FOREWORD

The subject matter of these conference proceedings comes in many guises. Some view it as the study of probability distributions with fixed marginals; those coming to the subject from probabilistic geometry see it as the study of copulas; experts in real analysis think of it as the study of doubly stochastic measures; functional analysts think of it as the study of Markov operators; and statisticians say it is the study of possible dependence relations between pairs of random variables. All are right since all these topics are isomorphic.

This diversity of viewpoints reflects a diversity of origins. Doubly stochastic measures arose, at least in part, from attempts to generalize the concept of doubly stochastic matrices to a continuous setting. Copulas were introduced because of an interest in the ways joint distribution functions are related to their marginals. Probability distributions with fixed marginals are a natural object in the study of nonparametric statistics; they also arise in problems that are not originally statistical in nature, e.g., in the determination of the optimal translocation of masses (the Monge-Kantorovich problem) as well as in the construction of probability metrics which are of increasing importance in the investigation of stability properties of stochastic models.

Over the years there has been a steady accumulation of results in all these areas, a growing realization of the fact that there are close links between them, and a gradual increase in the pace of research. For example, to anyone familiar with both definitions, it is quickly obvious that a copula is simply the joint distribution function of a doubly stochastic measure on the unit square, whence there is a one-to-one correspondence between these two concepts. In 1966 J. R. Brown showed that there is a "nice" homeomorphism between Markov operators and doubly stochastic measures on the unit square. Earlier, in 1959, A. Sklar, in response to a query by M. Fréchet, had shown that if H is a two-dimensional joint distribution function with one-dimensional margins F and G, then there exists a copula C satisfying $H(x, y) = C(F(x), G(y))$ and that the copula C is unique when the ranges of both F and G are each the entire unit interval $[0, 1]$. When the copula is unique, it can be associated with the type of dependence that exists between random variables having H as their joint distribution. When the copula fails to be unique (see A. W. Marshall's paper in this volume) the situation is far more complicated and one will be forced to work with the subcopulas which are unique. In case of uniqueness, the Fréchet-Hoeffding bounds for H may be looked upon as the first probabilistic explanation of the type of dependence corresponding to certain copulas. More recently there has been a growing realization by statisticians that copulas provide a unified way of studying many nonparametric

measures of dependence, e.g., Kendall's τ and Spearman's ρ. But to mention these results and insights is to barely scratch the surface of what has been done.

Broadly speaking, progress in this field may be divided into two categories: Advances by mathematicians and probabilists in understanding the theoretical structure of copulas, doubly stochastic measures, Markov operators, measures of dependence, etc., and a growing understanding by statisticians of the ways in which these insights can be exploited.

On the theoretical front, progress has been too heterogeneous to be briefly described, even if one limits oneself to the more outstanding advances. Here we mention only one additional result, namely, the fact that any type of statistical dependence (including statistical independence) between two random variables can be approximated arbitrarily closely by the dependence between two other random variables which are related to one another in a strictly deterministic fashion.

Several years ago I sent out a letter of inquiry to see if there was any interest in a conference devoted to these topics. Among the enthusiastic responses I received was one from G. Dall'Aglio who indicated that he had already given considerable thought to this matter, had already initiated some work to that end, and now wanted to go ahead with such a conference. I naturally deferred to his wishes and Dall'Aglio's conference took place in Rome, Italy, in April, 1990. The participants – statisticians, probabilists, and those interested in copulas – found the conference to be delightful, enlightening, stimulating, and, as demonstrated by the resulting published conference proceedings, fruitful beyond expectation. They also expressed strong interest in having another such conference, this time enlarged to include real and functional analysts.

It was in the context of this backdrop that the conference on "Distributions with Fixed Marginals, Doubly Stochastic Measures, and Markov Operators" (held in August, 1993 in Seattle, Washington, as one of the 1993 AMS-IMS-SIAM Joint Summer Research Conferences) became a reality.

We knew this conference would attract people with common interests but diverse expertise, many of whom until recently had little awareness of each others existence. So the goal of the conference was to stimulate growth of understanding of existing results and to produce new and important results by capitalizing on the differences in perspectives and tools of the participants. In our biased opinion, this conference was a major success.

After the opening address of the conference, in which Abe Sklar gave an inspiring, historical retrospective of his involvement with the subject of this conference, Ingram Olkin remarked that his experience with this subject had been perpendicular to Abe's. At the close of the conference Bert Schweizer,

who was asked to share some parting thoughts concerning the conference, referring to Ingram's remark, said, "If Ingram had said his experience was *diametrically opposed* to Abe's, their ideas would have cancelled each other out, but he said *perpendicular*, which means that, taken together, their ideas span a larger space than either one alone." This insightful observation said something important about the conference as a whole.

To give a more complete description of the conference, we have included a copy of the program on pages xiv–xvii.

This conference and these proceedings would not have been possible without the support of several institutions and a number of individuals. We desire to express our sincere gratitude to each one. Of course there is no way to explicitly thank everyone to whom thanks are owed. I therefore trust that those who are not mentioned below will forgive me.

First of all we thank the sponsor – the American Mathematical Society, Institute of Mathematical Statistics, and Society for Industrial and Applied Mathematics Committee on Joint Summer Research Conferences in the Mathematical Sciences. Certain individuals were particularly helpful. Until illness forced her to take a leave of absence, Carole Kohanski, the coordinator for the AMS, was of tremendous help to me in assuring me that everything was going along well and not to worry. After Carole went on leave, Chris Harkness assumed her responsibilities. Chris too was very helpful. The other person at the AMS who was very helpful and encouraging to me was James Maxwell. He always had the answers I needed and was willing to go the extra mile to help whenever called upon. A hearty thanks is given to all three.

This particular conference topic has received enthusiastic financial support from three agencies. The National Science Foundation and the National Security Agency provided the primary funding which made the conference possible. In addition, the International Science Foundation provided support for two of our participants from the former Soviet Union. I gratefully acknowledge their financial assistance.

The University of Washington is a marvelous setting for a conference. We appreciated the use of their magnificent facilities. Also, the University of Central Florida provided much appreciated support by way of postage, paper, phone bills, etc.; Lokenath Debnath, Chair of the Department of Mathematics, generously authorized use of departmental funds for this purpose.

Peter Fishburn, Joop Kemperman, Ingram Olkin, Ludger Rüschendorf, Bert Schweizer, and Mike Taylor all served on the Organizing Committee. This committee was very helpful to me all along the way. Some helped in the writing of the proposal for this conference; all gave me wise counsel from time to time; some encouraged me when I needed it. They deserve the credit

for whatever is good about the scientific organization of this conference. I owe special thanks to Ingram Olkin and Bert Schweizer who truly went far beyond all I could have hoped for in qualities one would like in members of an organizing committee. They were not just there when I needed guidance; they anticipated my needs and often called to help.

Mark Johnson and Piotr Mikusiński, two of my colleagues at the University of Central Florida, also deserve special thanks for their assistance during the writing of the proposal for this conference.

Mike Taylor, co-chair of the Organizing Committee, agreed from the beginning to edit the proceedings of the conference. He deserves special thanks for this monumental undertaking which he has cheerfully done with much devotion. The other editors – Ludger Rüschendorf and Bert Schweizer – have also generously devoted themselves to this effort. Thanks are due to all three.

Howard Sherwood