Preface

The aim of this memoir is to give an introduction to the statement and main ideas in the proof of the "Orbifold Theorem" announced by Thurston in late 1981 ([83],[81]). The Orbifold Theorem shows the existence of geometric structures on many 3-dimensional orbifolds, and on 3-manifolds with a kind of topological symmetry.

In July 1998, fifteen lectures on the Orbifold Theorem were presented by the authors at a Regional Workshop in Tokyo. We would like to take this opportunity of thanking the Mathematical Society of Japan who organized and supported this workshop. In particular we thank and express deep gratitude to Sadayoshi Kojima whose idea this meeting was and who provided the stimulus required to produce this manuscript in a timely fashion. His tremendous organizational skills helped to ensure the success of the meeting.

The first six lectures were of an expository nature designed to meet the needs of graduate students. It is the content of these first six lectures, somewhat expanded, that form the bulk of this memoir. The content of the final nine lectures where we discussed the proof of the orbifold theorem have been summarized, and a detailed account of this proof will appear elsewhere.

It is our intention that this memoir should provide a reasonably self-contained text suitable for a wide range of graduate students and researchers in other areas, and to provide various tools that may be useful in other contexts. For this reason we have included many examples and exercises.

We develop the basic properties of orbifolds and cone-manifolds. In particular many ideas from smooth differential geometry are extended to the setting of cone-manifolds. We have also included an outline of a proof of the orbifold theorem. There is a short account of Gromov's theory of limits of metric spaces (as re-interpreted using ϵ-approximations by Thurston), and a discussion of deformations of hyperbolic structures.

This memoir should provide the background necessary to understand the proof of the orbifold theorem which will appear elsewhere. The prerequisites include some acquaintance with hyperbolic geometry, differential geometry and 3-manifold topology.

We have borrowed heavily (in the sense of cut and paste, as well as other ways) from Hodgson's thesis [43] and also from his notes on the orbifold theorem [44].

The Orbifold Theorem is due to Thurston. It is one part of a major program to geometrize topology in dimensions two and three. Unfortunately it was also one of the least documented results. Thurston outlined his proof in graduate courses at Princeton in 1982 and again in 1984. Two of the authors attended these lectures. Our original intent was to write out in detail Thurston's proof. In the course of doing this, we found it easier to develop a somewhat different proof. However our proof is closely based on Thurston's. Our main contributions are the generalized Bieberbach-Soul theorem for non-compact Euclidean cone-manifolds in all dimensions, and the more combinatorial approach to the collapsing case. Our treatment of the Euclidean/spherical transition when there are vertices is also very different to Thurston's treatment. We thank Bill Thurston for creating and sharing his wonderful ideas. His presence at the Tokyo workshop helped ensure its success.

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