

**Institute of Mathematical Statistics**  
**LECTURE NOTES—MONOGRAPH SERIES**

# **Survival Analysis**

**Edited by**

**John Crowley and Richard A. Johnson**



**Institute of Mathematical Statistics**

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**Shanti S. Gupta, Series Editor**

**Volume 2**

# **Survival Analysis**

*Proceedings of the Special Topics Meeting sponsored by the  
Institute of Mathematical Statistics, October 26–28, 1981,  
Columbus, Ohio*

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## PREFACE

The 178th meeting of the Institute of Mathematical Statistics, held at Ohio State University in Columbus, Ohio, October 26-28, 1981, was organized as a Special Topics Meeting on Survival Analysis. The intent was to gather workers interested in the analysis of life length, from both reliability and biomedical applications, and to share progress and ideas across these disciplines. Survival analysis has been an active and exciting area of research for the past several decades, and one which is still gaining momentum today. The breadth of current activity in the field is illustrated in this Proceedings Volume, which includes invited papers from the meeting covering seven main topics:

### I. Counting Processes and Survival Analysis

This first paper reviews the application of counting processes and the associated martingales to the large sample theory for a broad class of problems in survival analysis, from one and two sample situations to regression.

### II. Nonparametric Inference for a Single Sample

Research on the one-sample problem is represented by three papers, covering a smooth version of the product-limit estimator, a generalization of the product-limit estimator to progressive censoring schemes, and an estimator of the hazard or failure rate.

### III. Proportional Hazards and Log-Linear Models

Papers in this group give a comparison of estimators of the ratio of hazard functions in the context of a proportional hazards model, and a comparison of least squares and partial likelihood approaches when a log-linear model and proportional hazards both hold.

A new algorithm for least-squares type estimation for parameters in a linear model (possibly after transformation) for censored survival data is also given and investigated.

#### IV. Regression Approaches

Other research presented on regression includes an analysis of the statistical aspects of the inverse Gaussian model, and of the Box-Cox transformation toward normality with censored data. General considerations regarding the errors in variables problem are also discussed.

#### V. Problems in System Reliability

Inference procedures with time-truncated life-test data from an exponential model and a mixture of exponentials are given, and the properties of a system with imperfect repair of components derived. General limit theorems for a class of life-testing problems are also presented.

#### VI. Multivariate Distributions and Competing Risks

General notions regarding the concept of negative dependence of random variables are given, as well as some aspects of the theory of possibly dependent competing risks. Two papers present and explore estimators of the bivariate distribution function with censored observations, one using a model with exponential hazards, the other taking a nonparametric point of view.

#### VII. Group Sequential Methods in Clinical Trials

Both large sample and Monte Carlo approaches are used to investigate the properties of various statistics as applied at several times during the course of a clinical trial.

There were 25 invited papers as well as 6 sessions for contributed papers.

The invited speakers were:

Per Kragh Andersen  
Statistical Research Unit  
Danish Medical and Social Research Councils

Richard Barlow  
University of California, Berkeley

Asit P. Basu  
University of Missouri, Columbia

Gouri K. Bhattacharyya  
University of Wisconsin, Madison

Henry W. Block  
University of Pittsburgh

Gregory Campbell  
Laboratory of Statistical & Mathematical Methodology  
National Institutes of Health

John Crowley  
Fred Hutchinson Cancer Research Center  
University of Washington

Kjell A. Doksum  
University of California, Berkeley

T.R. Fleming  
Mayo Clinic

Mitchell H. Gail  
National Cancer Institute

Joseph C. Gardiner  
Michigan State University

Richard A. Johnson  
University of Wisconsin, Madison

J.D. Kalbfleisch  
University of Waterloo

Jerome Klotz  
Ohio State University

Sue Leurgans  
University of Wisconsin, Madison

N.R. Mann  
University of California, Los Angeles

Paul Meier  
University of Chicago

Janet Myhre  
Claremont McKenna College

Ross Prentice  
Fred Hutchinson Cancer Research Center  
University of Washington

Frank Proschan  
Florida State University

Nozer D. Singpurwalla  
George Washington University

V. Susarla  
Michigan State University

Anastasios A. Tsiatis  
Harvard University  
Sidney Farber Cancer Institute

John Van Ryzin  
Columbia University

Marvin Zelen  
Harvard University  
Sidney Farber Cancer Institute

In a few instances the papers included here differ somewhat from the remarks given at the meeting because of prior publication elsewhere.

#### ACKNOWLEDGMENTS

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The excellent typing was done by Joy Hoggarth.

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## TABLE OF CONTENTS

	Page
COUNTING PROCESSES AND SURVIVAL ANALYSIS	
On the Application of the Theory of Counting Processes in the Statistical Analysis of Censored Survival Data	
Per Kragh Andersen	1
NONPARAMETRIC INFERENCE FOR A SINGLE SAMPLE	
Spline Smooth Estimates of Survival	
Jerome Klotz	14
A Nonparametric Estimator of the Survival Function Under Progressive Censoring	
Joseph C. Gardiner and V. Susarla	26
Fourier Integral Estimate of the Failure Rate and Its Mean Square Error Properties	
Nozer D. Singpurwalla and Man-Yuen Wong	41
PROPORTIONAL HAZARDS AND LOG-LINEAR MODELS	
Estimation of the Ratio of Hazard Functions	
John Crowley, P.-Y. Liu and Joseph G. Voelkel	56
On the Performance of Estimates in Proportional Hazard and Log-Linear Models	
Kjell A. Doksum	74
Multi-step Estimation of Regression Coefficients in a Linear Model with Censored Survival Data	
Hira L. Koul, V. Susarla and John Van Ryzin	85
REGRESSION APPROACHES	
Inverse Gaussian Regression and Accelerated Life Tests	
Gouri K. Bhattacharyya and Arthur Fries	101
Transformation of Survival Data	
Richard A. Johnson	118

	Page
REGRESSION APPROACHES (Continued)	
Covariate Measurement Errors in the Analysis of Cohort and Case-Control Studies	
Ross Prentice	137
PROBLEMS IN SYSTEM RELIABILITY	
Confidence Bounds for the Exponential Mean in Time-Truncated Life Tests	
N.R. Mann, R.E. Schafer and M.C. Han	152
Screen Testing and Conditional Probability of Survival	
Janet Myhre and Sam Saunders	166
Imperfect Maintenance	
Mark Brown and Frank Proschan	179
A Limit Theorem for Testing with Randomly Censored Data	
Hira L. Koul and V. Susarla	189
MULTIVARIATE DISTRIBUTIONS AND COMPETING RISKS	
Negative Dependence	
Henry W. Block and Thomas H. Savits	206
Some Recent Results in Competing Risks Theory	
Asit P. Basu and John P. Klein	216
Freund's Bivariate Exponential Distribution and Censoring	
Sue Leurgans, Wei-Yann Tsai and John Crowley	230
Asymptotic Properties of Several Nonparametric Multivariate Distribution Function Estimators Under Random Censorship	
Gregory Campbell	243
GROUP SEQUENTIAL METHODS IN CLINICAL TRIALS	
Group Sequential Methods for Survival Analysis with Staggered Entry	
Anastasios A. Tsiatis	257

	Page
GROUP SEQUENTIAL METHODS IN CLINICAL TRIALS (Continued)	
Procedures for Serial Testing in Censored Survival Data	
D.P. Harrington, T.R. Fleming and S.J. Green	269
Simulation Studies on Increments of the Two-Sample Logrank Score Test for Survival Time Data, with Application to Group Sequential Boundaries	
Mitchell H. Gail, David L. DeMets and Eric V. Slud	287

