A Uniform Theorem Proving Tableau Method for Modal Logic*

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Summary. In this paper, we propose a uniform theorem proving tableau method for a wide class of systems in propositional modal logic. The class is wide enough to include most well-known systems. In this method, for a given natural number μ, a modal formula θ is effectively transformed to a first-order formula $\Delta(\theta)_\mu$ without depending on the system addressed. The transformation is based on the idea of tableau methods. Now, if S is a system that is complete for a class of Kripke frames characterized by a first-order formula Σ, then $S \vdash \theta$ iff $\Sigma \supset \Delta(\theta)_\mu$ is provable in first-order logic for some μ. This method also raises questions that are interesting from a theoretical viewpoint.

1. Introduction

In this paper, we propose a uniform tableau method for a wide class of systems in propositional modal logic. Tableau methods are efficient ways of theorem proving, based on the idea of model elimination [4], [6]. These methods are known to be especially useful for modal logic. Tableau methods have been proposed for a number of well-known systems: K, D, T, B, S4, and S5. However, there are many systems to which tableau methods have not been proposed yet. For example, S4.1 and S4.2 are not covered. Moreover, each of the proposed methods requires a system dependent individual device to achieve a complete theorem proving procedure. At present, there is no general strategy for obtaining tableau methods for all systems.

The aim of this paper is to propose a uniform tableau method applicable to a wide class of systems. The class consists of complete normal systems which are complete for a class of Kripke frames characterized by a first-order formula. The class is natural and wide enough to include most well-known systems. In our method, a given modal formula θ is effectively transformed to a first-order formula $\Delta(\theta)_\mu$ for a given natural number μ. This transformation is independent of the system being addressed. We have the following correctness theorem.

If S is a system, complete for a class of Kripke frames characterized by a first-order formula Σ, then $S \vdash \theta$ iff $\Sigma \supset \Delta(\theta)_\mu$ is provable in first-order logic for some μ.

* This paper is in its final form and no similar paper has been or is being submitted elsewhere.

Our method is an extension of prefixed tableau methods [4], [5], [11]. Tableau methods are based on the idea of model elimination. In these methods, we assume an input formula θ is false in some world of a Kripke structure of a system and, according to the definition of Kripke structures, we create worlds, decide the truth of subformulas in those worlds and derive a contradiction: some formula is both true and false at the same time in some world. Hence, we conclude that θ is valid in all the Kripke structures; that is, θ is a theorem of the system. In a prefixed tableau method, in contrast, we do not create an actual world, but attribute world indexes to each formula generated during the proving procedure. Then, we select each pair of prefixed formulas whose formula parts are the same, but whose truths are different, and check if the indexes can be the same under special unifications derived from the accessibility relation of Kripke frames for the system. In [11], the unifications are given for systems K, D, T, K4, D4, S4, and S5. The unification method for these systems is efficient, but is restricted to only a few number of systems and requires a special device depending on systems. In our method, on the other hand, we introduce a more general notion for indexes by using Skolem functions and transform the condition of the contradictions to a first-order formula. As a result, we obtain a system independent theorem proving method for a wide class of systems in modal logic.

2. Preliminaries for Modal Logic

2.1 Syntax of modal logic

Definition 2.1. The alphabet used in this paper for a language $\mathcal{L}$ of modal logic is as follows.
logical connectives: $\wedge$, $\vee$, $\neg$, $\supset$, L
countably many propositional variables: p, q, r, . . .

Definition 2.2. *Well-formed formulas* (*wffs*) of the language $\mathcal{L}$ are defined recursively in the following.
1) Propositional variables are wffs.
2) If ϕ and ψ are wffs, then $\phi\wedge\psi$, $\phi\vee\psi$ and $\phi\supset\psi$ are all wffs.
3) If ϕ is a wff, then $\neg\phi$ and $L\phi$ are wffs.

As usual, we use brackets (and) for convenience.

2.2 Semantics of modal logic

Definition 2.3. Let W be a non-empty set and R be a binary relation on W $(R\subset W \times W)$. We call a pair $\langle W, R\rangle$ a *frame*. W and R are called a set of worlds and an accessibility relation on worlds, respectively. Let V be a binary relation on W and the set of all propositional variables of $\mathcal{L}$ (denoted by PV). That is $V \subset W \times PV$. We call a triple $\langle W, R, V\rangle$ a *Kripke structure* of $\mathcal{L}$ and V a *valuation.* We define models by using the Kripke structure below. In the following, we define the relation $M,w \models \phi$, where M is a Kripke structure $\langle W, R, V\rangle$, $w \in W$ and ϕ is a wff.

1) If p is a propositional variable , $M,w \models p$ iff $(w,p) \in V$.
2) $M,w \models \phi \lor \psi$ iff $M,w \models \phi$ or $M,w \models \psi$.
3) $M,w \models \phi \land \psi$ iff $M,w \models \phi$ and $M,w \models \psi$.
4) $M,w \models \neg\phi$ iff not $M,w \models \phi$ (We denote this by $M,w \not\models \phi$).
5) $M,w \models \phi \supset \psi$ iff $M,w \not\models \phi$ or $M,w \models \psi$.
6) $M,w \models L\phi$ iff for all v such that $(w,v) \in R$, $M,v \models \phi$.

When $M,w \models \phi$ for all $w \in W$, we write $M \models \phi$ and call the Kripke structure M a *(Kripke) model* of ϕ.

For convenience, we may write $M,w \models T\phi$ and $M,w \models F\phi$ for $M,w \models \phi$ and $M,w \not\models \phi$, respectively. Here, T and F denote "true" and "false", respectively.

A system S is called a *complete normal* system if there is a class of Kripke frames C such that the set of all the theorems of S coincides with the set $\{\phi \mid$ for all $\langle W, R\rangle \in C$ and for all V, $\langle W, R, V\rangle \models \phi\}$. Most well-known systems are complete normal.

2.3 First-order definable systems

We will define a class of systems to which our proving procedure is uniformly applicable.

Definition 2.4. Let Σ be a closed wff of FOL whose predicate symbols are among R and $=$. The class of frames $C(\Sigma)$ is defined to be $\{$frame $\langle W, R\rangle \mid \langle W, R\rangle \models \Sigma\}$. A system is called a *complete normal system defined by* Σ (we will call it Σ*-system* for short), if the set of all theorems of the system coincides with $\{\phi \mid$ for any $\langle W, R\rangle \in C(\Sigma)$ and any valuation V, $\langle W, R, V\rangle \models \phi\}$. A system is called *first-order definable* if it is a complete normal system defined by Σ for some Σ.

Examples:
If Σ is a tautology, then Σ-system is the system K.
If Σ is $\forall x R(x,x)$, then Σ-system is the system T.
If Σ is $\forall x R(x,x) \land \forall x \forall y \forall z(R(x,y) \land R(y,z) \supset R(x,z))$, then Σ-system is the system S4.
If Σ is $\forall x R(x,x) \land \forall x\, \forall y\, \forall z(R(x,y) \land R(y,z) \supset R(x,z)) \land \forall x\, \forall y\, \forall z(R(x,y) \land R(x,z) \supset \exists w R(y,w) \land R(z,w))$, then Σ-system is the system S4.2.
If Σ is $\forall x R(x,x) \land \forall x\, \forall y\, \forall z(R(x,y) \land R(y,z) \supset R(x,z)) \land \forall x\, \forall y\, \forall z(R(x,y) \land R(x,z) \supset R(y,z) \lor R(z,y))$, then Σ-system is the system S4.3.

3. A Uniform Theorem Proving Method

A *prefix* is a list of terms constructed from variables and functional symbols. A *signature* is a symbol of either T or F. A *prefixed formula* is a sequence of three elements in the following order: a prefix, a signature and a wff. For example, $[f, x, y, g(x, y), z]Fp \vee L(p \wedge q)$ is a prefixed formula.

[Procedure] A wff θ and a natural number μ are given and perform the following operations in the given order.

step 1. First, associate a new Skolem function symbol with each subformula of θ that has form $L\phi$ and occurs positively in θ; in other words, negatively in $F\theta$. We say that the Skolem function symbol *corresponds* to the subformula $L\phi$. This notion is used in the π-rule below.
Next, let $\mu = 1$ and E be the prefixed formula $[0]F\theta$. Here, 0 is a Skolem function with 0-ary.

step 2. Apply the following rules to the subformulas of E repeatedly until no rule is applicable to them. Then, let E' be the resulting expression.

$\langle\alpha$- rule$\rangle$
$sT\phi\wedge\psi \longrightarrow sT\phi \times sT\psi$
$sF\phi\vee\psi \longrightarrow sF\phi \times sF\psi$
$sF\phi\supset\psi \longrightarrow sT\phi \times sF\psi$
$sT\neg\phi \longrightarrow sF\phi$
$sF\neg\phi \longrightarrow sT\phi$

$\langle\beta$- rule$\rangle$
$sT\phi\vee\psi \longrightarrow (sT\phi + sT\psi)$
$sF\phi\wedge\psi \longrightarrow (sF\phi + sF\psi)$
$sT\phi\supset\psi \longrightarrow (sF\phi + sT\psi)$

(Here, s denotes a prefix.)

$\langle\nu$- rule$\rangle$
$sTL\phi \longrightarrow s*[x_1]T\phi\times...\times s*[x_\mu]T\phi$
Here, $x_1, .., x_\mu$ are μ pieces of new variables and * denotes a concatenation of lists.

$\langle\pi$- rule$\rangle$
$sFL\phi \longrightarrow s*[f(t)]\mathrm{F}\phi$
Here, f is the Skolem function symbol corresponding to subformula $L\phi$ and t is the last element of list s.

step 3. Apply the following rules to the subexpressions of E' repeatedly until no rule is applicable to them.
$S \times (P + Q) \longrightarrow S \times P + S \times Q$
$(P + Q) \times S \longrightarrow P \times S + Q \times \mathrm{S}$

Then, we get the resulting expression $S_{1,1}\times...\times S_{1,k_1}+...+S_{n,1}\times...\times S_{n,k_n}$. Here, S, P, Q, and $S_{i,j}$ denote prefixed formulas.

[Notations] We call each $S_{j,1}\times...\times S_{j,kj}$ above a *branch* of θ and denote the set of all branches of θ by $tab(\theta)_\mu$. A pair of prefixed formulas whose formulas are the same, but whose signatures are different, is called a *connection*. For a branch br, $co(br)$ denotes the set of all connections occurring in br. For a connection co of form $([g_1, .., g_m]Tp, [h_1, .., h_n]Fp)$, $rel(co)$ is defined to be the first-order formula $R(g_1, g_2) \wedge ... \wedge R(g_{m-1}, g_m) \wedge R(h_1, h_2) \wedge ... \wedge R(h_{n-1}, h_n) \wedge (g_m = h_n)$, where R is a binary predicate symbol.

step 4. Let $\nu(\theta)_\mu$ be $\wedge_{br \in tab(\theta)} \vee_{co \in co(br)} rel(co)$.
Let $\pi - set(\theta)_\mu$ be $\{R(g_1, g_2) \wedge ... \wedge R(g_{p-2}, g_{p-1}) \supset R(g_{p-1}, g_p) \mid$ prefix $[g_1, .., g_p, .., g_t]$ appears in some prefixed formula of some branch in $tab(\theta)$ and g_p is a function term$\}$.
Let $\pi(\theta)_\mu$ be the conjunction of all elements of $\pi - set(\theta)_\mu$.
Lastly, let $\Delta(\theta)_\mu$ be $\exists x_1 .. \exists x_p [\pi(\theta)_\mu \supset \nu(\theta)_\mu]$. Here, $x_1, .., x_p$ are all the variables in $\pi(\theta) \supset \nu(\theta)$.

step 5. If $\Sigma \supset \Delta(\theta)_\mu$ is a theorem of first-order logic, then output the sentence "θ is a theorem of Σ-system" and halt.
Otherwise, let $\mu = \mu + 1$ and go to step 2.

step 2 and **step 3** are based on the usual transformations of prefixed tableau methods. $rel(co)$ expresses that the formulas in the connection co are in the same world with opposite signatures. $\nu(\theta)$ expresses that every branch has some connection whose formulas belong to the same world. $\pi(\theta)$ expresses, by using function symbols, some information included in the given wff that is concerned with the existence of worlds. μ plays the same role as in [11]. That is, in the interpretation of the formula $L\phi$, we consider only μ worlds, instead of all worlds. $\Delta(\theta)_\mu$ can be written as $\forall x_1 .. \forall x_p \pi(\theta)_\mu \supset \exists x_1 .. \exists x_p \nu(\theta)_\mu$. We may abbreviate it as $\forall x \pi(\theta)_\mu$ for $\forall x_1 .. \forall x_p \pi(\theta)_\mu$.

We have the following correctness theorem.

Theorem 3.1 ((Correctness Theorem)). *θ is a theorem of the Σ-system iff $\Sigma \supset \Delta(\theta)_\mu$ is a theorem of first-order logic for some μ.*

4. Examples

By using examples, we will show how our procedure transforms any given wff θ of modal logic to the wff $\Delta(\theta)_\mu$ of first-order logic.

Example 1.
Input formula: $\theta = L(Lp \supset q) \vee L(Lq \supset p)$
$\mu = 1$

The process of transformation is as follows.
$[0]FL(Lp \supset q) \vee L(Lq \supset p)$
$[0]FL(Lp \supset q) \times [0]FL(Lq \supset p)$
$[0, 1]FL(Lp \supset q) \times [0]FL(Lq \supset p)$

$[0,1]F(Lp \supset q) \times [0,2]F(Lq \supset p)$
$[0,1]TLp \times [0,1]Fq \times [0,2]F(Lq \supset p)$
$[0,1]TLp \times [0,1]Fq \times [0,2]TLq \times [0,2]Fp$
$[0,1,w1]Tp \times [0,1]Fq \times [0,2]TLq \times [0,2]Fp$
$[0,1,w1]Tp \times [0,1]Fq \times [0,2,w2]Tq \times [0,2]Fp$

$$\begin{aligned}\pi(\theta)_1 &= R(0,1) \wedge R(0,2)\\ \nu(\theta)_1 &= [\underline{R(0,1)} \wedge R(1,w1) \wedge \underline{R(0,2)} \wedge (w1 = 2)] \vee [\underline{R(0,2)} \wedge R(2,w2) \wedge \\ &\quad \underline{R(0,1)} \wedge (w2 = 1)]\end{aligned}$$

The underlined atoms are redundant because they appear in $\pi(\theta)_1$ as a consequence.
$\Sigma \supset [R(0,1) \wedge R(0,2) \supset R(1,2) \vee R(2,1)]$
is the resulting transformed formula.

θ is an axiom of the system S4.3 and S4.3 is the complete normal system defined by $\forall x\ \forall y\ \forall z(R(x,y) \wedge R(x,z) \supset R(y,z) \vee R(z,y)) \wedge$ reflexiveness $\wedge$ transitivity.

Example 2.
Input formula: $\theta = L(p \vee \neg L \neg q) \supset (Lp \vee \neg L \neg q)$
$\mu = 1$

The process of transformation is as follows.
$Lp[0]FL(p \vee \neg L \neg q) \supset (Lp \vee \neg L \neg q)$
$[0]TL(p \vee \neg L \neg q) \times [0]F(Lp \vee \neg L \neg q)$
$[0,w1]T(p \vee \neg L \neg q) \times [0]F(Lp \vee \neg L \neg q)$
$[0,w1]T(p \vee \neg L \neg q) \times [0]FLp \times [0]F \neg L \neg q$
$([0,w1]Tp + [0,w1]T \neg L \neg q) \times [0]FLp \times [0]F \neg L \neg q$
$([0,w1]Tp + [0,w1]FL \neg q) \times [0]FLp \times [0]F \neg L \neg q$
$([0,w1]Tp + [0,w1]FL \neg q) \times [0]FLp \times [0]TL \neg q$
$([0,w1]Tp + [0,w1,f(w1)]F \neg q) \times [0]FLp \times [0]TL \neg q$
$([0,w1]Tp + [0,w1,f(w1)]F \neg q) \times [0,1]Fp \times [0]TL \neg q$
$([0,w1]Tp + [0,w1,f(w1)]F \neg q) \times [0,1]Fp \times [0,w2]T \neg q$
$([0,w1]Tp + [0,w1,f(w1)]Tq) \times [0,1]Fp \times [0,w2]T \neg q$
$([0,w1]Tp + [0,w1,f(w1)]Tq) \times [0,1]Fp \times [0,w2]Fq$
$[0,w1]Tp \times [0,1]Fp \times [0,w2]Fq + [0,w1,f(w1)]Tq \times [0,1]Fp \times [0,w2]Fq$

$$\pi(\theta)_1 = R(0,1) \wedge (R(0,w1) \supset R(w1,f(w1)))$$

$$\begin{aligned}\nu(\theta)_1 &= R(0,w1) \wedge \underline{R(0,1)} \wedge (\mathrm{w1{=}1}) \wedge R(0,w1) \wedge \underline{R(w1,f(w1))} \wedge R(0,w2) \wedge \\ &\quad (f(w1) = w2)\end{aligned}$$

The underlined atoms are redundant.
$\Sigma \supset \exists w[(R(0,1) \wedge (R(0,w) \supset R(w,f(w)))) \supset R(0,f(1))]$

is the resulting transformed formula.

It is known that θ is not a theorem of the system T, but of S4. Actually, for $\Sigma{=}\forall x R(x,x)$, $\Sigma \supset \exists w[(R(0,1)\wedge(R(0,w)\supset R(w,f(w)))) \supset R(0,f(1))]$ is not proved, but for Σ that includes transitivity, it is proved. Of course, the former does not mean that θ is not a theorem of T.

Example 3.
Input formula: $\theta = L(p \wedge q) \supset Lp \wedge Lq$
$\mu = 2$

The process of transformation is as follows.
$[0]FL(p \wedge q) \supset Lp \wedge Lq$
$[0]TL(p \wedge q) \times [0]F(Lp \wedge Lq)$
$[0,w1]T(p \wedge q) \times [0,w2]T(p \wedge q) \times [0]F(Lp \wedge Lq)$
$\vdots$
$[0,w1]Tp \times [0,w1]Tq \times [0,w2]Tp \times [0,w2]Tq \times ([0,1]Fp + [0,2]Fq)$

$\pi(\theta)_2 = R(0,1)\wedge R(0,2)$
$\nu(\theta)_2 = ((R(0,w1)\wedge \underline{R(0,1)}\wedge(w1 = 1)) \vee (R(0,w2)\wedge \underline{R(0,1)}\wedge(w2 = 1)))$
$\quad \wedge ((R(0,w1)\wedge \underline{R(0,2)}\wedge(w1 = 2)) \vee (R(0,w2)\wedge \underline{R(0,2)}\wedge(w2 = 2)))$
The underlined atoms are redundant.
$\Sigma \supset \exists w1 \exists w2\ (R(0,1)\wedge R(0,2) \supset ((R(0,w1)\wedge(w1 = 1)) \vee (R(0,w2)\wedge(w2 = 1)))$
$\quad \wedge ((R(0,w1)\wedge(w1 = 2)) \vee (R(0,w2)\wedge(w2 = 2))))$
is the resulting transformed formula.

θ is a theorem of any normal system. In fact, $\Delta(\theta)_2$ is proved, while $\Delta(\theta)_1$ is not a theorem.

Remark. We can simplify the proving procedure in some cases. First, we can prove that if some branch does not include any connection for some input μ, then, for any μ' larger than μ, there is always a branch not including any connection. Therefore, if we find a branch not including a connection, we can stop the calculation and conclude that θ is not a theorem. Secondly, as seen in example 1 above, by the completeness of paramodulation [3], if Σ does not include the predicate $=$ positively, we can remove each predicate of the form $t_1 = t_2$ in the formula ν by applying $mgu(t_1, t_2)$ to ν, when they are unifiable and their variables appear only in the conjunct. If t_1 and t_2 are not unifiable, then we can remove the conjunct that includes the predicate $t_1 = t_2$ from ν.

5. Soundness and Completeness of the Procedure

The formal proof of soundness and completeness of the procedure is long. We give only the outline of the proof.

As for the soundness, we must show that if $\Sigma \supset \Delta(\theta)_\mu$ is a theorem of first-order logic for some μ, then θ is a theorem of Σ-system. This is achieved by purely using the definition of models of modal logic and first-order logic.

As for the completeness, we must show the reverse. We will show the contraposition of it. That is, if $\Sigma \supset \Delta(\theta)_\mu$ is not a theorem for any μ, then θ is not a theorem of Σ-system. Hence, all we have to do is construct a Kripke structure M of the system and locate a world w such that $M,w \models \mathrm{F}\theta$ from the precondition.

First, we prove that $\Delta(\theta)_\mu \supset \Delta(\theta)_{\mu+1}$ is valid. Then, by the compactness theorem, the precondition means that the set of formulas $\{\Sigma,\neg\Delta(\theta)_1,\neg\Delta(\theta)_2, \neg\Delta(\theta)_3,...\}$ has a model. Moreover, by Löwenheim Skolem theorem, we can assume that the model is countable. Let the model be (W,I), where W is the underlying set and I is the interpretation of the predicate symbol R and the Skolem functions appearing in $\{\Delta(\theta)_\mu\}_{1\leq\mu}$. Then, we can employ $\langle W, R^I\rangle$ as the Kripke frame of the model we are seeking, where R^I is the interpretation of R by I. We index the worlds with natural numbers.

In the next step, we give a valuation to the frame. We construct partial valuations, step by step, depending on μ. First, we place the signed formula $F\theta$ at the world 0^I. We consider that partial valuations make θ false at 0^I. Next, we decompose the formula $F\theta$ just as done in **step 2** of the proving procedure. For example, if the formula has the form $F\phi\vee\psi$, then we place the two formulas $F\phi$ and $F\psi$ at the world. If the formula has the form $F\phi \wedge \psi$, then we split the partial valuation and place $F\phi$ at the world for one of the partial valuations and $F\psi$ for the other. If a placed formula has the form $TL\psi$, we place $T\psi$ to the first μ pieces of the accessible worlds according to their indexes. If a placed formula has the form $FL\psi$, we place $F\psi$ at the accessible world determined by the interpretation of Skolem functions. We then iterate this procedure.

As a result, we obtain maps (partial valuations) V^i_μ from W to the power set of the set of signed subformulas of θ $(1 \leq i \leq n_\mu)$, where $V^i_\mu(w)$ is the set of all signed formulas placed at the world w in the branch corresponding to V^i_μ. Here, we can prove that for any μ there is a partial valuation V^i_μ without contradiction: for any world w and any formula ϕ, $F\phi \in V^i_\mu$ and $T\phi \in V^i_\mu$ cannot be simultaneously valid. This is due to the fact that (W,I) is a model of $\{\Sigma,\neg\Delta(\theta)_1,\neg\Delta(\theta)_2, \neg\Delta(\theta)_3,...\}$. The proof is straightforward from the construction of the partial valuations, but long and tedious. We define the extension relation $V^i_{\mu1} \lhd V^j_{\mu2}$ $(\mu1 \leq \mu2)$ as $V^i_{\mu1}(w)\subset V^j_{\mu2}(w)$ for any world w. Note that if $V^i_{\mu1} \lhd V^j_{\mu2}$ and $V^j_{\mu2}$ has no contradiction, then $V^i_{\mu1}$ also has none. Therefore, by this relation, the partial valuations without contradiction form an infinite tree with a finite number of branches at each node. Then, by

König's lemma, we can find an infinite chain of partial valuations $V_1 \lhd V_2 \lhd \ldots$, such that each V_μ has no contradiction. Let V_∞ be the inductive limit of this sequence. In other words, $S\psi \in V_\infty(w)$ iff $S\psi \in V_\mu(w)$ for some μ, where S is either T or F. We define the valuation V as $\langle W, R^I, V \rangle, w \models p$ if $\mathrm{T}p \in V_\infty(w)$ for any propositional variable p. Then we can prove that if $S\phi \in V_\infty(w)$ then $\langle W, R^I, V \rangle, w \models S\phi$ for any formula ϕ, where S is as above. This is shown by formula induction. The key point is that $T\phi$ is now placed at every world accessible from one where $TL\phi$ is placed, by taking the limit of the chain. Therefore, in particular, $\langle W, R^I, V \rangle, 0^I \models F\theta$, and we have thus obtained a desired model.

6. Termination of the Procedure

The proving method for modal logic proposed here transforms the statement "a given wff of modal logic is a theorem of a certain system" to the statement "one of some countably many wffs of first-order logic is a theorem". Therefore, this procedure is semi-decidable, but not decidable in general. This corresponds to the fact that there is a first-order definable system that is not decidable [7]. On the other hand, many well-known first-order definable systems are known to be decidable. It is desirable that our proving method guarantees termination for those systems. A simple way to realize this is to find two computable functions ω and d from the set of all wffs to the set of natural numbers, which have the following properties.

(1) For a given θ, $\Sigma \supset \Delta(\theta)_\mu$ is a theorem for some μ iff $\Sigma \supset \Delta(\phi)_\mu$ is a theorem for some μ less then $\omega(\theta)$.
(2) The proof search of $\Sigma \supset \Delta(\theta)_\mu$ $(\mu \leq \omega(\theta))$ can be restricted to the bounded Herbrand space, the depth of whose terms are less then $d(\theta)$.

These functions have been obtained for K, D, T, S4, and S5, by analyzing their tableaux. The basic idea is to find the effectively calculated finite part of their tableaux such that if all branches have connections, we can find a connection in the finite part for all branches. To find the finite part, we capture the periodicity of their tableaux. The method depends on each system and we have not yet found a general framework for it.

7. Discussion

There is another approach to a uniform proving method called the translation method [8], which is applicable to the same class of systems. In this method, a modal formula is directly translated to a first-order formula by introducing predicate symbols with 1-ary for each propositional variable and an accessibility relation symbol. Our method, in contrast, uses only two predicate symbols:

an accessibility relation symbol and =. It is also known that in the translation method, a transformed formula grows exponentially. Recently, in [9] and [10], this method was improved by introducing special Skolem functions and axioms on them. Nonetheless, the method requires a special modification for systems not containing D and then loses efficiency and generality.

Example 1 in section 4 shows that the procedure has some relationship with Correspondence Theory [2]. For Sahlqvist's formulas, the procedure seems to give the condition on accessibility relations corresponding to an input formula. But, at present, we have no idea about for what class of systems this method gives the corresponding conditions mechanically.

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