

Contents

Preface to Volume 1.	xi
Preface to Volume 2.	xiii
General Introduction.	xv
Notations.	xvii
Chapter 1.	1
Part 1. The group Γ and its ζ -function.	1
§§ 1 – 5. Discrete subgroup Γ .	1
§§ 6 – 8. The ζ function of Γ .	5
§§ 9 – 13. Lemmas for the proof of Theorem 1.	7
§14. The proof of Theorem 1 assuming Lemmas 2, 3.	13
§§15 – 19. Proofs of Lemmas 2, 3.	15
§20. Regular cycles on $\Gamma_{\mathbb{R}}^0 \backslash \mathfrak{H}$.	24
§§21 – 23. Estimation of the roots of $\zeta_{\Gamma}(u)$.	24
§24. Concluding remarks on Chapter 1, Part 1.	27
Part 2. Detailed study of elements of Γ with parabolic and elliptic real parts; the general formula for $\zeta_{\Gamma}(u)$.	28
§§25 – 28. Study of elements of Γ with parabolic real parts.	29
§§29 – 34. Study of elements of Γ with elliptic real parts.	37
§§35 – 38. The ζ function of Γ in the general case.	53
Chapter 2.	63
Introduction to Part 1 and Part 2.	63
Part 1. The G_p -fields over \mathbb{C} .	65
§§ 1 – 4. The G_p -fields.	65
§§ 5 – 10. Analytic construction of G_p -fields over \mathbb{C} .	69
§§11 – 17. The full automorphism group of L over \mathbb{C} .	77
Part 2. Full G_p -subfields over algebraic number fields.	83
§18. Main results.	83
§§19 – 20. Reducing Theorem 5 to Theorem 4.	83
§§21 – 26. Preliminaries for the proof of Theorem 4.	85
§§27 – 30. More lemmas.	90
§31. Proof of Theorem 4 (Conclusion).	96
§§32 – 34. Variations of Theorems 4, 5.	98
§§35 – 36. The fields k_0 and $F = \mathbb{Q}((\text{tr } \gamma_{\mathbb{R}})^2 \gamma_{\mathbb{R}} \in \Gamma_{\mathbb{R}})$.	103

Introduction to Part 3A and Part 3B.	106
Part 3A. The canonical S -operator and the canonical class of linear differential equations of second order on algebraic function field L of one variable over \mathbf{C} , and their algebraic characterizations when L is "arithmetic".	109
§§37 – 38. The S -operators.	109
§§39 – 42. The canonical S -operator on algebraic function field of one variable over \mathbf{C} , and its algebraic characterization in ample (arithmetic) cases.	110
§§43 – 44. The canonical class of linear differential equations of second order on algebraic function fields over \mathbf{C} , and its algebraic characterization in ample (arithmetic) cases.	120
Part 3B. Unique existence of an invariant S -operator on "arithmetic" algebraic function fields (including G_p -fields) over any field of characteristic zero.	126
§45. Unique existence of invariant S -operator on ample (arithmetic) L/k .	126
§§46 – 47. Proofs of Lemma 14 _k and Theorem 10.	128
§48. Some corollaries and applications of Theorem 10.	133
Chapter 3.	137
Part 1. Some properties of Γ .	137
§§ 1 – 7. The vanishing of $H^1(\Gamma_{\mathbf{R}}, \rho_n)$ and its consequences.	138
§§ 8 – 9. Applications of Theorem 3 ; the deformation variety.	144
§§10 – 14. Study of V_0 ; the field $F = \mathbf{Q}((\text{tr } \gamma_{\mathbf{R}})^2 \gamma_{\mathbf{R}} \in \Gamma_{\mathbf{R}})$.	146
Chapter 4.	153
Part 1. Examples of Γ .	153
§§ 1 – 3. Examples of Γ .	153
§§ 4 – 5. That k_0 contains F .	156
Chapter 5.	159
Part 1. Elliptic modular functions mod p and $\Gamma = PSL_2(\mathbf{Z}^{(p)})$.	159
§§ 1 – 5. Elliptic modular functions mod p and $\Gamma = PSL_2(\mathbf{Z}^{(p)})$.	159
§§ 6 – 7. Deuring's results.	162
§ 8. Proof of Theorems 1, 1'.	164
§§ 9 – 10. A corollary and an announcement of generalizations.	166
Part 2. Non-abelian classfields attached to subgroups of $\Gamma = PSL_2(\mathbf{Z}^{(p)})$ with finite indices.	168
§§11 – 14. Preliminaries on elliptic curves; results of Igusa and Shimura.	170
§§15 – 16. The group Γ^* and the extension \mathfrak{K}/\bar{k} .	175
§§17 – 21. A fundamental theorem.	179
§§22 – 25. Decomposition of ordinary prime divisors of \bar{k} in \mathfrak{K} .	186
§§26 – 27. Decomposition of supersingular prime divisors of \bar{k} in \mathfrak{K} .	190

§28.	Decomposition of the infinite prime divisor of \bar{k} in \mathfrak{R} .	193
§§29 – 30.	Reformulation in terms of non-abelian classfields; Main Theorems $(\Gamma^*-1) \sim (\Gamma^*-3)$, and Conjecture Γ^* .	195
§§31 – 33.	Supplements to Main Theorems and to Conjecture Γ^* .	198
Supplements.		207
Bibliography.		215
Author's Notes (2008).		217
Additional References.		229