Contents

Foreword i

Preface iii

1 Topological vector spaces 1

1.1 Topological vector spaces. 1

1.2 Linear operators 16

1.3 Countably Hilbertian nuclear spaces. 29

2 Probability measures 45

2.1 Probability measures 45

2.2 Weak convergence of probability measures. 55

2.3 Minlos and Sazonov theorems 61

2.4 $C([0,T],\Phi')$ and $D([0,T],\Phi')$ 71

2.5 Probability measures on $D([0,T],\Phi')$ 80

3 Stochastic integrals 85

3.1 Martingales 86

3.2 Φ'-Wiener process 93

3.3 Continuous stochastic integral 109

3.3.1 Stochastic integral 109

3.3.2 Representation theorems 113

3.4 Discontinuous stochastic integral 119

4 SPDE 127

4.1 Introduction 127

4.2 Space-time Ornstein-Uhlenbeck SDE 129

4.3 SPDEs 134

4.4 Nonlinear stochastic cable equation 141

4.5 O-U SDE's driven by Poisson random measures 144

5 SDE in Hilbert space **149**
5.1 Diffusion equations in Hilbert spaces 149
5.2 Stochastic evolution equations 154

6 Stochastic differential equations **171**
6.1 Weak convergence theorems 172
6.2 Existence of a weak solution 189
6.3 Strong solution 199

7 Environmental pollution **211**
7.1 Introduction 211
7.2 Pollution with Poisson deposits 213
7.3 Pollution along a river 221
7.4 Pollution with a tolerance level 229
7.5 Filtering problem 230

8 Diffusion processes **239**
8.1 Martingale problem of a diffusion equation 240
8.2 Weak solutions of diffusion equations 248
8.3 Strong solution of diffusion equation 253
8.4 Applications of diffusion approximation 260
8.5 Examples of nuclear-space-valued SDE's 273

9 Interacting systems **279**
9.1 McKean-Vlasov equation 280
9.2 Interacting systems 284

10 Large deviations **299**
10.1 LDP for a class of random variables 299
10.2 Application to SDE in conuclear space 308
10.3 Application to SPDEs 320
10.4 Reaction-diffusion SPDEs 331

Bibliography **335**

Index **340**