

Contents

Foreword	vii
List of Tables	xi
List of Notations	xii
Introduction	1
0.1. Historical remarks and our main principle of classification of log del Pezzo surfaces of index ≤ 2	1
0.2. Classification of log del Pezzo surfaces of index ≤ 2 and K3 surfaces.....	4
0.3. Final classification results for log del Pezzo surfaces of index ≤ 2	5
Chapter 1. Log del Pezzo surfaces of index ≤ 2 and Smooth Divisor Theorem	12
1.1. Basic definitions and notation.....	12
1.2. Log terminal singularities of index 2.....	13
1.3. Basic facts about log del Pezzo surfaces.....	14
1.4. Smooth Divisor Theorem.....	15
1.5. Reduction to DPN surfaces of elliptic type.....	18
Chapter 2. General Theory of DPN surfaces and K3 surfaces with non-symplectic involution	20
2.1. General remarks.....	20
2.2. Reminder of basic facts about K3 surfaces.....	22
2.3. The lattice S , and the main invariants (r, a, δ) , (k, g, δ)	29
2.4. Exceptional curves on (X, θ) and Y	32
2.5. The root invariant of a pair (X, θ)	39
2.6. Finding the locus X^θ	43
2.7. Conditions for the existence of root invariants.....	44

2.8.	Three types of non-symplectic involutions of K3 surfaces	48
Chapter 3.	DPN surfaces of elliptic type	50
3.1.	Fundamental chambers of $W^{(2,4)}(S)$ for elliptic type	50
T.1.	Table 1	52
3.2.	Root invariants, and subsystems of roots in $\Delta^{(4)}(\mathcal{M}^{(2)})$ for elliptic case	56
3.3.	Classification of non-symplectic involutions (X, θ) of elliptic type of K3 surfaces	58
T.2.	Table 2	64
3.4.	Proof of Classification Theorem 3.6	68
3.5.	Classification of DPN surfaces of elliptic type	89
T.3.	Table 3	93
3.6.	Application: On classification of plane sextics with simple singularities	100
T.4.	Table 4	103
Chapter 4.	Classification of log del Pezzo surfaces of index ≤ 2 and applications	104
4.1.	Classification of log del Pezzo surfaces of index ≤ 2	104
4.2.	Example: Enumeration of all possible types for $N = 20$	112
4.3.	Application: Minimal projective compactifications of affine surfaces in \mathbb{P}^2 by relatively minimal log del Pezzo surfaces of index ≤ 2	114
4.4.	Dimension of the moduli space	115
4.5.	Some open questions	116
Appendix	117
A.1.	Integral symmetric bilinear forms. Elements of the discriminant forms technique	117
A.2.	Classification of main invariants and their geometric interpretation	122
A.3.	The analogue of Witt's theorem for 2-elementary finite forms	126
A.4.	Calculations of fundamental chambers	128
References	142
Index	147