Preface

Quasilinear hyperbolic systems describe many physical phenomena. Particularly, important examples occur in gas dynamics, shallow water theory, plasma physics, combustion theory, nonlinear elasticity, acoustics, classical or relativistic fluid dynamics and petroleum reservoir engineering. Solutions to quasilinear hyperbolic systems model wave propagation in fluid dynamics or continuum mechanics. By means of investigating the *generalized null condition*, we systematically study the global existence and the blow-up phenomenon, in particular, the life span of C^1 solutions to Cauchy problem for general quasilinear hyperbolic systems with small and decaying initial data. Roughly speaking, when system satisfies the generalized null condition, the C^1 solution globally exists on $t \geq 0$. Basing on this, we further investigate the large time behaviour of the global C^1 solution. If the system does not satisfy the generalized null condition, then, in general the C^1 solution blows up in a finite time: the first order derivatives of the C^1 solution tend to the infinity as t goes to this finite time, however the solution itself remains bounded. Furthermore, we give the asymptotic behaviour of life span of the C^1 solution and illustrate that envelope of characteristics of the same family will appear and singularities occur at the starting point of the envelope, i.e., the point with minimum t-value on the envelope. Moreover, two mechanisms of breakdown of C^1 solutions to quasilinear hyperbolic systems are discussed. Some successful applications of our general theory are given to the quasilinear canonical system related to the Monge-Ampère equation, the system of nonlinear three-wave interaction in plasma physics, the nonlinear wave equation with higher order dissipation, the system of one-dimensional gas dynamics with higher order damping, the system of motion of an elastic string, the system of plane elastic waves for hyperelastic materials and the nonlinear wave equation with scalar operators of higher order.

Taking this opportunity, I would like to express my warm thanks to Professor Y. Giga and Professor Ta-tsien Li for their constant support and encouragement. I also thank Professor M. Tsuji for his help. Special thanks are due to the referees for their pertinent comments and valuable suggestions.

I should like to thank the JSPS for offering me a postdoctoral fellowship. This work was partially supported by the National Natural Science Foundation of China, and the Grant-in-Aid for Scientific Research for JSPS Foreign Researcher in Japan, provided by the Japan Ministry of Education, Science and Culture.

Kong De-xing
4 August, 2000
Shanghai, China