Institute of Mathematical Statistics
LECTURE NOTES–MONOGRAPH SERIES

Essays on the Prediction Process

Frank B. Knight
University of Illinois
at Champaign-Urbana

Volume 1

Shanti S. Gupta, Series Editor

Institute of Mathematical Statistics

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Hayward, California

Institute of Mathematical Statistics

Lecture Notes Series

Editor, Shanti Gupta, Purdue University

International Standard Book Number 0-940600-00-5

Printed in the United States of America.

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PREFACE. This work comes at a stage when the literature on the prediction process consists of only six papers, of which two are by the present author and the other four are in the Strasbourg Seminaire des Probabilités. None of these papers is simple to read, much less to understand. Accordingly, our work has been cut out for us to make the prediction process comprehensible to more than a few specialists. One way of doing this, it would appear, is to present the subject separately in several different contexts to which it applies. Thus for a reader interested mainly in a certain aspect, that part may be studied independently, while for one wishing to have a fuller understanding, the force of repetition of a common theme in different settings may serve to deepen the effect.

Accordingly, the present work consists of four distinct papers based on a common theme. No attempt is made to exhaust the subject, but at the same time the purpose is not just to illustrate. The first and most fundamental paper is an introduction to the method. It has been kept as simple as possible in order to make it more accessible. Besides organizing and explaining the subject, it provides some elements not in the previous literature and which are needed to understand the fourth essay. On the other hand, a few of the most difficult known results on the prediction process, in part depending heavily on analytic sets, are not included in the results of this paper. The attempt has been to make the subject self-contained and as concrete as possible, by avoiding unnecessary mathematical abstractions and artificial methods of proof.

The second essay presents what is perhaps the simplest non-trivial type of stochastic process: one consisting simply of the arrival time (or lifetime) of a single instantaneous event. To a surprising degree, this already illustrates and clarifies the method. One sees in clear distinction the two basic types of processes involved. On the one hand, we have the direct model of the physical phenomenon, where t represents physical time and we allow $-\infty \le t \le \infty$. On the other hand, we have the prediction process based on the model, in which t represents observer's time and we require $0 \le t \le \infty$.

This essay uses two results of the Strasbourg school, as well as several of the associated methods, but they are largely confined to the beginning and the end. It should be possible to gain an understanding of the main idea by taking for granted these results as stated.

The third essay gives an application of the method to ordinary Markov processes. Like the second it is written to be read independently of the first, and it does make some demands on the literature of the subject. In a sense it represents a concession to traditional formalism. The problem is to apply the prediction process (which is always Markovian) to a given Markov process without permitting any change in the given joint distribution functions. This has the double intent of providing new insight into the usual regularity assumptions for Markov processes, and of clarifying the meaning and role of the prediction process.

The fourth essay brings the method to bear on three basic classes of processes: square integrable martingales, uniformly integrable martingales, and potentials of class D. In accordance with essay one, the study of each class is reduced to that of a corresponding Markov process. Thus for example the "potentials" do actually become Markovian potential functions in the usual sense of probabilistic potential theory. Several basic applications are made, including the orthogonal decomposition of square-integrable martingales, and the Doob-Meyer decomposition of Class D potentials. Of some general interest is the Lévy system of a prediction process. This is shown to exist in complete generality, not in any way limited to martingales. It is then applied to an arbitrary process to yield simultaneously the compensators (or dual previsible projections) of all of the integrable, adapted increasing pure-jump processes. Finally, the class of continuous martingales which are germ-diffusion processes (i.e., have an autonomous germ-Markov property), is investigated briefly.

In this essay, more than previously, a basic contrast with the Strasbourg approach to the same subject matter becomes apparent. While the latter approach studies the class of all martingales (or supermartingales, etc.) with respect to a given probability measure and adapted family of σ-fields, the prediction process approach studies the class of all martingale (or supermartingale) probabilities with respect to a fixed canonical definition of the process and σ-fields.

One acknowledgment and one word of caution should be given in conclusion. Essays 1 and 2 have profited from the careful reading and criticism of Professor John B. Walsh. In particular Theorem 1.2 of Essay 1 owes its present formulation largely to him. On the cautionary side, our numbering system permits one consecutive repetition of a number when this corresponds to a different heading. Thus, Theorem 1.2 is followed by Definition 1.2, but it might have been preceded instead. However, since no number is used more than twice, we thought that the present more informal system was justified in preference to the usual monolithic progression.