Institute of Mathematical Statistics
LECTURE NOTES–MONOGRAPH SERIES

The Likelihood Principle
(Second Edition)

James O. Berger
Purdue University

Robert L. Wolpert
Duke University

Volume 6

Shanti S. Gupta, Series Editor

THE LIKELIHOOD PRINCIPLE:

A REVIEW, GENERALIZATIONS, AND STATISTICAL IMPLICATIONS

SECOND EDITION

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To my parents, Orvis and Thelma

James Berger

To my wife, Ruta

Robert Wolpert

PREFACE

This monograph began with research designed to provide a generalization of the Likelihood Principle (LP) to quite arbitrary statistical situations. The purpose of seeking such a generalization was to partially answer certain criticisms that had been levied against the LP, criticisms which seemed to prevent many statisticians from seriously considering the LP and its implications. The research effort seemed worthwhile because of the simplicity, central importance, and far reaching implications of the LP.

Background reading for the research revealed a wider than expected range of published criticisms of the LP. In an attempt to be complete and address all such criticisms, the research paper expanded considerably. Eventually it seemed sensible to enlarge the paper to a monograph. This also allowed for discussion of conditioning ideas in general and for a review of the implications of the LP. It was decided, however, to stop short of a general review of conditional *methods* in statistics. In particular, the monograph does not discuss the many likelihood based statistical methodologies that have been developed, although references to these methodologies will be given. This limitation was, in part, because such an endeavor would be far too ambitious, and, in part, because we feel (and indeed argue in Chapter 5) that Bayesian implementation of the LP is the correct conditional methodology.

The mathematical level of the monograph is, for the most part, kept at a nontechnical level. The main exception is the generalization of the LP in Section 3.4, which is (necessarily) presented at a measure-theoretic level, but can be skipped with no loss in continuity. Also, the monograph

presupposes no familiarity with conditioning concepts. Indeed Chapter 2 provides an elementary review of conditioning, with many examples.

This second edition was produced under the rather severe constraint that the original manuscript, used for photo-offset printing, was inadvertantly destroyed; only the photos were kept. Thus changes could only be made by retyping entire pages or inserting new pages. A list of corrections that were too minor to justify the retyping of an entire page is given at the end of the monograph. Inserted pages received decimal page numbers: e.g. 74.1, 74.2. A list of additional references was added, and new discussions were kindly contributed by M. J. Bayarri and M. H. DeGroot, Bruce Hill, and Lucien Le Cam.

Substantial changes or additions were made in Sections 3.1, 3.5, 4.2.1, 4.4, and 4.5. The changes in Section 4.4 correct a glaring oversight in the first edition: the failure to emphasize the misleading conclusions that can result from violation of the Likelihood Principle in significance testing of a precise hypothesis. Another very weak part of the first edition was Section 3.5, which discussed prediction, design, and nuisance parameters. The new material incorporates recent substantive insights from the literature.

Numerous other minor changes and literature updatings were made throughout the monograph. We did not attempt complete coverage of recent literature, however.

We are grateful to a number of people for valuable discussions on this subject and/or for comments and suggestions on original drafts or the first edition of the monograph. In particular, we would like to thank George Barnard, M. J. Bayarri, Mark Berliner, Lawrence Brown, George Casella, Morris DeGroot, J. L. Foulley, Leon Gleser, Prem Goel, Clyde Hardin, Bruce Hill, Jiunn Hwang, Rajeev Karandikar, Lucien Le Cam, Ker-Chau Li, Dennis Lindley, George McCabe, Georges Monette, John Pratt, Don Rubin, Herman Rubin, Myra Samuels, Steve Samuels, and Tom Sellke. We are especially grateful to M. J. Bayarri and M. H. DeGroot for an exceptionally complete and insightful set of corrections and comments on the first edition. We are also grateful to Shanti Gupta for the encouragement to turn the material into a monograph.

Thanks are also due to the Alfred P. Sloan Foundation, the National Science Foundation (Grants MCS-7801737, MCS-8101670A1, and DMS-8702620), and the Center for Stochatic Processes at the University of North Carolina for support of the research in the monograph. Finally, we are extremely grateful to Norma Lucas, Teena Chase, and Betty Gick for excellent typing of the manuscript.

March, 1988

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