

# CONTENTS

ART.	PAGE
INTRODUCTION . . . . .	1
I. THE RESULTANT	
2. Resultant of two homogeneous polynomials . . . . .	4
6. Resultant of $n$ homogeneous polynomials . . . . .	7
8. Resultant isobaric and of weight $L$ . . . . .	11
8. Coefficient of $a_r^{L_r} \dots a_n^{L_n}$ in $R$ is $R_r^{l_r l_{r+1} \dots l_n}$ . . . . .	11
8. The extraneous factor $A$ involves the coefficients of $(F_1, F_2, \dots, F_{n-1})_{x_n=0}$ only . . . . .	11
9. Resultant is irreducible and invariant . . . . .	12
10. The vanishing of the resultant is the necessary and sufficient condition that $F_1 = \dots = F_n = 0$ should have a proper solution . . . . .	13
11. The product theorem for the resultant . . . . .	15
11. If $(F_1, \dots, F_n)$ contains $(F'_1, \dots, F'_n)$ , $R$ is divisible by $R'$ . . . . .	15
12. Solution of equations by means of the resultant . . . . .	15
12. The $u$ -resultant resolves into linear homogeneous factors in $x, u_1, u_2, \dots, u_r$ . . . . .	16
II. THE RESOLVENT	
15. Complete resolvent is a member of the module . . . . .	20
15. Complete resolvent is 1 if there is no finite solution . . . . .	21
17. Examples on the resolvent . . . . .	21
18. The complete $u$ -resolvent $F_u$ . . . . .	24
18. $(F_u)_{x=u_1x_1+\dots+u_nx_n=0} \text{ mod } (F_1, F_2, \dots, F_k)$ . . . . .	24
19. All the solutions of $F_1 = F_2 = \dots = F_k = 0$ are obtainable from true linear factors of $F_u$ . . . . .	25
20. Any irreducible factor of $F_u$ having a true linear factor is a homogeneous whole function of $x, u_1, \dots, u_n$ . . . . .	26
21. Irreducible spreads of a module . . . . .	27
22. Geometrical property of an irreducible spread . . . . .	28

## III. GENERAL PROPERTIES OF MODULES

ART.		PAGE
23.	$M/M' = M/⟨M, M'⟩$ . . . . .	30
23.	If $M'M''$ contains $M$ , $M'$ contains $M/M'$ . . . . .	31
24.	Associative, commutative, and distributive laws . . . . .	31
25.	$⟨M, M'⟩[M, M']$ contains $MM'$ . . . . .	32
26.	$M/M'$ and $M/⟨M/M'⟩$ mutually residual with respect to $M$ . . . . .	32
28.	$M/⟨M_1, M_2, \dots, M_k⟩ = [M/M_1, M/M_2, \dots, M/M_k]$ . . . . .	33
28.	$[M_1, M_2, \dots, M_k]/M = [M_1/M, M_2/M, \dots, M_k/M]$ . . . . .	33
30.	Spread of prime or primary module is irreducible . . . . .	34
31.	Prime module is determined by its spread . . . . .	34
32.	If $M$ is primary some finite power of the corresponding prime module contains $M$ . . . . .	35
33.	A simple module is primary . . . . .	36
34.	There is no higher limit to the number of members that may be required for the basis of a prime module . . . . .	36
34.	Space cubic curve has a basis consisting of two members . . . . .	37
35.	The L.C.M. of primary modules with the same spread is a primary module with the same spread . . . . .	37
36.	If $M$ is primary $M/M'$ is primary . . . . .	37
37.	Hilbert's theorem . . . . .	38
38.	Relations between a module and its equivalent $H$ -module . . . . .	39
38, 42.	Condition that an $H$ -module $M$ may be equivalent to $M_{x_n=1}$ . . . . .	39
38.	Properties of an $H$ -basis . . . . .	40
39.	Lasker's theorem . . . . .	40
40.	Method of resolving a module . . . . .	42
41, 44.	Conditions that a module may be unmixed . . . . .	43
42.	Deductions from Lasker's theorem . . . . .	44
42.	When $M/M'$ is $M$ and when not . . . . .	44
42.	No module has a relevant spread at infinity . . . . .	44
43.	Properties of the modules $M^{(r)}$ , $M^{(s)}$ . . . . .	45
44.	Section of prime module by a plane may be mixed . . . . .	47
46.	The Hilbert-Netto theorem . . . . .	48
UNMIXED MODULES . . . . .		49
48.	Module of the principal class is unmixed . . . . .	49
49.	Conditions that $(F_1, F_2, \dots, F_r)$ may be an $H$ -basis . . . . .	50
50.	Any power of module of principal class is unmixed . . . . .	51
51, 52.	Module with $\gamma$ -point at every point of $M$ . . . . .	52

ART.	PAGE
52. When a power of a prime module is unmixed . . . . .	53
53. Module whose basis is a principal matrix is unmixed . . . . .	54
SOLUTION OF HOMOGENEOUS LINEAR EQUATIONS . . . . .	58
NOETHER'S THEOREM . . . . .	60
56. The Lasker-Noether theorem . . . . .	61
IV. THE INVERSE SYSTEM	
58. Number of modular equations of an $H$ -module of the principal class . . . . .	65
59. Any inverse function for degree $t$ can be continued . . . . .	67
59. Diagram of dialytic and inverse arrays . . . . .	67
59. The modular equation $1=0$ . . . . .	69
60, 82. The inverse system has a finite basis . . . . .	69
61. The system inverse to $(F_1, F_2, \dots, F_k)$ is that whose $F_i$ -derivates vanish identically . . . . .	70
62. Modular equations of a residual module . . . . .	70
63. Conditions that a system of negative power series may be the inverse system of a module . . . . .	71
64. Corresponding transformations of module and inverse system . . . . .	71
65. Noetherian equations of a module . . . . .	73
65. Every Noetherian equation has the derivate $1=0$ . . . . .	73
65. The Noetherian array . . . . .	75
66. Modular equations of simple modules . . . . .	75
PROPERTIES OF SIMPLE MODULES . . . . .	77
67. A theorem concerning multiplicity . . . . .	77
69. Unique form of a Noetherian equation . . . . .	79
71. A simple module of the principal Noetherian class is a principal system . . . . .	80
72. A module of the principal class of rank $n$ is a principal system . . . . .	81
73. $\mu = \mu' + \mu''$ . . . . .	82
74. $\mu'_v + \mu''_{v'} = \mu_v = \mu_{v'}$ , where $v' + v'' = \gamma - 1$ . . . . .	83
75. $H_m = 1 + \mu_1 + \mu_2 + \dots + \mu_m$ . . . . .	83
76. $H'_v - H''_{v'} = H'_{v+v'} - H_{v'} = H_v - H''_{v+v'}$ , where $v' + v'' = \gamma - 2$ . . . . .	84
MODULAR EQUATIONS OF UNMIXED MODULES . . . . .	85
77. Dialytic array of $M^{(v)}$ . . . . .	86
78. Solution of the dialytic equations of $M^{(v)}$ . . . . .	88

ART.	PAGE
79. Unique system of $r$ -dimensional modular equations of $M$ . . . . .	89
79. The $n$ -dimensional equations . . . . .	89
80. Equations of the simple $H$ -module determined by the highest terms of the members of an $H$ -basis of $M^{(r)}$ . . . . .	89
81. If $R=1$ and $M$ is unmixed, $M$ is perfect . . . . .	90
82. If $M^{(r)}$ is a principal system so is $M$ . . . . .	90
82. A module of the principal class is a principal system . . . . .	90
83. $M^{(r)}$ and $M$ are principal systems if the module determined by the terms of highest degree in the members of an $H$ -basis of $M^{(r)}$ is a principal system ; not conversely . . . . .	91
84. Modular equations of an $H$ -module of the principal class . . . . .	92
85. Whole basis of system inverse to $M^{(r)}$ . . . . .	93
86. Modules mutually residual with respect to an $H$ -module of the principal class . . . . .	94
87. The theorem of residuation . . . . .	96
88. Any module of rank $n$ is perfect . . . . .	98
88. An unmixed $H$ -module of rank $n - 1$ is perfect . . . . .	98
88. An $H$ -module of the principal class is perfect . . . . .	98
88. A module of the principal class which is not an $H$ -module is not necessarily perfect . . . . .	98
88. A prime module is not necessarily perfect . . . . .	98
89. An $H$ -module $M$ of rank $r$ is perfect if the module $M_{x_{r+2}=...=x_n=0}$ is unmixed . . . . .	99
90. A perfect module is unmixed . . . . .	99
90. The L. C. M. of a perfect module of rank $r$ and any module in $x_{r+1}, \dots, x_n$ only is the same as their product . . . . .	99
91. Value of $H_l$ for a perfect module . . . . .	99
92. If $M, M'$ are perfect $H$ -modules of rank $r$ , and if $M$ contains $M'$ , and $M_{x_{r+1}=...=x_n=0}$ is a principal system, $M/M'$ is perfect . . . . .	100
NOTE ON THE THEORY OF IDEALS . . . . .	101