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Empirical Processes

Peter Gaenssler

*Mathematical Institute
of the University of Munich*

Institute of Mathematical Statistics

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Shanti S. Gupta, Series Editor

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**Institute of Mathematical Statistics
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PREFACE

The present notes are based on lectures given at the Department of Statistics, University of Washington, Spring 1982, when the author was on leave from the Mathematical Institute, University of Munich, as a Visiting Professor at the University of Washington, supported by the National Science Foundation under Grant (MCS) 81-02568.

The last section of these lecture notes is supplemented by a course on Functional Central Limit Theorems held at the University of Munich during the Summer term in 1982.

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Munich, August 1982; revised June 1983,

PETER GAFNSSLER

TO MY WIFE INGRID

with admiration and thanks
for her patience with me

