SPACE DISTRIBUTION OF SMALL DARK NEBULAE

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1. Introduction

The National Geographic Society—Palomar Observatory Sky Atlas has been used to compile a catalogue of dark nebulae [4]. In this catalogue the galactic coordinates, projected surface areas (in square degrees), and estimates of the opacities of the clouds are listed. The most opaque clouds are designated by opacity 6. These objects were selected on the condition that no stars were visible within the measured surface area of the cloud. This paper deals only with the distribution of the opacity 6 clouds, and attempts to reproduce the observed distribution of apparent sizes of those objects by a simple model.

2. Model I

Model I is based on the following idealized conditions of the region surrounding the sun.

(1) Stars are assumed to be distributed at random in space with a mean density of S_0 stars/pc^3.

(2) It is assumed that we are able to detect all stars not obscured by the opacity 6 clouds.

(3) The opacity 6 clouds are assumed to be spherical, and completely opaque so that no stars can be seen through the nebulae.

(4) There is assumed to be no lower limit to the measurable size of a nebula.

(5) It is assumed that no nebulae eclipse each other.

(6) All opacity 6 nebulae are assumed to have the same area Ω pc^2 of their orthogonal projection on a plane perpendicular to the line of sight.

(7) The opacity 6 clouds are assumed to be distributed at random in space, with a mean density of C_0 clouds/pc^3.

On these assumptions we want to calculate the probability of observing a nebula of measured surface area of ω' square degrees and classify this object as an opacity 6 cloud. This probability is given by

$$P = \pi^*(0)\pi^c(0)\pi'(1), \tag{2.1}$$

where

$\pi^*(0)$ is the probability that there are no stars in the volume element subtended by the cloud,

$\pi^c(0)$ is the probability that there are no clouds in the same volume element,

$\pi'(1)$ is the probability that one cloud is located at a distance $\xi = \Omega^{1/2}/\omega^{1/2}$ pc, if ω is in strdn.

On assumption of (1) we find

$$\pi^*(0) = e^{-n_0^*}, \tag{2.2}$$

where $n_0^* = \xi\Omega S_0/3$.

For the evaluation of $\pi^c(0)$ we must consider not the probability that there are no cloud centers in the volume element $\xi\Omega/3$ but the probability that there are no cloud centers within a volume element $7\xi\Omega/3$, since within this volume element if there were clouds they would partially eclipse the cloud we are considering. (The author is indebted to Dr. W. Bühler for this point.) Thus,

$$\pi^c(0) = e^{-7\xi\Omega C_0/3}. \tag{2.3}$$

Now the probability that one cloud is located at $\xi \pm \Delta\xi/2$ in the volume $\Omega\Delta\xi$ is

$$\pi'(1) = n_0' e^{-n_0'}, \tag{2.4}$$

where $n_0' = \Omega\Delta\xi C_0$.

By expanding the exponential and carrying only first order terms in $\Delta\xi$, we find

$$\pi'(1) = C_0\Omega\Delta\xi. \tag{2.5}$$

Thus, the probability of finding one cloud in the volume $\Omega\Delta\xi$ is

$$P(\xi)\Delta\xi = C_0\Omega e^{-\frac{1}{3}\Omega\xi(S_0 + 7C_0)}\Delta\xi, \tag{2.6}$$

or since $\xi = (\Omega/\omega)^{1/2}$, the probability of finding one cloud of angular size ω strdn in $\Delta\omega$ is

$$P(\omega)\Delta\omega = P[\xi(\omega)]\left|\frac{d\xi(\omega)}{d\omega}\right|\Delta\omega \tag{2.7}$$

or

$$P(\omega)\Delta\omega = \frac{C_0}{2}\Omega^{3/2}\exp\left\{-\frac{1}{3}\Omega^{3/2}\omega^{-1/2}(S_0 + 7C_0)\right\}\omega^{-3/2}\Delta\omega. \tag{2.8}$$

The actual number of clouds predicted to be counted would be the probability $P(\omega)\Delta\omega$ multiplied by the total number of these volume elements $4\pi/\omega$. Thus the predicted counts $f_p(\omega)\Delta\omega$ are given by

$$f_p(\omega)\Delta\omega = 2\pi C_0\Omega^{3/2}\omega^{-5/2}\exp\left\{-\frac{1}{3}\Omega^{3/2}\omega^{-1/2}(S_0 + 7C_0)\right\}\Delta\omega \tag{2.9}$$

The observations do not cover the entire sky, so an incompleteness factor must be applied. On the assumption of symmetry, this factor is taken to be that segment of the celestial sphere unobservable at Palomar—all southern declinations greater than $-35°$. That is, since some 21.3 per cent of the sky is not visible from Palomar, we will include a correction factor of 0.787 to the predicted counts.

Table I shows the comparison of the predicted counts with the observed distribution $F_{Ob}(\omega)$. The parameters of the predicted distribution were estimated by trial and error to bring the two distributions into reasonable agreement. No least squares solution was made.

TABLE I

COMPARISON BETWEEN MODEL I PREDICTED COUNTS AND OBSERVED DISTRIBUTION, USING VALUES

$$\Omega^{3/2}(S_0 + 7C_0)/3 = 5 \times 10^{-3}$$
$$2\pi C_0 \Omega^{3/2} \Delta\omega = 6.54 \times 10^{-12}$$
$$\Delta\omega = 1.52 \times 10^{-6} \text{ strdn}$$

ω' (square degrees)	$e^{-5\times 10^{-3}\omega^{-1/2}}$	$f_p(\omega)\omega$	$F_{Ob}(\omega)$
0.003	0.0061	38.0	47
0.008	0.0408	22.5	33
0.013	0.0813	13.3	8
0.018	0.119	8.70	13
0.023	0.153	5.92	3
0.028	0.174	4.63	3
0.033	0.206	3.29	4
0.038	0.230	2.60	8
0.043	0.252	2.08	4
0.048	0.273	1.70	1
0.053	0.289	1.40	1
0.058	0.304	1.19	1

On the basis of model I, we find

$$\Omega^{3/2}(S_0 + 7C_0) = 1.5 \times 10^{-2}, \quad C_0\Omega^{3/2} = 7.38 \times 10^{-7}. \tag{2.10}$$

Since $C_0 \ll S_0$, we have

$$\Omega^{3/2}S_0 = 1.5 \times 10^{-2} \tag{2.11}$$

or

$$C_0 = 4.56 \times 10^{-5} S_0^{-1/2} \text{ pc}^{-3}, \quad \Omega = 6.08 \times 10^{-2} S_0^{-2/3} \text{ pc}^2, \tag{2.12}$$

which makes these objects of sizes of the order of 0.14 pc or 30,000 a.u. in radius, with a space density about 10^{-4} that of stars.

If we look at the projected galactic latitude distribution of the opacity 6 nebulae, we see that they are confined for the most part to $\pm 10°$ of the plane. We have catalogued opacity 6 clouds for ω's $\sim 10^{-3}$ square degrees, which corresponds to distances of the order of 450 pc on the basis of model I. There are two assumptions in model I regarding uniform space distributions—that of stars (assumption 1) and clouds (assumption 7). According to Bok and MacRae [3], there are fluctuations in the mean star density as much as a factor of two, as determined in certain selected areas. The space density ρ of certain types of stars is represented [1] in the form $\rho = \rho_0 e^{-z/\beta}$, where z is the height above or below the galactic plane, and β is a constant which is a function of the type of star, and varies from 50 pc (for Cepheids and open clusters) to 3000 pc (for extreme population II objects). If we arbitrarily use the value listed for M5 through M8 stars $\beta = 500$ pc, we find that the mean density has dropped by a factor of 2.7 at $z \sim 500$ pc. Thus, S_0 could easily vary by a factor of 3. However, if we assumed that the mean line of sight value of S_0 for $70° \leqq |b| < 90°$ is 1/3 the value of S_0 for $0° \leqq |b| \leqq 40°$ then the ratio for the number of opacity 6 nebulae catalogued in each of these areas should be

$$\exp\left\{\frac{1}{3}\Omega^{3/2}\omega^{-1/2}[S_0(0°) - S(80°)]\right\} \tag{2.13}$$

or, using the values of model I, the factor becomes $\exp\{-5 \times 10^{-3}\omega^{-1/2}\}$, that is to say, the probability of observing a nebula in high galactic latitudes would be substantially greater than observing it in the plane. Obviously, this is not the case and the problem of detection of such a cloud in a star poor field (neglected in model I) is the dominant factor.

3. Model II

For our model II we will retain the assumptions (1) through (6) of model I but replace (7) (which assumed a random distribution of nebulae) by

(7) assume that the nebulae are confined to a plane centered about the galactic plane, with a thickness of $2h$ pc.

With this modification we must reevaluate the probability of cataloguing an opacity 6 nebula. The star distribution is still assumed Poisson. Therefore,

$$\pi^*(0) = e^{-\xi\Omega S_0/3}, \tag{3.1}$$

but the nebular distribution must be changed to

$$\begin{aligned} \pi^c(0) &= e^{-7\xi\Omega C_0/3} && \text{for} \quad \xi \leqq h \csc |b|, \\ \pi'(1) &= C_0\Omega\Delta\xi && \text{for} \quad \xi \leqq h \csc |b|, \\ \pi'(1) &= 0. && \text{for} \quad \xi > h \csc |b| \\ & && \text{or} \quad \omega < (\Omega/h^2)\sin^2 b. \end{aligned} \tag{3.2}$$

We see from this that there is a lower limit to ω defined by the equation

$$\omega = (\Omega/h^2)\sin^2 b. \tag{3.3}$$

Figure 1 shows the distribution of ω' as a function of $\sin^2 b$, the lower lines represent the limits as determined from the Ω value of model I for two different values of h.

The actual number of clouds predicted to be counted would be the probability $P(\omega)\Delta\omega$ multiplied by the total number of volume elements $V(\omega)$ for which $P(\omega)$ is not equal to zero, which is the same as model I for $\xi < h$ (or $\omega > \Omega/h^2$), but for $\omega < \Omega/h^2$ we can no longer consider an entire sphere, but must use only the solid angle subtended by the segment of a sphere cut out by two parallel planes of separation $2h$. In this case, we have for $|b| \leqq \sin^{-1}(h|\xi)$

(3.4) $$V(\xi|b) = \frac{1}{\omega}\left[4\pi - 4\pi\left(1 - \frac{h}{\xi}\right)\right]$$

or

(3.5) $$V(\omega|b) = 4\pi h\omega^{-1/2}\Omega^{-1/2}.$$

Using equations (2.9), (3.3) and (3.5), we can now try to adjust the values of

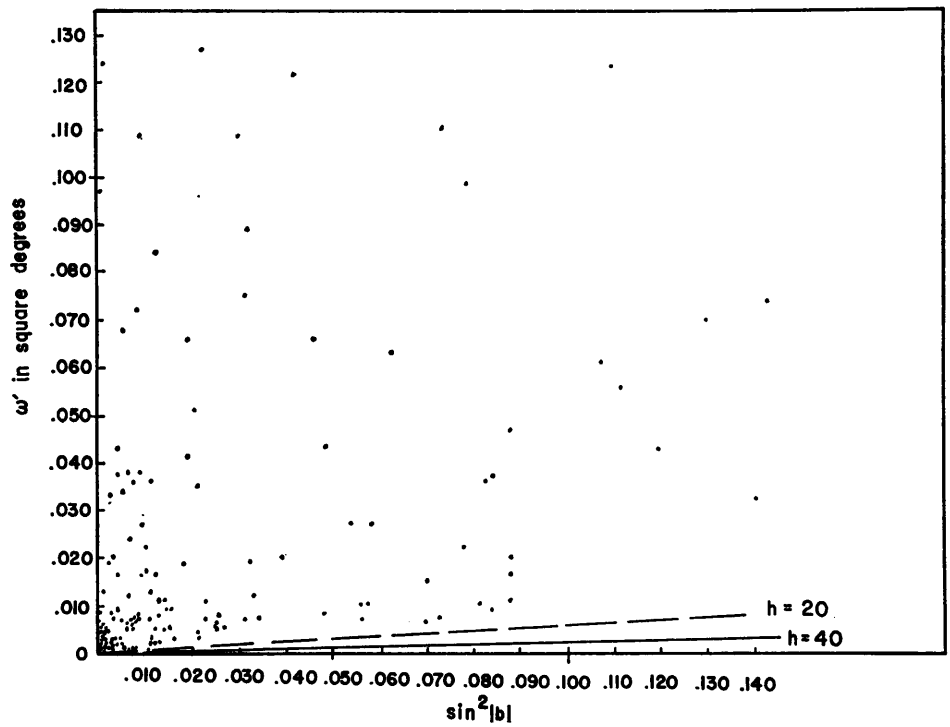

FIGURE 1

Distribution of observed surface area ω' in square degrees with $\sin^2 b$. The solid and dashed lines are lower limits defined by a plane parallel layer of nebulae of size given in model I and of two different layer thicknesses 2 h, measured in parsecs.

h, Ω, S_0 and C_0 to represent the catalogued nebulae. The following values were used for table II

(3.6) $$\Delta\omega = 1.52 \times 10^{-6}\,\text{strdn},\quad \Omega^{3/2}(S_0 + 7C_0)/3 = 2.5 \times 10^{-3},\quad h = 50\,\text{pc},$$

to adjust the fit such that

(3.7) $$C_0 S_0^{-2/3} = 7.41 \times 10^{-5}.$$

The lower limit of the ω, $\sin^2 b$ distribution is now represented by

$$\omega' \text{ square degrees} = 0.0208 \sin^2 b \tag{3.8}$$

and approximately coincides with the $h = 40$ line of figure 1. It appears that we could probably have selected an h about a factor of two smaller.

TABLE II

MODEL II

Frequency Distribution $f_p(\omega)$ and Observed Counts $F_{Ob}(\omega)$

ω' (square degrees)	$f_p(\omega)$	$F_{Ob}(\omega)$
0.003	56.0	47
0.008	21.4	33
0.013	11.3	8
0.018	7.2	13
0.023	5.0	3
0.028	3.6	3
0.033	2.8	4
0.038	2.3	8
0.043	1.6	1
0.048	1.6	1
0.053	1.3	1
0.058	1.1	1

The comparison between the predicted counts and observed counts is shown in figure 2.

The physical parameters of interest describing the clouds are $\Omega = 1.78 \times 10^{-2} S_0^{-2/3}$ pc^2 (if we use the fact that $S_0 \gg C_0$) and $C_0 = 3.71 \times 10^{-3} S_0^{2/3} h^{-1}$.

Thus, the actual numerical values of the sizes and space densities of the clouds depend on our choice of S_0 and h (table III).

TABLE III

CLOUD PARAMETERS ON MODEL II

	$S_0 = 1$ pc^{-3}		$S_0 = 0.1$		$S_0 = 0.01$	
h pc	Ω pc^2	C_0 pc^{-3}	Ω	C_0	Ω	C_0
50	1.8×10^{-2}	7.4×10^{-5}	0.08	1.6×10^{-5}	0.3	5.0×10^{-6}
25	1.8×10^{-2}	1.6×10^{-4}	0.08	3.2×10^{-5}	0.3	1.1×10^{-5}
10	1.8×10^{-2}	3.7×10^{-4}	0.08	8.0×10^{-5}	0.3	2.5×10^{-5}

We conclude, then, that on the basis of model II small dark nebulae are present in the interstellar medium of sizes of the order of less than one parsec. If we assume that the absorption in these nebulae is at least 5 magnitudes (otherwise we should see some background stars through them), then the masses of these clouds can be determined [5] by $M = 1.5 \times 10^{-5}(\Delta m)\omega'\xi^2$ solar masses,

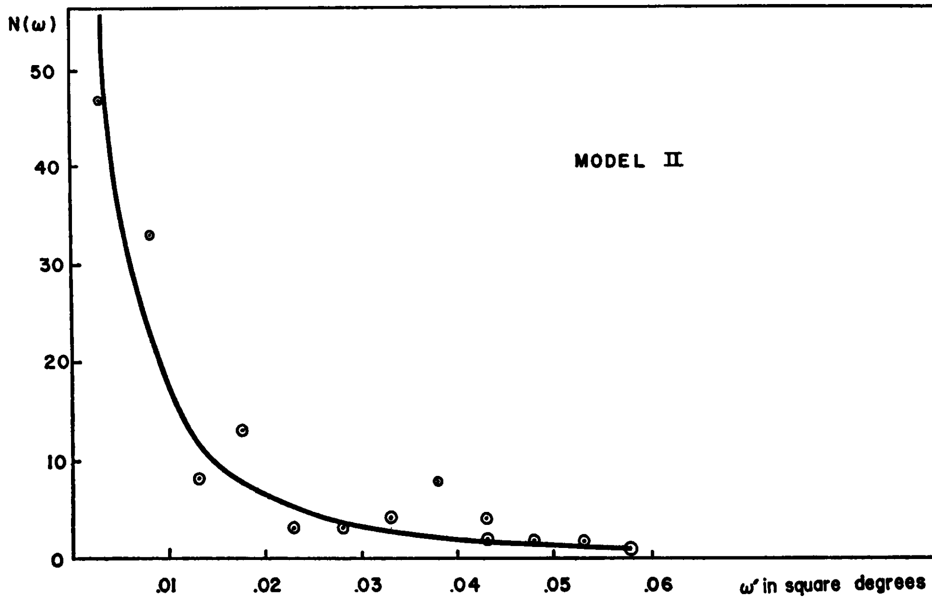

FIGURE 2

Comparison between the observed counts of nebulae of measured surface area ω' square degrees with the predicted variation based on model II.

where Δm is the absorption in magnitudes, ω' is the surface area in square degrees and ξ is the distance parsecs. On model II we may estimate the minimum mass of the clouds to be $M_c = 4.9 \times 10^{-2}(\Delta m)\Omega$, or using a $\Delta m \geqq 5$ we have

$$M_c \geqq 4.4 \times 10^{-3} S_0^{-2/3} M_\odot. \tag{3.9}$$

This gives the mass of the grains in the clouds on the assumption that they are typical interstellar grains. The absorption in the opacity 6 clouds is probably at least 5 magnitudes, but we have no way at the present time of setting an upper limit on the magnitude extinction through the cloud. Thus, we can only say that masses from $0.004 S_0^{-2/3}$ solar masses up are consistent with the counts, but we cannot give a quantitative measure until the actual value of the absorption within a cloud is known and until the gas/dust ratio is established for such objects.

The frequency in space of these clouds is also deduced on the assumptions of model II. If these objects represent some early evolutionary stage of a star, then the ratio of clouds to stars could be an estimate of the time spent in this early stage,

$$\frac{C_0}{S_0} = \frac{3.7 \times 10^{-3}}{h S_0^{1/3}} \tag{3.10}$$

and if we take $h = 25$ pc and an $S_0^{1/3} \sim 1.0$, then

$$\frac{C_0}{S_0} \sim 10^{-4}. \tag{3.11}$$

For a total lifetime of 10^{10} years, this gives a "globule stage" of the order of 10^6 years.

On the basis of model II we may ask how would the frequency of clouds be predicted on the hypothesis that the opacity 5 stars are those which show several (say, 1 or 2) foreground stars, but which otherwise were those described by model II. The only change would be that $\pi^*(0)$ should now be $\pi^*(n)$, where n is the number of foreground stars. Since

$$\pi^*(n) = \frac{n_0^{*n}}{n!} e^{-n_0^*}, \tag{3.12}$$

we have

$$f_p(\omega|\text{opacity } 5)/f_p(\omega|\text{opacity } 6) = \sum_{n=1}^{2} \frac{(S_0\Omega^{3/2}\omega^{-1/2}/3)^n}{n!}. \tag{3.13}$$

Using the numerical values adopted in model II,

$$S_0 \sim 1, \qquad \frac{1}{3}\Omega^{3/2} \sim 2.5 \times 10^{-3}. \tag{3.14}$$

We find the evaluations of the ratio of equation (3.13) shown in table IV.

TABLE IV

FREQUENCY DISTRIBUTION OF OPACITY 5 CLOUDS

ω' (square degrees)	$f_p(\omega\|\text{opacity } 5)/f_p(\omega\|\text{opacity } 6)$ $n = 1$	$n = 2$	$f_p(\omega\|\text{opacity } 5)$	$F_{Ob}(\omega\|\text{opacity } 5)$
0.003	0.821	0.337	65	130
0.008	0.505	0.127	14	65
0.013	0.396	0.078	5.4	34
0.018	0.337	0.056	2.8	39
0.023	0.297	0.044	1.7	18
0.028	0.269	0.036	1.1	14
0.033	0.248	0.031	0.8	8
0.038	0.232	0.027	0.6	5
0.043	0.218	0.024	0.4	2
0.048	0.206	0.021	0.4	10
0.053	0.196	0.019	0.3	2

It appears that at least 10 per cent of the clouds of opacity 5 could be represented on this model. The remaining 90 per cent clouds (or 50 per cent of the smaller area opacity 5 clouds) must be those clouds for which the assumptions in model II do not hold—in particular the assumption (3), which states that the clouds are completely opaque.

The clouds of lower opacity cannot be predicted on this model. Once we reach values of foreground stars such that the factorial in the denominator of (3.13) predominates, we predict very few observable clouds on model I. Thus, the clouds of opacities 1 to 4 represent primarily clouds for which the assumptions in model I do not hold; probably the most significant departures from these assumptions are that the clouds are not completely opaque and not of so small a size as the data on the opacity 6 clouds indicate. No analysis of semitransparent clouds will be considered here; the usual Wolf diagram type of studies of individual clouds are probably more significant for these types of objects.

We have not discussed the longitude variation of the clouds. On the basis of model II, we expect a random distribution with longitude as long as $|b| \leq \sin^{-1}(h|\xi)$. This is not observed, however. The region of Orion is singularly void of these dark globules. The histogram of figure 3 shows that there

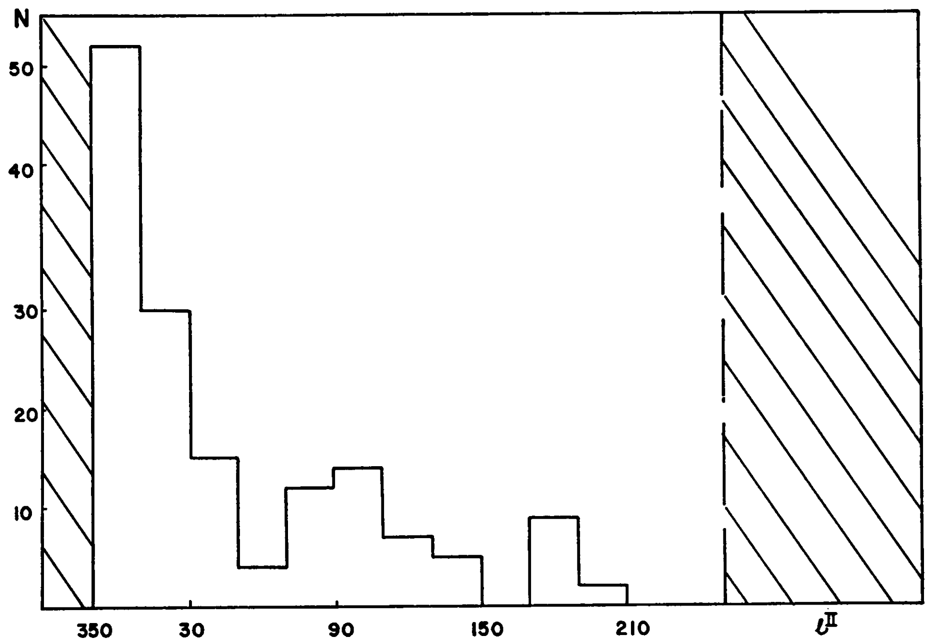

FIGURE 3

Galactic longitude distribution of 6 clouds.
The region between 350° to 330°ℓ^{II} contains the Sco-Oph complex;
the region of 180° < ℓ^{II} < 230° contains Orion.

are essentially no such objects in this area. No simple model could predict these marked changes in longitude; it may be that the extensive H II region of Orion has prevented the formation of such opaque clouds.

The maximum number is found at longitudes of the order of 0°, which represents the group of clouds in the Sco-Oph complex. Thus, there is a clustering of nebulae at a region in space corresponding to this dark cloud complex, that is, at distances of the order of 200 pc [2]. This would also give rise to an overabundance of objects about 0.046 square degrees for an $S_0 = 0.1$. This may be reflected in the "hump" in the observed distribution between 0.035 and 0.045, and is confirmed by the fact that some 30 per cent of the clouds catalogued in this $\Delta\omega$ interval are found in the Sco-Oph region, while no clouds of areas less than 0.005 square degrees are found in this region.

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