

# Contents

<b>Preface</b>	<b>vii</b>
<b>1 Necessary conditions for well-posedness</b>	<b>1</b>
1.1 Lax-Mizohata theorem . . . . .	1
1.2 Ivrii-Petkov condition . . . . .	3
1.3 Implications of well posedness . . . . .	7
1.4 Proof of Ivrii-Petkov condition . . . . .	9
<b>2 Hyperbolic double characteristics</b>	<b>19</b>
2.1 Hamilton map . . . . .	19
2.2 Ivrii-Petkov-Hörmander condition . . . . .	21
2.3 Hyperbolic quadratic form . . . . .	24
<b>3 Noneffectively hyperbolic characteristics</b>	<b>27</b>
3.1 Elementary decomposition . . . . .	27
3.2 Case $\text{Im } F_p^2 \cap \text{Ker } F_p^2 = \{0\}$ . . . . .	28
3.3 Case $\text{Im } F_p^2 \cap \text{Ker } F_p^2 \neq \{0\}$ . . . . .	34
3.4 Vector field $H_S$ . . . . .	40
3.5 Elementary decomposition revisited . . . . .	43
<b>4 Noneffectively hyperbolic Cauchy problem I</b>	<b>51</b>
4.1 $C^\infty$ well-posedness . . . . .	51
4.2 Pseudodifferential operators . . . . .	52
4.3 Energy estimates . . . . .	53
4.4 Levi condition . . . . .	56
4.5 Strict Ivrii-Petkov-Hörmander condition . . . . .	61
4.6 An example . . . . .	67
<b>5 Noneffectively hyperbolic Cauchy problem II</b>	<b>71</b>
5.1 $C^\infty$ well-posedness . . . . .	71
5.2 Parametrix with finite propagation speed of wave front sets . . . . .	72
5.3 Preliminaries . . . . .	74
5.4 Microlocal energy estimates . . . . .	76
5.5 Finite propagation speed of $WF$ . . . . .	82

<b>6</b>	<b>Well-posedness in the Gevrey classes</b>	<b>91</b>
6.1	Gevrey well-posedness . . . . .	91
6.2	Preliminaries . . . . .	92
6.3	A lemma on composition . . . . .	101
6.4	Energy inequality . . . . .	111
<b>7</b>	<b>Behavior of bicharacteristics</b>	<b>121</b>
7.1	Results . . . . .	121
7.2	Hamilton system and formal solutions . . . . .	123
7.3	A singular initial value problem and bicharacteristics . . . . .	129
<b>8</b>	<b>Optimality of the Gevrey index</b>	<b>133</b>
8.1	Non solvability in $C^\infty$ and the Gevrey class . . . . .	133
8.2	Construction of solutions . . . . .	134
8.3	Proof of non solvability . . . . .	139
<b>9</b>	<b>Not well-posed results</b>	<b>143</b>
9.1	Introduction . . . . .	143
9.2	Asymptotic solutions . . . . .	144
9.3	Lemmas . . . . .	147
9.4	A priori estimates . . . . .	152
9.5	Proof of not well-posed results . . . . .	154
<b>10</b>	<b>Appendix</b>	<b>157</b>
10.1	Symplectic vector space . . . . .	157
10.2	Darboux theorem . . . . .	159
10.3	Homogeneous Darboux theorem . . . . .	162