*, B. III, pp. 282-285, 304; pp. 352-355.

46 AT I, pp. 335-337.

1632, after describing a simple instrument for measuring the ratio of refraction and establishing the law of sines by numerical verification, he adds significantly a paragraph which we quote at length:

> "If you have not yet thought of the means of making this experiment, since I know you have better things to do, perhaps this will seem easier to you than the instrument described by Vitellius. In any case, I can very well be wrong, for I used neither the one nor the other, and the only experiment I have ever made in this matter is to have had a glass formed, five years ago, a model of which was traced by Mr. Mydorge himself, and, when it was made, all the sun's rays which passed through it converged in one point, exactly at the distance I had predicted. Which assured me, either that the artisan had fortunately failed or that my reasoning was not false."[47]

This glass and its making is again described in detail in a letter to Const. Huyghens in December, 1635, and is there identified as an hyperbolical glass.[48] Hence we can infer that Descartes, having discovered the law of sines, not by experiment, verified it by an hyperbolical glass where the hypothesis was already built into the instrument. For, in the *Dioptrique*, published in 1637, he will prove that the convergence of the rays follows mathematically from this law for the hyperbolical glass, and also in this treatise he will give a derivation of the law from the principles of motion, principles which will be involved in the theory of percussion published later and whose general laws we have discussed above.

Let us analyze carefully this derivation of the law of refraction in the *Dioptrique*, so ridiculed by Mach, but actually the same as given, *mutatis mutandis*, by Newton in the *Principia* and which formally has been renewed in our own day by the de Broglie theory and the Germer-Davisson experiment of 1927. Light, for Descartes, follows upon the motion of very small particles moving in the plenum, but, since for him it is instantaneous and acts from a luminous point in all directions, we must consider the inclination or tendency of the particles to move which corresponds to the sensation of light.[49] To proceed to his deduction, Descartes assumes that the inclination to motion follows the same laws as motion itself or, as he will later say, the potential can only be considered as an act.

In the case of reflection, we must imagine that the light particles rebound as a ball from the perfectly hard earth. The ball rebounds with the same speed as it strikes but its direction is changed because the part of its motion parallel to the earth has no reason to be changed. From this follows the equality of the angle of incidence and reflection.

The same principles are to be applied more or less to explain the law of refraction. To this end, we must make clearly the distinction between the speed or quantity of motion and the determination or direction of the motion, a distinction which shall be carried into the laws of percussion with fatal consequences. We suppose that the ball, moving with a certain speed in the first medium, reaches the point of incidence with the second medium. Then by virtue of its contact with this second medium, its speed changes in a definite ratio, independent of the angle of incidence. So much for the speed; now

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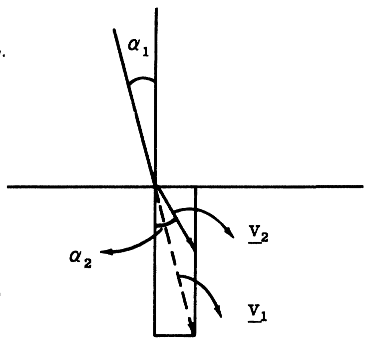
47 AT I, p. 239 n.
48 AT I, pp. 335-337.
49 *La Dioptrique*, AT VI, pp. 87-88.

for the determination or direction. We imagine the action of this second medium to be that of a very thin cloth spread parallel to the surface of refraction. We have described its action on the speed; strangely enough it changes it independently of the angle of incidence. But since it does not oppose the ball in the direction parallel to the surface the velocity or determination of the ball in that direction is unchanged while in the direction perpendicular to the surface the resulting velocity must accommodate itself to the given speed in the second medium and the unchanged horizontal component. Representing this in the modern notation of vectors instead of using Descartes' figure, we have immediately from the drawing, where b represents the fixed horizontal component,




$$\frac{b}{|\underline{v}_1|}\bigg/\frac{b}{|\underline{v}_2|} = \frac{\sin\alpha_1}{\sin\alpha_2} = \frac{|\underline{v}_2|}{|\underline{v}_1|} = \mu,$$

where $\mu$ is a constant independent of the angles, and where we see that the sines are *inversely* as the speeds.[50]

Newton, who read and pondered this whole treatise, as we know from his letter to a friend, Feb. 23, 1668/69 and particularly in his letter to Oldenburg, Dec. 21, 1675,[51] deduced precisely this same formula in the *Principia*,[52] where he supposes that there act, at the surface of refraction, for a zone of very small depth, forces (in the sense of Newton) perpendicular to the surface whose magnitude varies only as the depth. Dividing this zone into subzones of constant force, Newton uses for each the parabolic path of Galileo in the manner of Apollonius, and is able to show that the ratio of the sines of the angle of incidence and emergence is independent of the angle of incidence. Presumably then one passes to the limit.[53] Thus the force acting at the surface gives Newton the two assumptions of Descartes:

(1) the change in the perpendicular component of the velocity,

and

(2) a speed in the second medium with a constant ratio to the speed in the first independent of the angle of incidence.

The modern version of Newton's derivation is much simpler, using the work-energy equation, which was, of course, unknown to Descartes, and not used in Newton's *Principia*. It is the work-energy equation which gives the constant speed in the second medium, and the perpendicular surface forces which give the change in the perpendicular component.

After this analysis of his law of sines, Descartes, in *La Dioptrique*, VIII;[54] proves geometrically that the ellipse and hyperbola are anaclastic curves.

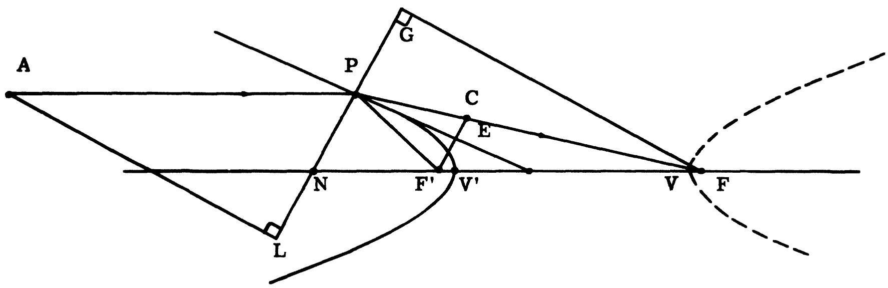
50 *Ibid.*, AT VI, pp. 96-104.

51 *The Correspondence of Newton*, Vol. I, Cambridge, 1959, p. 405.

52 Book I, Sect. XIV.

53 For the details of this geometrically elaborate proof, few of which are supplied by Newton himself, the curious reader can consult the notes of the editors, Le Seur and Jacquier, in the Latin Glasgow edition of 1833, p. 413.

54 AT VI, pp. 179-181.




For taking PA = PF where F and F' are the hyperbola's foci, with PA parallel to the axis and PL perpendicular to the tangent at P, he shows triangles PNQ, PFG, PLA, PNF similar respectively to triangles, PLA, PNM, FGN, CF'F, where F'C is parallel to PL. Using the property that angle F'PE equals angle EPF, it is easy to show

$$\frac{AL}{FG} = \frac{VV'}{FF'} \text{ or } \frac{\sin APL}{\sin FPG} = \frac{2a}{ae} = \frac{1}{e};$$

and therefore the hyperbola is anaclastic if the index of refraction is equal to the inverse of the eccentricity. As is well known, Descartes proceeds, in his *Geometrie*, to find by his new algebraic methods, more general anaclastic curves.

In a like manner, Newton, in the passage of the *Principia* already discussed, passes to the consideration of general anaclastic curves, and, by the method of limits used for the planetary orbits, he boasts to have deduced in one proposition "all those figures which Descartes has exhibited in his *Optics* and *Geometry* relating to refractions."

If one asks how Descartes found the law of sines, it is fairly clear that experiment in the crude sense did not immediately enter into consideration except as the verification of an anaclastic curve. After all Kepler had struggled with the tables of refraction of Vitellius and with Tycho's tables of refraction of the sun, moon and fixed stars, with limited success; Descartes was not likely to tangle with observations in the same way. But he seems to have adopted the simpler theoretical ideas of Kepler and to have adapted them to his own way of thinking to find this simple law. But Kepler is far more involved in this affair then we have yet seen.

Descartes, in the *Dioptrique*, already anticipates the objection to his mechanical analogy of the ball intercepted by a thin cloth. For light is deflected downwards by the apparently denser medium, water, in the passage from the rarer medium, air. According to the figure of the ball and the cloth, it should be deflected upwards; or, as Descartes pictures it, we need a tennis racquet smacking it down. Descartes explains this by an analogy with the phenomena of percussion: a hard body loses more of its motion to a soft body than to a hard one.[55]

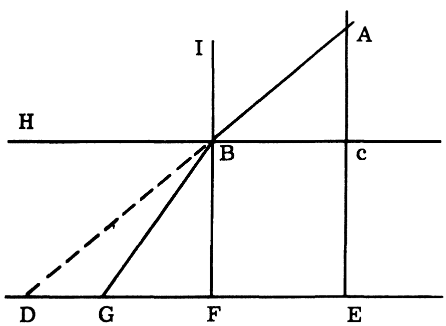
[55] AT VI, p. 103.

Before turning to the criticisms of Fermat in the face of these strange explanations, let us report a letter of Descartes to Mersenne, March 31, 1638.

> "He who accuses me of having borrowed from Kepler the ellipses and hyperbolas of my *Dioptrique*, must be ignorant or malicious; for as for the ellipse, I don't remember Kepler's speaking of it, or if he speaks of it, it is assuredly to say that it is not the anaclastic curve he seeks, and as for the hyperbola, I remember very well he pretends to show expressly that it is not the anaclastic curve he is looking for, although he says it is not much different. . . This does not prevent my admitting that Kepler was my first master in optics, and that I believe him to have been the one who knew most up until now."[56]

Hence we have it from Descartes himself that Kepler was his master, but the contemporaries of Descartes could have accused him - if they did not - of borrowing far more than the anaclastic curves from Kepler. The whole idea of the ball and the piece of cloth is prefigured in the first chapter of Kepler's treatise *Paralipomena in Vitellionem* of 1604.

It is there pointed out that light which is immaterial and hence moves at infinite speed, has however an aspect in which it can be likened to moving things. It cannot be impeded by media in the same way as ordinary local motions; light can only be impeded by a body in so far as it has a surface, not in so far as it has three dimensions. Therefore, explains Kepler, we can analyze refraction in analogy with a small ball meeting the surface of the water BC. We divide the motion of the ball into its two components IB and BH, one perpendicular to the surface and one parallel to the surface. The ball is first impeded by the opposition of the medium perpendicular to its surface, but later it is also impeded in the direction parallel to the surface. But, in the case of light, because of its peculiar property of reacting only to the surface, it is impeded in the direction parallel to the surface only. Hence, on entering a denser medium such as water or crystal, from a rarer medium air, the light is bent downwards towards the perpendicular. There is no explicit assumption concerning a constant speed in the second medium independent of the angle of incidence.[57]




Descartes, as Kepler, assumes that the explanation of light should be in terms of the laws of motion of bodies, even though this is by analogy; as Kepler, he assumes that the action of impeding or accelerating takes place only at the surface; like Kepler, he keeps one component fixed although a different one; but, unlike Kepler, he wishes to mechanize completely the behavior of light introducing the motion of the cloth and the hidden percussions of his plenum. Finally, Descartes comes out with a greater speed

56 AT II, pp. 85-86.
57 Kepler, *Ges. Werke*, B. II, pp. 26-27.

in the denser medium while Kepler comes out with a less. And, if we follow through Kepler's theory as Descartes did his, assuming speed in the second medium always the same, independent of the angle of incidence, we would have

$$\frac{b}{|\underline{v}_2|} \Big/ \frac{b}{|\underline{v}_1|} = \frac{\cos \alpha_1}{\cos \alpha_2} = \frac{|\underline{v}_1|}{|\underline{v}_2|} = \mu.$$

Consistent with his fundamental aims, Descartes tries to reduce color and the phenomenon of dispersion to the rotation of the particles of light,[58] while Kepler leaves this outside of mechanical and quantitative considerations.[59]

In a letter to Mersenne, September 1637,[60] Fermat objects to the confusion of tendency to motion with motion as the confusion of potentiality with act, and consequently to speaking of the modification of speeds by forces when the speed is infinite. And secondly he asks why the motion cannot be decomposed differently. To this Descartes replies, in a letter to Mersenne, Oct. 1637,[61] that, while whatever is in the potentiality is not necessarily in the act, yet whatever is in the act is in the potentiality, and that the potentiality here can only be judged by the act. He repeats his original comparison of the action of light to the blind man with a stick where the reactions of one end of the stick are felt immediately at the other end, although the analysis should be in terms of the concomitant motion. And finally, he answers that the other possible decompositions of the motion are abstractions and have nothing to do with the given situation.

Fermat replies to Mersenne in December 1637,[62] with deeper objections. He understands that one can consider as different the motion and its determination, but this distinction made by Descartes seems to be removed in the use; for the determination is here finally determined by the speed. And when the ball enters the second medium, the medium impedes it in all directions so that the component parallel to the surface should be equally affected; Fermat does not understand the assumption of surface tension only. And finally he proposes a solution of his own. Making Descartes' assumption that only the perpendicular component is affected by the second medium at the surface, Fermat would add vectorially the velocity of the first medium to a velocity perpendicular to the surface to give the velocity vector in the second (adding, says Fermat, according to the manner of Archimedes and the ancients). This however, gives a constant ratio $\sin(\alpha_1 - \alpha_2)/\sin \alpha_2$, on supposing the normal component constant and independent of the angle of incidence; and now the speed in the second medium varies with the angle of incidence.

Descartes' reply, in his letter to Mydorge, March 1, 1638,[63] adds little to the controversy, only restating his original position so as to make his assumptions clearer than in the *Dioptrique*.

The matter is apparently dropped until after Descartes' death, when Fermat writes in August 1657 to de la Chambre, on the occasion of the

58 *Les Météores*, AT VI, pp. 331-333.
59 Kepler, *loc. cit.*, p. 23.
60 *Oeuvres de Fermat*, T. II, Paris, 1894, pp. 106-112.
61 *Op. cit.*, II, pp. 112-115.
62 *Op. cit.*, II, pp. 116-125.
63 AT II, pp. 17-21.

publication of the latter's book, where he had postulated that nature always acts in the simplest way. This reminds Fermat that the ancients had used this principle to explain the law of reflection. How can it be used to prove the law of refraction which Fermat feels Descartes has failed to do? For he is sure neither of his proof nor of the law itself. Obviously the notion of shortest distance no longer suffices, and Fermat brings in the notion of the relative resistances of the two media. It is necessary to find the path of least resistance, and it is easy to show it is not a straight line when the resistances are not equal. Fermat here uses resistances instead of speeds because, as a concession to de la Chambre, he is still accepting the infinite speed of light.

In two letters to Clerselier, the representative of Cartesianism, in March and June, 1658,[64] Fermat renews his attacks on the Cartesian theory, pointing out that the ball should be constantly impeded by the second medium in all directions, and that common sense would indicate, for all Descartes' concealed percussions, that the speed of light be less in the denser medium. But it is only on Jan. 1, 1662,[65] in a letter to de la Chambre, that he announces his formal proof that, assuming different constant speeds in the two media, the path of least time between two given points astride the surface of refraction, gives precisely the law of sines of Descartes, but with the lesser speed in the denser medium. Fermat, proud also of a new success for his method of maxima and minima, feels now that the law of sines is really established, and he asserts that *recently* he has been assured on all sides and especially by Mr. Petit that experiments agree exactly with Descartes' proportions, so long did it take experiment to catch up with the Baroque imagination.

Clerselier replies that such a principle as that found by Fermat is a moral law and not physical, and as such cannot be the cause of any natural effect.[66]

The irony of history is that two young men, Huyghens and Leibniz, passing first through the fire of Cartesianism, will use this principle of Fermat for light: - the first will establish it in 1678 (to be published in his *Traité de Lumière* in 1690) by a truly Cartesian theory of vibrations in a new Cartesian plenum, the aether; and the second will use it to transcend the Cartesian world with a new kind of explanation in physics and a new kind of matter. We have given this history of the law of sines for refraction not only to show how Descartes' concept of matter is applied and how it will stand as a moment of the dialectic of ideas, but also to throw light on the remark of Leibniz in the *Discours de Métaphysique* composed in 1685:

> "Also I hold that Snell who is the first inventor of the rules of refraction would have waited a long time to find them if he had wished to seek first how light is formed. But apparently he followed the method which the ancients used for catoptrics, which is in fact by final [causes]. . . And this Mr. Snell, as I believe, and after him (although without knowing anything about him), Mr. Fermat have applied more ingeniously to refraction. For when the rays observe in the same media the same proportion of the

64 *Oeuvres de Fermat*, II, pp. 354-359.
65 *Op. cit.*, II, pp. 457-463.
66 *Op. cit.*, II, pp. 464-472. This controversy is also reported in P. Mouy, *op. cit.*, pp. 55-60.

sines, which is also that of the resistances of the media, it turns out that it is the easiest path or at least the most determinate for passing from a given point in one medium to a given point in the other. And the demonstration of this same theorem that Mr. Descartes wished to give by means of efficient causes is far from being as good. At least, there is room to suspect that he would have never have found them in that way, if he had learned nothing in Holland of Snell's discovery."[67]

Modern scholarship does not know how Snell discovered it and has no reason to believe Descartes knew of Snell's discovery until after he found it himself. In one sense, however, Leibniz is right: Descartes discovered it by final causes just as much as did Fermat. But the final cause in Descartes' case was the determination to reduce all phenomena of light to his ideas of extension and motion, and such an heuristic principle of being should properly find its place in the mind-world constructed by Leibniz.

The Cartesian derivation and formula, revamped by Newton, defeated by Fresnel's elegant mathematization of Huyghen's theory (1817-1822) and Fizeau's measurement of the phase velocities in different media (1860), comes to life again, through Hamilton's synthesis (1825-1830) in the theory of de Broglie (1924) where the parallelism of the momentum 4-vector with the wave 4-vector for light is extended to particles having a particle speed less than that of light, so that the particle in its particle aspect obeys the law of Descartes and in its wave aspect that of Fermat and Huyghens. In the experiment of Davisson and Germer (1927) which verified this theory, a bundle of cathode rays was reflected from nickel crystals. The index of refraction was determined as greater than one from vacuum to metal crystals. Hence from the wave point of view, where w is phase velocity.

$$\frac{\sin \alpha_1}{\sin \alpha_2} = \frac{w_1}{w_2} > 1;$$

and from the particle point of view, where v is particle velocity,

$$\frac{\sin \alpha_1}{\sin \alpha_2} = \frac{v_2}{v_1} > 1;$$

the particles are accelerated, therefore, on entering the crystal precisely as Descartes assumed and are impeded on coming out. This was checked experimentally by measuring the potential in the work-energy equation for the Descartes-Newton formula, that is,

$$\frac{|\underline{v}_2|}{|\underline{v}_1|} = \sqrt{1 + \frac{U}{1/2\, m\, v_1^2}}.$$

The derivation of the law of sines for the refraction of light is certainly incomplete in the eyes of Descartes since it involves unresolved and hidden percussions. It is only an example of a partial application of the fundamental laws of the impact of perfectly hard bodies from which all the phenomena should be deduced. In fact, the question of the refraction of light appears

67 *Edition Lestienne*, Paris, 1929, pp. 66-67.

neither in *Le Monde* nor the *Principia;* it is only a special treatise designed to show the application of his method in a piece of prize research.

Hence we turn now to the second problem, that of the percussion of perfectly hard bodies. A perfectly hard body for Descartes is one whose parts are at rest with respect to each other, and soft bodies are those where the parts are moving with respect to each other.[68] No other definition was open to him, since binding forces are not clear and distinct ideas, and are not entertained even as a point of departure for mathematical relations as in the freer tradition of Kepler. But for many, of course, in the Kepler tradition, perfectly hard bodies are those which act elastically, that is, react on each other by an extremely rapid contraction and dilatation. For such people, of whom we have a case immediately below, the perfectly hard bodies of Descartes might well be perfectly soft.

In *Le Monde,* after the general laws of motion, Descartes says he could give as immediate deductions several laws of percussion useful for reducing all phenomena to the first principles of his physics, but that he defers this to another time. In the first and Latin edition of the *Principia* (1644), there appear the famous seven laws of percussion for perfectly hard bodies, with Descartes' own explanations added in the French edition of 1647. But before we study them let us see, aside from the refraction of light, what history they have in Descartes' thought.

Strangely enough in the *Journal* of Beeckman for Nov. 23 to Dec. 25, 1618 at a time when he and Descartes were meeting together for the discussion of problems in physics and mathematics, there appear seven laws of percussion for perfectly soft bodies. Since Beeckman as we have already said, moves in the tradition of Kepler where forces between particles are a primary consideration, the body which rebounds, that is, the elastic body, is one whose parts are not relatively at rest and which contains vacuous pores capable of retraction and expansion. Hence for him, the atoms of his theory (for he accepts them) correspond to Descartes' perfectly hard bodies, except that for Beeckman they are perfectly soft in the sense that they do not rebound but move off together after impact.[69] This theory of Beeckman will indeed be followed by Wallis,[70] and is not an isolated case in Baroque theories. These laws are all correct from the point of view of classical mechanics (except for certain discontinuities); and well they might be, since Beeckman assumes the law of inertia, the conservation of momentum (with mass distinct from volume as for Kepler), and that the bodies are perfectly "inelastic." These laws parallel perfectly those of Descartes which are not correct according to classical mechanics. We therefore translate them here, with our own analytical transcription after each one, for which we use the following notation: $v_1$ and $v_2$ are velocities (positive or negative) in some straight lines before impact and $v_1'$ and $v_2'$ the corresponding ones after impact, where $m_1$ and $m_2$ are masses. As we have said, the reader will notice in the following laws of Beeckman, always

$$v_1' = v_2', \; m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'.$$

1. "And suppose any body at rest is hit by any body in motion. That which was at rest will move with the body in motion in this way: If they are

68 *Principia*, II, 54-55, AT VIII, pp. 70-71.
69 *La Corresp. du P. Mersenne*, II, pp. 633 and following.
70 *Oeuvres Complètes de Huyghens*, XVI, p. 175 and note 17.

equal in corporeality [mass], each will move twice as slowly as the one in motion moved. . . ."

If $m_1 = m_2 = m$, $v_1 = 0$, and $v_1' = v_2'$, then

$$mv_2 = mv_1' + mv_1', \; v_1' = \frac{1}{2}v_2.$$

2. "If the body at rest has twice the mass of the one hitting it, then 2/3 of the speed of the one in motion is subtracted. . ."

3. "If, on the other hand, the one in motion is twice as large, 1/3 of the speed of the one in motion is subtracted and both move off at 2/3 its speed; hence, *the ratio of the sum of the two bodies to the body first in motion is as the first speed of that body to the [final] speed of both."*

   If $v_1 = 0$, and $v_1' = v_2'$, then $m_2 v_2 = m_1 v_1' + m_2 v_1'$ and $\frac{m_1 + m_2}{m_2} = \frac{v_2}{v_1'}$ which is the formula for the first three laws.

4. "If equal bodies meet with equal speeds, they immediately come to rest. . ."

   If $v_1 = -v_2$, $m_1 = m_2 = m$, then $mv_1 + mv_2 = 2mv_1' = 0$.

5. a. "But the motions of unequal speed are added, and each moves with half of the whole speed if they were moving in the same direction."

   b. "But, if they come against each other, the less speed is subtracted from the greater and each moves with half the difference in the direction of the greater speed; for the less motion is cancelled and what remains distributed."

   If $m_1 = m_2 = m$, $v_1' = v_2'$, then $mv_1 + mv_2 = 2mv_1'$,

$$v_1' = \frac{1}{2}(v_1 + v_2).$$

6. "But, if a body twice as large meets the other with the same speed it loses half its speed since it carries it off with itself [using 4]; the other half is bisected and both move off with a fourth of the first speed of the larger body [using 2]."[71]

   If $m_1 = 2m_2$, $v_1 = -v_2$, $v_1' = v_2'$, then $2m_2 v_1 - m_2 v_1 = 3m_2 v_1'$ and $v_1' = 1/3\, v_1$.

7. "But, if the smaller of these were twice as slow, only a fourth part is subtracted from the motion of the larger [4]. For, if the larger body were bisected, one part would be equal to the slower body; and this takes away from the former only half its speed; therefore, there remains in this respect a fourth of the whole. Therefore both joined together will move in the following way: the fourth part of the greater body because of the impact of the smaller, has to be immobile [4]; therefore so far there remain three unaffected parts of speed. These must move the smaller body and a fourth of the greater as if both were at rest. Since therefore the unaffected parts are to these as 3 to 3, the unaffected parts will have the ratio to the whole of 3 to 6. Therefore the twice larger and twice speedier body is moved along with the other, at half the former speed."

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71 It is fairly obvious that, according to the reasoning of Beeckman, this should read "a third," since there is left at rest a mass double the mass moving.

$$\text{If } m_2 = 2m_1,\ 2v_1 = -v_2,\ v_1' = v_2', \text{ then } m_1 v_1 - 4m_1 v_1 = m_1 v_1' + 2m_1 v_1',\ -3v_1 = 3v_1' \text{ and } v_1' = -v_1'.$$

There follows, then, the statement: "And with these rules assumed, motion in a vacuum can never be understood as accelerating, but all things must strive for rest by equal impacts. Whence it follows that God Almighty alone has been able to conserve motion by moving together the greatest bodies with the least speed which constantly resuscitate and vivify in succession the others which strive always for rest." And in the margin, Beeckman wrote his law of entropy: "Motion in a vacuum never increases but always decreases. Why therefore is there not a universal rest?"[72]

This dying world of Beeckman, kept alive only by God's predestined choice of an elect of bodies, a kind of Calvinist world innately dead and artificially redeemed by an exterior decree, will be avoided by Descartes where the world moves on by itself after an original and unique creation. This will result simply from the fact that, for Descartes' definition of matter as pure extension, the perfectly soft bodies of Beeckman are equated with the perfectly hard bodies of the Cartesian theory which in turn are equated with the perfectly elastic bodies suggested by experience.

It will be noticed that, of course, the laws of Beeckman will hold for all observers moving at constant velocity with respect to each other. He therefore does not need to pick out a reference space absolutely at rest as the only space where his laws could be verified.

In a letter to de Beaune, April 30, 1639,[73] Descartes says that his physics is entirely a mechanics, but that he has really never gotten around to formulating his laws of speed. He goes on to give an extremely clear and forceful statement of the law of inertia which had not been so stated by Galileo, as Koyré has gone to great pains to show in his *Etudes Galiléennes*.[74] He then states the universal law of the conservation of the absolute quantity of motion, and illustrates with the example of the ball hitting the earth without rebound so that the earth's greater extension absorbs the ball's motion; this is obviously a limiting case of Beeckman's laws.

In a letter to Mersenne, Dec. 25, 1639,[75] he again states the law of inertia and now refers to a law of percussion of bodies which after impact move off together, precisely the first two laws of Beeckman. In other letters to Mersenne from March 11, 1640 to April 26, 1643, Descartes treats of certain cases of the laws of percussion, particularly of his own Rule 5, which he will discuss in its place.

We are now ready to analyze the seven rules of percussion of Descartes as they appear full blown in the *Principia*, embodying, as they do, the general laws of inertia and of the conservation of the absolute quantity of motion and purporting to be the laws for perfectly hard bodies in the sense of Descartes, those which have no relatively moving parts. But what really makes them represent the elastic bodies of ordinary experience is the peculiarly Cartesian law of the conservation of the absolute quantity of motion as opposed to Beeckman's law of the conservation of momentum, but it is also this law

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72 *Corresp. du P. Mersenne*, II, p. 635.
73 AT II, p. 543.
74 *Galilée et la loi d'inertie*, III, pp. 69 and following.
75 AT II, p. 627.

which makes them for the most part false from the point of view of classical mechanics. We shall state them as they appear in the Latin edition but in our own analytic notation, since the original text is so generally available, placing after them the number of the corresponding rule of Beeckman.[76]

By m we shall denote the volume of the perfectly hard body, v speed before impact and v' after. Always, as we have said

$$m_1|v_1| + m_2|v_2| = m_1|v_1'| + m_2|v_2'|,$$

as opposed to Beeckman's

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2';$$

and the distinction of determination and speed will play its role as prefigured in the law of refraction. The rules are:[77]

1. If $m_1 = m_2 = m$, and $v_1 = -v_2$, then $v_1' = -v_1$, $v_2' = -v_2$.
2. If $m_1 = m_2$, and $v_1 = -v_2$, then $v_2' = -v_2 = v_1'$ [6]
3. If $m_1 = m_2 = m$, $|v_1| = |v_2|$ but opposite in sign, then $v_1' = v_2'$ with the sign of $v_1$ and $|v_1| + |v_2| = |v_1'| + |v_2'|$. [5b]
4. If $m_2 > m_1$, $v_2 = 0$, then $v_2' = 0$, $v_1' = -v_1$ [2]
5. If $m_1 > m_2$, $v_2 = 0$, then $v_2' = v_1'$ with sign of $v_1$, and $m_1 v_1' + m_2 v_2' = m_1 v_1$ [3]
6. If $m_1 = m_2$, $v_2 = 0$, then $m_2 |v_2'| = \frac{1}{4} m_1 |v_1|$, $m_1 |v_1'| = 3/4\, m_1 |v_1|$; and $v_1'$ has sign of $v_2$, but $v_2'$ has sign opposite of $v_2$. [1]

7a. If $m_2 > m_1$, $v_2 < v_1$ and both positive, $v_1 - v_2 > m_2 - m_1$, then $v_2' = v_1'$, $m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$.

b. If the same holds, but $v_1 - v_2 < m_2 - m_1$, then they reflect in opposite directions keeping their respective speeds. [7]

In the first three, we see operating immediately the principle of distinction of determination of direction and quantity of motion. It is obvious that the direction after impact is determined by the direction of the body having the greater quantity of motion before impact. In the first, there is no difference, so they rebound each in the opposite direction. The objection of Fermat and the reply of Descartes in this same matter with respect to refraction could be repeated here.

In the next three, we see operating the principle that rest is an opposite state to motion, more opposite, according to Descartes, than motions in opposite directions. It is here that we can understand Leibniz' contention that rest is not the opposite of motion but is continuous with it. Again, the direction after impact is decided, in cases (4) and (5), by comparing the combi-

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76 See Cornelis de Waard, *Corresp. du P. Mersenne*, II, pp. 633-634.
77 *Principia*, II, 46-52, AT VIII, pp. 68-70.

nation of volume and rest with that of volume and speed, where rest increases proportionally with the opposing speed, as Descartes says in his explanation in the French edition of 1647.

In (6), we have a complicated combination of all of these principles. Descartes explains it, in the French edition, in this way:[78]

"For, since it is necessary either that $m_1$ push $m_2$ without rebounding and thus impart to $m_2$ half of its motion; or that it rebound without pushing $m_2$ and thus retain all its motion; or finally that it rebound keeping only a part of the half of its motion, and push it by transferring the rest: it is evident, since they are equal and thus there is no more reason for it to rebound than to push $m_2$, these two effects must be equally divided."

The reader may, as an exercise, work out (7) for himself.

It has been noticed, of course, that there is a discontinuity between (6) and (3); but motion for Descartes is not necessarily continuous and he points out, in the case of (5), that $m_2$ passes instantaneously from rest to a speed $v_1'$ common with $m_1$ and in the same direction with $v_1$. For it can never have a speed between zero and $v_1'$.[79] No example could show better the difference between the perfectly hard bodies of Descartes and the perfectly elastic bodies of the classical theory.

These rules can never be directly verified since there is no possible isolation of two particles, nor even a probability of approximating such an isolation, in the plenum of Descartes where a rare medium simply means one with more relative motion between its very small parts. Indeed, there is not even a perfectly hard body since there can be relative motion in every volume no matter how small. Before the publication of the *Principia*, an objection was made to (5), since experience shows that the smaller body moves off faster than the larger; and Descartes replies in a letter to Mersenne, Feb. 23, 1643, that this is because the bodies are not perfectly hard and the plane not perfectly smooth.[80]

It is obvious that rule (4) was to raise serious doubts, for the case of a large ball suspended in the air and hit by a smaller one is too clear to everyone in spite of all the hidden percussions which the Cartesian theory can bring into play in the fluid action of the air.

In a letter to Clerselier, Feb. 17, 1645, Descartes announces another general principle to explain these rules:

"When two bodies meet which have in them incompatible modes, there must be some change in these modes which makes them compatible, but this change must always be the least there can be, that is to say, if, when a certain quantity of these modes is changed, they can become compatible, then there will be no change of a greater quantity. And we must consider in movement two different modes: The one is the motion alone or speed, and the other is the determination of this motion in a certain direction, which two modes are changed with as much difficulty the one as the other."

And he goes on to show how this principle explains rules (4), (5), and (6).[81]

78 AT IX, p. 52.
79 *Descartes to Mersenne*, Nov. 14, 1642, AT III, pp. 592-593.
80 AT III, p. 634.
81 AT IV, p. 185.

It is slightly ironical that this minimal principle is addressed to Clerselier who, later, as a leader of the Cartesian school, will upbraid Fermat for his.

Another very serious problem, not immediately raised, but which will be noticed, with the most fruitful results, by Huyghens, is that the rules of Descartes can only hold for an absolute motion and an absolute rest; except for the first one, they are not invariant for systems having constant velocity with respect to each other, although Beeckman's are. The opposition of motion and rest is at the center of this problem. The whole question of absolute and relative motion in the *Principia* is unclear, since Descartes on the one hand stresses the relativity of change of place, while on the other the fundamental law of the conservation of motion and the very idea that God created at once a certain definite quantity which is conserved would seem to demand an absolute reference point. The meaning of the phrase "considered as at rest" quoted in the definition of local motion from the *Principia* in the beginning of this paper, in view of these considerations, can only be a puzzle. It is quite clear that, for political purposes, Descartes in this treatise does not want to make a decision between Ptolemy and Copernicus, although he held firmly for the earth's motion which he considered the basis of all his physics:

> "I am not sorry that the [Protestant] ministers fulminate against the earth's movement; that will perhaps invite our [Catholic] preachers to approve it. . . nothing has prevented me up until now from publishing my *Philosophy* except the prohibition against the earth's motion which I could never separate from it since all my physics depends on it."[82]

Therefore it would seem these rules would apply only in a system at rest with respect to the sun, and from them must be deduced the astronomical systems in an infinite world space, as well as the laws of falling bodies here on the earth. No wonder Descartes almost despaired of being able to find the precise laws of gravity.

Descartes, like Kepler and unlike Galileo, aspires to explain all motions both terrestrial and celestial from one set of laws, but, unlike Kepler, he was unable to play with alternative mathematical theories.

> "With respect to physics," he says, in a letter to Mersenne, March 11, 1640, "I would think to know nothing, if I could only say how things can be, without showing that they cannot be otherwise; for, having reduced it to the law of mathematics, it is possible, and I believe I can do it in all the little I believe I know, although I have not done so in my Essays [*Dioptrique, Méteores, Géometrie*], because I did not wish to expound there my principles and as yet I see nothing which invites me to expound them in the future."[83]

True, Kepler has a similar preoccupation when he declares it insufficient for a theory only to save the appearances, but his archetypal reason is just far enough away to allow him more freedom of maneuver.

But these rules of percussion given by Descartes in his final work which he admitted himself presented certain difficulties,[84] and which seem so strangely impossible to the modern reader, belonging to an imaginary world

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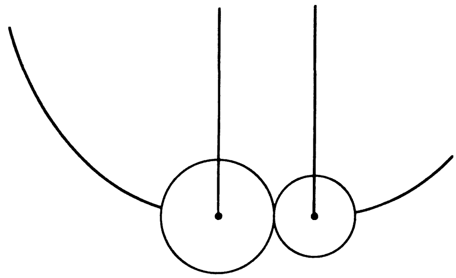
82 Letter to Mersenne, Dec. 1640, AT III, p. 258.
83 AT III, p. 79.
84 AT IV, p. 187.

of thought foreign to our immediate experience, were to lead by their stimulation, as we have said, to the famous treatise on percussion of Huyghens to be finally incorporated in Newton's *Principia* as a justification for his third law. Although the *De Motu Corporum ex Percussione* of Huyghens was only published in 1690, we have notes and projects which show us that Huyghens had essentially solved all the problems involved in 1656.[85] The verification of the laws of Huyghens was accomplished by the well known experiment of two spheres hanging as two pendulums which are just in contact when at rest. Since Huyghens himself extended Galileo's law that the speeds acquired on an inclined plane are proportional to the square roots of the height of fall in a vacuum to the case for any smooth curve,[86] the speeds with which they hit could be ascertained from the height of their fall, and the respective speeds of rebound judged by the heights to which they rise.



Huyghens reports a famous seance in his room in London in 1661 with Wallis, Wren, and others[87] where he successfully applied his theorems and whose echoes are found in Newton's *Principia,* Book I, Scholium to Law III.

We discuss this theory from the published treatise.[88] Huyghens starts with five hypotheses:

1. The law of inertia of Descartes.
2. Rule 1 of Descartes: that two equal bodies meeting from opposite directions with equal speeds rebound with the same speeds.

So far Huyghens is precisely Cartesian; but in the next one, he will exactly find the general weakness of the Cartesian theory and correct it:

3. The laws must hold for all systems of coordinates moving at constant velocity with respect to each other.
4. If $m_1 > m_2$, $v_2 = 0$, then $v_1' < v_1$ and $v_2' = \delta$.

This follows the general form of Descartes' Rule 5, but corrects it by being less determinate.

5. If $v_1 = v_1'$, then $v_2 = v_2'$, a special case of the conservation of momentum.

Huyghens is careful to say "for whatever cause the hard bodies rebound" and so avoids committing himself to any definite position.

To these is added, in Prop. VIII, the mechanical axiom that the center of mass of the system cannot rise. From these hypotheses and this axiom,

85 *Oeuvres Complètes de Huyghens,* XVI, pp. 93 and following. These appendices to the treatise contain earlier versions, dating from 1652 to 1656.

86 The method of Huyghens (1660) which consisted in breaking up the smooth curve into successive inclined planes, *Oeuvres Complètes,* XVII, p. 133, is criticized by Newton in a letter to Oldenburg, June 1673, Corresp. I, p. 290.

87 *Oeuvres Complètes de Huyghens,* XVI, pp. 172-173.

88 *Oeuvres Compl. de Huyghens,* XVI, pp. 33-91.

Huyghens deduces what he considers the most significant principle of all, the principle of the conservation of kinetic energy. (Prop. XI)

$$m_1 v_1^2 + m_2 v_2{}^2 = m_1 (v_1')^2 + m_2 (v_2')^2.$$

For us, who are accustomed to see this principle deduced, in a special case, from the laws of Newton, it is hard to realize that it was deduced long before the publication of the *Principia* in 1687, from the subtle modification we have seen of the Cartesian laws of percussion.

So much for the first two problems. As for the third, it is well known that the problem of falling bodies was for Descartes a very derivative one. In 1619, Beeckman asked Descartes to find the distances as a function of the times for constantly accelerated bodies, and Descartes gave him a perfect mathematical solution but a completely erroneous physical interpretation while Beeckman records for himself the correct one.[89] Descartes again refers to this solution in a letter to Mersenne in 1629.[90] We shall not stop here to analyze his later theory of gravity leading to the well known vortex motions of the last works.[91]

We have given the Cartesian background in such detail because the Leibnizian concept of matter cannot be understood without it. Again and again Leibniz states that his world view was attained in passing through the Cartesian mechanism to a domain beyond it. We have only small space left, in this paper, to delineate this Leibnizian world. Every one of the major theses of Leibnizian metaphysics can be traced to a corresponding law of mechanics since the world of mechanics is symbolic of the real world, and, by properly reading the symbols, we can properly know the world it represents.

On the other hand, our thesis transcends perhaps the debate begun by Couturat in an article[92] where, with Russell, he claims that the metaphysics of Leibniz follows directly from his logic of analytic propositions and has little to do with his mechanics. Rivaud, replying to this in his article of 1914[93] and using Leibnizian texts just edited, claims that all Leibniz thinking proceeds from the principle of harmony and he inclines to the traditional view of a strong dependence of the Leibnizian metaphysics on the Leibnizian mechanics. Guéroult, in his *Dynamique et métaphysique leibniziennes*, Paris, 1934, defends in detail the traditional view. When one considers how often Leibniz declares that mathematics and mechanics are only symbols of a deeper reality, it is not likely that he would raise logic to that eminence. It would be exactly consonant with the Leibnizian theory of monadic perception and thought to consider logic, mathematics, mechanics, and metaphysics as symbolic systems representative of an idea which none expresses completely. And it would not be foreign to the tradition in which Leibniz thinks, to feel that metaphysics standing by itself is likely to cut a poor figure, communicating little to anyone. And Couturat himself admits[94] that our academic history of philosophy has been completely sterile in trying to teach Descartes,

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89 AT X, pp. 58, 75 and following; p. 219 and following. See Koyré, *Etudes Galiléennes* II, pp. 108-111.

90 AT I, pp. 71-72.

91 See AT II, pp. 544, 593.

92 "Sur la Métaphysique de Leibniz," *Revue de Métaphysique et de Morale*, 1902.

93 "Textes inédits de Leibniz publiés par Ivan Jagodzinsky," *Ibid.*, 1914.

94 "Le Système de Leibniz d'après M. Cassirer," *Ibid.*, 1903, p. 85, n. 1.

Leibniz, and Kant without Kepler, Galileo, and Newton. We would suggest that the creation of a system of physics or mathematics is often more an exercise of the science of metaphysics than the official discussions of professional metaphysicians. But better putting aside even these general statements, we can follow the Leibnizian development and divine for ourselves the view of the world it entails.

Starting as Descartes did with the thinking subject, the *cogito ergo sum*, Leibniz assumes a universe of thinking subjects, the only substances besides God, since the Cartesian substance, extension, is dissolved into a pure symbolic representation of this world of thinking subjects. Already in his first important dynamical treatise of 1671, the *Hypothesis Physica Nova*, strangely composed because it precedes Leibniz' entrance into the discipline of mathematics with the helping hand of Huyghens in Paris in 1672, there appears the sentence, "All body is momentary mind or lacking recollection,"[95] which one might translate "all body is mind appearing under the conditions of time," a phrase which was later to be recalled with pleasure by no less a person than John Bernoulli in his correspondence with Leibniz, June, 1695.[96] Further, the notion of *conatus*, used also in Descartes' *Principia* to name the tendency of a body in circular motion to move off at a tangent, becomes a prominent term, symbolizing as it does the nascent force intruding into the apparent world of time, space, and motion. This same treatise, divided into two parts, the theory of concrete and the theory of abstract motions, sets out, in its awkward way, the predominant themes of Leibniz' life work: In the concrete part, the problem of the planetary motions and the problem of the refraction of light, and, in the theoretic part, the problem of the percussion of bodies.[97] We see here the Cartesian view that all the appearances must be reduced to the fundamental laws of percussion between primitive bodies (what ever these may be), a general doctrine which Leibniz, as well as Huyghens, will carry to the bitter end against the notion of action at a distance of the Newtonian school. But, in this treatise, for Leibniz, the bodies of the Huyghens-Wren experiments are the bodies of a concrete situation,[98] while the primitive bodies of the abstract motion are those of Beeckman and Wallis which move off together after collision. But this will not be the later doctrine; for any idea of primitive bodies will disappear. At least he has here accepted the law of conservation of momentum in a special case, a law which he will later generalize and which will become the symbol for the doctrine of pre-established harmony, a profound doctrine, usually understood in a dull and shallow fashion.

In the letter to Honoratius Fabri, about the same time, we find the praise of Descartes but also the statement that many things must be amended in his doctrines and in particular the doctrine that body and extension are identical. For why, it is asked, can a body be in several places if body and space are the same?[99] In the *Demonstrationes Novae de Resistantia Solidorum* of 1684, it is remarked that the science of mechanics seems to have two aspects, one the power of acting or moving, the other the power of being passive or of resisting. These will symbolize for Leibniz the active thought and

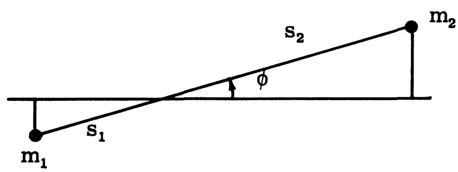
95 *Gesammelte Werke*, Gerhardt, *Math. Schriften*, VI, p. 69.
96 *Virorum Celeberrimorum, Leibnitii et Johan, Bernoullii Commercium Philosophicum et Math.*, 1745, I, p. 62.
97 *Math. Schrift.*, VI, p. 72.
98 *Ibid.*, p. 29.
99 *Ibid.*, pp. 94-95.

the passive thought or sensations of Descartes,[100] just one step in many to transform the Cartesian world of extension to the Cartesian world of thought, to transform kinematics to dynamics.

It is around 1676 that Leibniz translates two dialogues of Plato, the *Phaedo* and the *Theaetetus*.[101] Nothing could be more significant than the almost pre-established harmony of his choice of dialogues and the direction his own thought has taken and will take. For the *Phaedo* will give him the two kinds of causes: (1) The necessary conditions to be transformed into the principle of contradiction, and (2) the sufficient reasons or conditions of unity and of the good.[102] The *Theaetetus*, together with the *Sophist*, is the dialogue of Platonic dynamism which will furnish the other side of the Leibnizian system, the plenum of forces.[103]

In the *Brevis Demonstratio* of 1686, that is, *The Short Demonstration of the Memorable Error of Descartes and Others concerning the Natural Law, according to which they wish always the same quantity of motion to be conserved,* Leibniz gives an early exposition of his profound interpretation of the Huyghens' theory of percussion taken together with Galileo's law that the square of the speed acquired by falling bodies from rest on the surface of the earth is proportional to the height of fall, and that a body will rise as well as fall according to this proportion. Since Huyghens had proved, from his assumptions about perfectly "hard" percussions, that the sum of the squares of the speeds before and after impact is conserved, and, since this law together with the law of conservation of momentum holds for all observers moving at constant velocity with respect to each other, while, as we have seen, the corresponding Cartesian law of the conservation of the quantity of absolute motion does not, therefore Leibniz insists that the conservation of momentum and the conservation of kinetic energy hold universally, and that they are the symbols of the real world of thinking subjects. The origin of Descartes' error, he claims, lies in his generalizing of the derivation of the law of the lever by differential displacements (or, as one would say later, virtual displacements or velocities):

$m_1v_1 = m_2v_2$, $m_1s_1 \sin\phi = m_2s_2 \sin\phi$,




to actual speeds. This is a false leap; these virtual speeds are what Leibniz calls "dead" forces as opposed to the "live" forces which produce, not the tendency to motion, but the actual motion, and which are the integral of these:[104]

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100 *Ibid.*, p. 106.

101 See note in *Math. Schrift.*, VI, p. 9.

102 Leibniz quotes the passage concerning the νοῦς of Anaxagoras and the reasons for Socrates' remaining in prison, not only in the *Discours de Métaphysique* but also in the *Principium quoddam generale non in math. tantum* etc., *Math. Schrift.* VI, pp. 134-135.

103 Leibniz to Reymond, Jan. 10, 1714, *Phil. Schrift.* III, pp. 606-607. Also *Epistola ad Hanschium*, 1707, Erdmann II, p. 445, Besides, there is a constant Leibnizian reference to the αὐτοκίνητον ἀθάνατον of the Phaedrus. See also *De Primae Philosophiae Emendatione, Phil. Schrift.* IV, p. 469.

104 *Math. Schrift.*, VI, pp. 117-118, and also the *Beilage*, pp. 119-122. See also *Système nouveau, Phil. Schrift.*, IV, pp. 486-487.

$$\int m\, v\, dv = 1/2\ m\, v^2 + c,$$

the '1/2' being usually omitted.

Hence motion, symbolized by speed, cannot characterize anything, since what has motion for one observer, may not have it for another. And so it comes about that one of Descartes' fundamental modes of extension, an active thought, is dissolved by Leibniz as inadequate except as it appears conjointly, and not simply, in the two conservation laws mentioned above. This is explained very carefully in a later treatise, the *Specimen Dynamicum pro Admirandis Naturae Legibus etc.* of 1695 where the conflict of Descartes' conservation law with his notion of the relativity of motion as defined in the *Principia* is brought up. The Leibnizian dissolution of the Cartesian primitive idea of motion is accompanied by the dissolution of extension and time. In extension, there appears resistance to motion, as exemplified in the case of Descartes' Rules 4 and 5 of percussion whether one accepts the conclusions of Descartes or of Huyghens; the inertial property of body as defined by Kepler is here evident.[105] But for Leibniz, the notion of extension is indifferent to motion,[106] and the fact that the slower body overtaken by the faster changes the speed of the faster is evidence of this diremption between body and extension. In the earlier dispute with Malebranche,[107] whereas Malebranche considers two congruent parts of space as two clear and distinct ideas, Leibniz can see no note or requisite by which they can be distinguished unless they are considered as relations within another world which can furnish a ground for their differentiation. This is the source of the famous Principle of Indiscernibles: space, time, and motion are not adequate to distinguish individuals, although their combinations in the true laws of mechanics will show the existence of the only individuals which could possibly be the centers of active and passive thought.

The dissolution of the idea of extension is reported in several places in Leibniz in the following way.[108] Extension is *infinitely divisible in act.* But strangely enough, Leibniz cannot consider the parts resulting from this division as being points, since points cannot give the continuum of expansion. [It would be interesting to speculate on the position of the Cantorian Continuum in the system of Leibniz.] But there must be real unities, that is, indivisibles, to support this extension, which are like points but are not points and are beyond extension. These are the intellectual units or monads, all different in the only way intellectual beings can be different, by thinking different thoughts, each representing actively and passively the universe of monads from its own point of view. The vectorial law of the conservation of momentum, as I have said, is the symbol of this unity of the monads from the dynamical point of view of percussion; for there will be another and deeper symbol of this unity revealed later. And the problem of mind and body is resolved as this correspondence of symbol and symbolized. The law of the conservation of kinetic energy, on the other hand, represents the sum of the active forces of the monads or of their active thoughts, given at creation by a divine fulguration, subject to an intellectual development within each monad,

105 *Math. Schrift.*, VI, pp. 247-248.

106 *De ipsa Natura, sive de Vi Insita Actionibusque Creaturarum* (1698), *Phil Schrift.* IV, p. 510.

107 Leibniz, *Philosophisches Briefwechsel, Sämtliche Schriften und Briefe*, Reihe II, B. II, Preuss. Ak., 1926, pp. 254-259.

108 *Système nouveau de la nature, Phil. Schrift.* IV, p. 478. *De ipsa Natura*, IV, p. 511.

within the system of them all as symbolized by the laws of the transfer of motion in space and time.[109]

The monad is a substance in the radically Cartesian sense of a thinking subject[110] and the bastard notion of a not-thinking substance, extension, which held such an uneasy position in the Cartesian system has become a phenomenon, that is, a symbol with Leibniz:

> "The unity of a clock which you mention is quite different for me from that of an animal, the latter able to be a substance endowed with a true unity like what one calls the I in ourselves."[111]

To return to the world of symbols, the world of pure mechanical laws, Leibniz argues against the Cartesian rules of percussion in three ways: (1) they do not obey the law of the conservation of momentum; (2) combined with the law of falling bodies, they lead to two absurdities in not obeying the law of the conservation of kinetic energy, one in the loss of energy which introduces an asymmetry in the effects from fall and rise, and the other in the increase leading to a perpetual motion machine;[112] (3) they violate the Principle of Continuity, according to which in geometry, continuity in the data requires continuity in the consequences, a principle to be applied also in physics.[113] The analogy is here drawn between the projective properties of the conic sections and Rules 1 and 2 of Descartes.

The enunciation of the law of the conservation of kinetic energy by Leibniz will be challenged by Huyghens and John Bernoulli in an exchange of letters which will lead to fruitful clarifications.

Huyghens, in July, 1692,[114] proposes to Leibniz that the primary bodies are the perfectly hard bodies of Descartes except that they are atoms distinct from extension, bodies having indeed infinite hardness. The reply of Leibniz can be foreseen. There can be no atoms, since the phenomenal world of bodily extension has been dissolved into a monadic support, whose qualitative differentiation in the world of thought can only be represented in the world of mechanics by the infinite differentiation of the degrees of hardness so that two bodies macroscopically have the same hardness only statistically. Furthermore Leibniz sees no way of getting elasticity from perfectly hard bodies in the Cartesian sense of bodies with no internal motion; on the contrary, every body must contain subtle motions to infinity,[115] so that just as gravity can give back exactly the speeds absorbed from the rising body to the same body falling again through the same height to conserve kinetic energy, so the elastic body must receive and give back the same kinetic energy it received on impact, which entails that every body, no matter how small, is never completely at rest within itself. Moreover, already in the *Essai de dynamique sur les lois du mouvement,* Leibniz had shown that the percussion of hard nonelastic bodies is impossible since it would necessarily violate the

109 *Discours de Métaphysique*, Edit. Lestienne, pp. 50-58.

110 *De ipsa Natura, Phil. Schrift.*, IV, pp. 509-510. See also Letter to Boyle, 1687, *Phil. Schrift.*, IV, p. 48.

111 *Eclaircissement du Nouveau Système*, Ibid., IV, p. 494.

112 *Illustratio Ulterior Objectionis contra Cartesianam Naturae Legem, Math. Schrift.*, VI, pp. 123-125.

113 *Principium Quoddam Generale non in Mathematicis Tantum* etc., *Math. Schrift.*, VI, pp. 129-131.

114 *Math. Schrift.*, II, pp. 139-158.

115 *Ibid.*, VI, p. 228.

Principle of Continuity or the principles of conservation.[116] Hence again the metaphysical world of monads is in harmony with the symbols of percussive motions.

The attack of Bernoulli is against the conservation of kinetic energy itself. Leibniz could not state the law in its more classic form of kinetic and potential energy. For haunted, as was Huyghens, by the necessity of avoiding all actions at a distance in mechanical formulas (strange that action at a distance could not be considered, in its strict mathematical formulation, as a symbol of the intellectual relations of the monads!), Leibniz assumes that the equivalence of the kinetic energy with the height attained by a rising body is really a conservation of energy of motion by the same sort of concealed motions as Descartes imagined.[117] But Bernoulli remarks in 1695 that the proportionality of height and the squares of the speeds is an accidental relation,[118] and yet the demonstration of the universality of the law is taken from this. Quite other proportionalities could be set up for bodies moving in impeding media; and after all for Bernoulli, Leibniz, and Huyghens as well as Descartes, gravity is the macroscopic phenomenon of such a medium. Leibniz replies with a notable distinction:[119] In all cases where the situation is such that the surrounding medium can give back exactly (except for small differences) what it has taken from the motion of the body, then it will act so as to conserve the square of the speeds. There is no proof for this except to allege two known cases, that of gravity and that of elasticity in percussion. It would have been gratifying if he had shown it for the inverse-square field. In other situations, there is dispersion of kinetic energy and no return; but, of course, the kinetic energy is conserved in the dispersed motions which are beyond our vision. Since in the tradition of Beeckman and Descartes, at least, heat and light are explained by the very rapid motion of subtle particles, it is likely these transformations of energy were included in the law in the mind of Leibniz although they do not seem to have been stated explicitly. The most obvious case of such an apparent loss of kinetic energy is in the percussion of completely inelastic bodies as pointed out in the *Essai de dynamique*.[120] Leibniz is at no loss to explain that the law of conservation of momentum, because of its vectorial character, holds even when the other apparently does not.

It should be remarked that, although Leibniz denies the Cartesian identity of extension and body or mass, the same is not true of the Cartesian proportionality of matter and volume:

> "Interim non assero in rigore philosophico eandem materiae quantitatem majus minusve volumen occupare posse, imo contrarium verius puto; nobis vero hic sufficit, ad sensum sic videri, etsi fortasse materia levior revera sit spongiosior, nec volumen suum exacte impleat, interfluente

116 *Ibid.*, VI, p. 229.

117 Leibniz tried to build a mathematical theory for Kepler's laws much like Huyghens, using the latter's formula for centrifugal acceleration in circular motion, with a plenum and without action at a distance. See *Tentamen de Motuum Coelestium Causis*, *Math. Schrift.*, VI, pp. 144 and following. For a discussion of such theories, see Hall, "Cartesian Dynamics," in *Archive for History of Exact Sciences*, I, 1961.

118 *Leibnitii et Bernoulli Comm.*, I, p. 63.

119 *Ibid.*, I, pp. 68-69.

120 *Math. Schrift.*, VI, p. 230; *Leibnitii et Bernoullii Comm.*, I, p. 147.

alia materia tenuiore, sed quae ad rem non facit, nec mobile de quo agitur constituit, nec in motu ejus computata."[121]

Which is essentially the explanation of Descartes.

So far we have shown the development of the Leibnizian dynamics from the Cartesian and Huyghenian laws of percussion and the consequent metaphysics of monads. But in the same *Essai de dynamique*, Leibniz made a fateful definition.[122] Trying to find some way of saving Descartes' law of the conservation of absolute motion, $m|v|\Delta s$ where $\Delta s$ is the arc-length or distance traveled, or, as Leibniz analyzed it, the formal effect, $m\,\Delta s$, multiplied by the promptness of the effect, $|v|$, which Leibniz shows to be conserved in the percussion of perfectly elastic bodies. Later, in the *Dynamica de Potentia et Legibus Naturae Corporeae*, it is defined[123] as

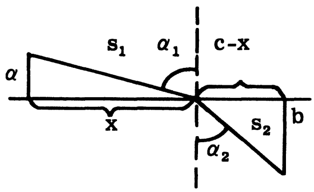
$$A = \int \overline{dt \int \overline{dm\, v^2}}.$$

In the last years of his life, this definition will play a role in his concern for minimal and maximal laws, to lay bare the very foundations of the actual world chosen from all the possible worlds subject only to the law of contradiction.

We have quoted the passage from the *Discours de Métaphysique* (about 1686) on the Principle of Fermat for the refraction of light. Already in the *Unicum Opticae, Catoptricae et Dioptricae Principium* of the *Acta Eruditorum*, June, 1682, Leibniz had used this principle but with a different twist, considering the denser medium to give the greater resistance and the greater speed on the analogy of a river contained by stronger banks; he thus derived the formula of Descartes.[124] The Fermat principle is a principle of least time, we recall, so that, in the familiar proof,

$$t = \frac{s_1}{v_2} + \frac{s_2}{v_2}$$

$$= \frac{\sqrt{a^2 + x^2}}{v_1} + \frac{\sqrt{b^2 + (c-x)^2}}{v_2}$$




is minimized. But, in 1744, Maupertuis presented to the Academy of Paris a new principle, the Principle of Least Action, using Leibniz' definition.[125] Applying it to the problem of refraction, he deduced the formula of Descartes-Newton; for

$$A = s_1 v_1 + s_2 v_2 = v_1 \sqrt{a^2 + x^2} + v_2 \sqrt{b^2 + (c-x)^2}.$$

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121 *Dynamica de Potentia et Legibus Naturae Corporae*, Pars I, Cap. 2, *Math. Schrift.*, VI, p. 297.

122 *Math. Schrift.*, VI, p. 220.

123 *Ibid.*, p. 431.

124 I have not been able to read this work, so I rely on the account of Fleckenstein and Euler in *Leonhardi Eulerii Opera Omnia, Commutationes Mechanichae*, Series Secunda, V, 1957, p. xxxviii and p. 184.

125 Maupertuis, *Accord des différentes Loix de la nature*, in *L. Euleri Opera Omnia, op. cit.*, pp. 274-281.

Minimizing, we have

$$0 = v_1 x \sqrt{b^2 + (c-x)^2} - v_2 (c-x) \sqrt{a^2 + x^2}$$

or

$$\frac{v_1}{v_2} = \frac{\sin \alpha_2}{\sin \alpha_1}.$$

In 1746, Maupertuis presented to the Berlin Academy[126] the application of this principle to the law of the lever and to the laws of percussion, and, in 1748, Euler extended it to more general cases.

But in 1750, Kōnig alleged that, in a letter to Hermann in 1707, Leibniz had announced his application of a law of least and greatest action to several mechanical situations. A copy of this letter was later published by Kōnig.[127] Since Kōnig could never produce the original, this letter was declared a forgery by the Berlin Academy, but modern scholarship has indirectly established the great probability of its genuineness.[128]

That such a principle should crown the life work of Leibniz, his mechanics and his metaphysics, is the fitting end of the development of such a system. Nor is it isolated. We know that Leibniz was working with John Bernoulli on the solution of the brachystochrone or the path of least time for a particle under gravity from June, 1696,[129] and that he was applying its results to his metaphysics. Indeed, the *Tentamen Anagogicam* is written for the sole purpose of showing the necessity for architectonic principles in mechanics and the insufficiency of geometry alone. It is here[130] he remarks that the best of forms is found not only in the whole but also in every part of the whole, just as the curve of least time between two given points is the curve of least time between any two other points of that curve no matter how near. And at the end, after having extended the principle of the most determined path to reflection and refraction at curved surfaces, he adds: "But I have found also other laws of nature, very beautiful and very general, and yet quite different from those which are usually used, and always dependent on architectonic principles."[131]

Later in a letter to Basnage,[132] Bernoulli himself makes another step to bridge the gap between mechanics and the refraction of light. Remarking first that the brachystochrone curve, the cycloid, which he has found, is shown by his method to be unique (a significant fact for Leibniz' metaphysical interpretation) against the conjecture of Tschirnhaus, he says that this

126 *Les Loix du mouvement et du repos déduites d'un principe mêtaphysique, ibid.*, pp. 282-302.

127 *Lettre de Mr. de Leibniz, Ibid.*, pp. 264-267.

128 W. Kabitz "Ubereine in Gotha aufgefindene Abschrift des von S. König in seinem Streit mit Maupertuis," *Sitz. Ber. Kgl. Preussische Ak. der Wiss.*, 1913, pp. 632-638. See also Fleckenstein, *Vorwort des Herausgebers, Eulerii Opera Omnia, op. cit.*, p. xxxiii.

129 *Leibnizii et Bernoullii Comm.*, I, pp. 160 and following. See also Caratheodory, "The Beginning of Research in the Calculus of Variations," in *Osiris*, III, 1938, Appendix I, pp. 235-236.

130 *Phil. Schrift.* VII, p. 270.

131 *Ibid.*, p. 279. See Also Guéroult, *op. cit.*, p. 215 et seq.

132 *Johann Bernoullii Opera Omnia*, I, p. 194; reprinted in Appendix II of Caratheodory's article cited above.

same curve and method can solve not only the problem of quickest descent under gravity but also the problem posed by Huyghens in his *Traité de Lumière*[133] of the path of a ray of light and a wave of light in a constantly varying medium.

> "For I show a wonderful thing, that if a diaphanous medium beginning at a luminous point and descending vertically changes in rarity proportionally as the speeds acquired by a heavy body, the curve of quickest descent will be exactly the same as the ray of light, that is, both will be the roulette or cycloid. . . So that these two speculations, taken from two such different parts of mathematics as dioptrics and mechanics, have between them an absolutely necessary and essential relation."

In mathematical notation, we have on the one hand,

$$t = \int dt = \int \frac{dt}{ds} \cdot \frac{ds}{dx} \cdot dx = \int \frac{ds}{dx} \cdot \frac{1}{v} \cdot dx = \int \frac{ds}{dx} \cdot \frac{1}{\sqrt{2gy}} \cdot dx,$$

and, on the other, assuming $n = \frac{c}{v}$ as with Fermat and Huyghens,

$$t = \int n ds = \int \frac{c}{v} \cdot \frac{ds}{dx} \cdot dx.$$

It is not surprising then that, in the midst of such conjectures and such theories, Leibniz wrote in 1697 the deepest statement perhaps of what is popularly called the doctrine of the best of all possible worlds, the *De Rerum Originatione Radicali*. It is this, with the appropriate background in mathematics and mechanics, which will lead us to see the height and depth of the Leibnizian optimism, rather than the more eloquent and popular correspondence with Clarke. For there is nothing soft and rhetorical about this Leibnizian doctrine; it is the necessary consequence of the theory of monads, supporting a powerful theory of mechanics, and finally manifesting itself, as it should, in this very same mechanics, the complete symbol of the transcendental world, bringing to its end the way begun by the conservation of momentum and the pre-established harmony. Just as the supposition of all possible paths leads to the distinguished path, so the consideration of all possible worlds leads to the actual world.

> "Hence it is very clearly understood from the infinite combinations of possibles and possible series that one exists by which the most of essence or possibility is brought to exist. There is always in things a principle of determination which must be sought by a maximum or minimum, so that the maximum effect is set forth with the least expense."[134]

That is, the maximum of being must be realized with the least use of time and space. In terms of the primitive Leibnizian intent, we can paraphrase this perhaps, the most thought with the greatest economy of symbol.

> "It is as in certain games when all places on the table must be filled according to certain laws, where, unless you use a certain artifice, excluded finally from hostile spaces, you are forced to leave empty more places

133 *Treatise on Light*, Thompson's trans., 1912, pp. 45-52.
134 *Phil. Schrift.* VII, pp. 303-304.

than you could or would. But there is a certain plan by which the maximum repletion is most easily obtained."

The examples of the easiest and shortest paths are produced as those to be realized so that one sees "how in the beginning of things a certain Divine *mathesis* or metaphysical mechanism is exercised and the determination of the maximum takes place."

We have said, from the beginning, that Descartes consciously created a dream world, a construction beyond the world of every-day experience, using the identity of extension with matter as an attempt to exorcise that which seemed most opposed to thought, so that what appeared as a very primitive device has proved quite otherwise, developing, as we have seen it, through the inherent wealth of mathematical forms, to the deeper and more completely exorcised world of Leibniz where

> "our interior feelings . . . which are in the soul itself and not in the brain or in the body's subtle parts, being only phenomena consequent upon external things or rather truly appearances and like well regulated dreams, these internal perceptions in the soul must come to it by its own original constitution."[135]

A world indeed to which the double abyss, the two infinities of Pascal, is only an entrance, as Leibniz himself says,[136] where space and time only symbolize the archetypal symbolic act of the monads with respect to each other. But if the exorcism has been so nearly complete and so powerful in its consequences, it is because there has nowhere been a flight from the symbols themselves. Time, space, and motion, and the mathematical forms have been pressed for everything they contained, and the pre-established harmony of symbol and symbolized pushed to the extreme. That is what we have tried to show, using for the most part two examples from physics and unfortunately bypassing the purely mathematical evolution where Leibniz considered he had dissolved the Cartesian concern with algebraic operations by the introduction of transcendental methods. Such a purely mathematical investigation would doubtless show the same fruitful symbiosis of symbol and symbolized, and a universal characteristic which in arbitrary marks would mirror thought itself. Indeed one might guess that no system in the grand style can do otherwise and it is no accident that Leibniz felt the charismatic power of Plato's αὐτοκίνητον ἀθάνατον and of its madness of recollection whose significance for the wretched learned is that the history of Western thought lies doubtless in the history of Western astronomy and mechanics; whatever is divorced from this is verbal illusion, a persistent illusion indeed so long as no one has shown why the everlasting monad in its transcendent activity is tied to the well founded phenomena, caught with the symbols of space, time, motion and number. For in this and only this lies the concept of matter that one thing signifies another, and, as Plato saw in the Sophist, this indeed constitutes from another point of view οὐσία, so that as the forms signifying each other in virtue of the one are the ground of the signifying by appearances, so the monads mirroring each other are the ground for the significance of space and time and their relations in the distinguished world which exists.

135 *Système nouveau*, *Ibid.*, p. 484.
136 *Textes inédits*, II, publiés par Gaston Grua, 1948, pp. 553-555.