MASSES OF GALAXIES: SINGLES AND MEMBERS OF MULTIPLE SYSTEMS

THORNTON PAGE
WESLEYAN UNIVERSITY

1. Introduction

The masses of galaxies are important in several areas of astronomy and physics. In cosmology the mean mass is used to derive the average density of matter in the universe, a quantity which is related to the curvature of space in the cosmological models of general relativity. In any theory of the origin and evolution of galaxies, the masses are important in the dynamical aspects. Also, the wide range in mass estimates must be explained by a statistical theory of the origin of galaxies.

In principle, mass can be detected and measured by the Einstein redshift and the gravitational deflection of light, as well as by its dynamical effect on other masses. In practice, the Einstein redshift can be separated from Doppler redshift in only a few cases of no cosmological significance; gravitational deflection is unlikely to be useful [54]; and all determinations of the masses of galaxies have so far been based on simpler Newtonian mechanics. It is true that lower limits to the mass can be established in terms of emission lines of hot gases (in optical spectra) and cold hydrogen (in radio spectra). Moreover, many authors assume a relationship between luminosity L and mass M in the form of stars. This amounts to assuming a "normal" value of the ratio M/L, even though this ratio is known to vary from less than 0.01 to over 1000.

Statistics have been involved in practically all phases of these studies, and one of the basic problems concerns observational selection [50]. The luminosities of nearby galaxies range from 10^8 to 10^{12} suns, and it is clear that only the most luminous ones are observed at large distances. Moreover, they have a wide variety of forms, and there is further selection due to confusing distant galaxies of circular projection with foreground stars on photographs.

Projection introduces a second statistical problem, since most galaxies appear to have an axis of symmetry similar to that of a disk or oblate spheroid. Each is viewed in one projection at an unknown angle to the axis. Masses are determined from motions perpendicular to the plane of projection (radial velocities), generally on such simplifying assumptions as these: (1) the average internal motions in a galaxy are circular and in the equatorial plane; (2) the velocities of individual galaxies in a cluster are directed at random; (3) the orbits of double galaxies are circular, randomly oriented, and equally likely to be viewed at any angle to the line of centers; (4) the only forces involved are gravitational; and (with a few exceptions) (5) the observed forms, groupings, and distributions are relatively stable over long periods of time.

Distributions of luminosities, sizes, distances, and derived masses of galaxies are not only confused by the effects of selection noted above, and by possible systematic errors introduced as a result of the assumptions listed, but also by fairly large observational errors, by small sample sizes, and by interdependent errors. The distance of a galaxy, for example, is often inferred from its apparent brightness compared with its assumed luminosity. Its dimensions and mass are also derived from this distance, so that correlations between mass, dimensions, and luminosity are subject to bias.

2. Masses of single galaxies

A summary of mass determinations was collected at a special conference organized by Neyman, Scott, and myself ([49] p. 619). Corrections and additions have since been made (Holmberg [42]) as shown in table I.

The first column of table I gives the NGC catalog number of the galaxy (or Messier number or Vorontsov-Velyaminov number). In the second column, m_{pg} is the total photographic magnitude (in general, larger m_{pg} implies lower accuracy). The morphological types indicate forms from Ir (irregular) through Sc, Sb, Sa (spirals), SBc, SBb, SBa (barred spirals), and S0 (smooth lenticulars) to E (ellipticals) of projected ellipticity 0.7 (E7) to 0 (circular E0). The corrected redshift radial velocity V is relative to the Milky Way nucleus, and is used as a distance indicator; $D = V/100$ in Mpc except when $V < 300$ km/sec. Under Method, Ls stands for optical spectra taken with a long slit extending across the galaxy image to determine rotation; H II stands for separate optical spectra giving orbital velocities of ionized hydrogen gas clouds about the center of a galaxy; 21 cm stands for radio Doppler shifts used to determine rotation; circular orbits refers to the double galaxy analysis presented in the next section; and stat refers to the statistical studies of stellar radial velocities showing the rotation of our Milky Way galaxy. References refer to the list at the end of this paper with the abbreviations de V for de Vaucouleurs, BBP for Burbidge, Burbidge, and Prendergast, Z H for Zwicky and Humason, v d B for van den Bergh, Min for Minkowski, and D-A for Duflot-Augard. The last two columns give the estimated mass M and mass luminosity ratio M/L, both in solar units. Both of these estimates are subject to r.m.s. errors of 50 per cent or more; the least accurate values are enclosed in parentheses. In the previous listings by the authors cited, by Holmberg [42] and Page [51], distances were based on Hubble's Law, $D = V/H$, with $H = 75$ km/sec Mpc (BBP), 80 km/sec Mpc (Holmberg), and 100 km/sec Mpc (Page). All the mass estimates are proportional to the inverse of H used, and the M/L estimates are proportional to the value of H used, except in a few cases (such as M31 and LMC) where other distance indicators have been used. In table I they have

TABLE I

MASSES OF GALAXIES
(IN SOLAR UNITS, DISTANCES BASED ON THE HUBBLE LAW WITH H = 100 km/sec Mpc)

Galaxy NGC (* = IC)	m_{pg}	Type	$V/100$	Method	Reference	$M/10^{10}$	M/L
55	7.9	IrSc	1.0	H II	de V [36]	3.	2.
1613*	10.0	Ir I	(0.)	21 cm	Volders [69]	0.03	4.
3034 (M82)	9.6	Ir II	3.2	Ls	Mayall [46]	1.	9.
3556	10.6	ScIr	7.6	Ls	BBP [17]	1.	1.
6822	9.7	Ir I	0.7	21 cm	Volders [69]	( )	
— LMC	0.5	Ir S	(0.)	21cm, H II	de V [35] [36]	1.1	5.
— VV254		Ir	45.9	Ls	BB [7]	9.8	
Mean Ir		7 Ir	0.7 to 45.9			2	5
157	11.2	Sc	18.4	Ls	BBP [24]	4.4	1.5
253	6.9	Sc	1.0	Ls	BBP [26]	20.	2.
598 (M33)	6.2	Sc	(0.)	21cm, H II		1.	3.
613	11.0	SBc	14.9	Ls	BBRP [32]	10.	10.
1084	11.1	Sc	14.5	Ls	BBP [2]	0.8	1.
1365	10.5	SBc	15.1	Ls	BBP [5]	2.5	1.5
2146	11.3	Sc	9.9	Ls	BBP [15]	1.3	1.5
2903	9.5	Sc	5.1	Ls	BBP [1]	4.0	2.1
3646	11.8	Sc	42.0		BBP [23]	20.	4.
4631	9.7	Sc	6.5	Ls	de V [2]	2.4	1.8
5144 (M51)	8.6	Sc	(4.)	Ls	BB [9]	4.3	11.
5248	11.0	Sc	11.4	Ls	BBP [25]	4.	1.5
5457 (M101)	8.5	Sc	4.2	21 cm	Volders [68]	(1.0)	(17.)
6503	10.7	Sc(dwf)	3.5	Ls	BBCRP [10]	0.13	0.8
7320	13.	Sc	10.7	Ls	BB [6]	(4.4)	(8.)
Mean Sc		15 Sc	0.5 to 42.0			6	2
Mean Ir Sc		22 Ir, Sc	0.5 to 45.9			5	3
16 double systems	10. to 13.	2 Ir 32 S	6. to 76.	circular orbits	Page [52] and Table II	4.0	3.2
224 (M31)	4.3	Sb	(0.8)	21 cm, H II	[43] [71]	34.	8.4
1068 (M77)	10.	Sb(em)	12.0	Ls (em)	BBP [14]	2.0	2.7
1097	10.4	SBb	12.1	Ls(em)	BB [5]	0.6	0.5
3031 (M81)	8.1	Sb	(0.8)	H II	Münch [48]	12.	6.
3504	11.6	SBb	14.7	Ls (em)	BBP [20]	0.8	1.
3521	9.6	Sb	6.4	Ls (em)	BBCRP [11]	8.	5.
4258	8.9	Sb	5.3	Ls	BBP [30]	10.	2.4
5005	10.5	Sb	10.8	Ls (em)	BBP [21]	10.	2.5

TABLE I (Continued)

Galaxy NGC (* = IC)	m_{pg}	Type	$V/100$	Method	Reference	$M/10^{10}$	M/L
5055 (M63)	9.3	Sb	6.0	Ls (em)	BBP [16]	4.5	2.
5383	12.4	Sb	23.7	Ls	BBP [27]	4.	7.
7479	11.6	Sb	26.6	Ls	BBP [19]	(>0.8)	0.5
Milky Way	—	Sb	—	21cm, stat	Schmidt [60]	18.	5.6
Mean Sb		12 Sb	0.8 to 22.6			10	4
2782	12.5	Sa	25.1	Ls	D-A [34]	11.	7.5
3623 (M65)	10.2	Sa	6.4	Ls	BBP [22]	10.	7.2
7469	12.7	Sa (em)	50.2	Ls	BBP [29]	0.8	0.5
Mean Sa		3 Sa	6.4 to 50.2			20	7
Mean Sb Sa		15 Sb, Sa	0.8 to 50.2			11	4.5
Mean Ir S		37 Ir, S	0.5 to 50.2			7	3.5
221 (M32)	9.7	E2	(0.8)		Fish [39] [57]	0.3	11.
3115	10.1	E7	4.2		[48] [57]	15.	46.
3379	10.5	E1	7.5		Fish [39] [57]	13.	20.
4111	11.6	ES0	8.4		Poveda [57]	4.	14.
4278	11.2	E	6.2		Poveda [57]	5.	14.
4406 (M86)	10.3	E3	(0.)		Fish [39]	96.	39.
4472 (M49)	10.	E1	8.6		Fish [39]	110.	19.
4486 (M87)	9.6	E0	11.9		Fish [39] [57]	260.	60.
5128	8.	Epec	4.0	Ls (em)	BB [4]	15.	13.
Mean E		9E	0.8 to 11.9			70	30
28 double systems	9. to 14.	33E 13S0	7. to 48.	circular orbits	Page [52] and Table II	60.0	90.
Mean groups		4S, 4E	3. to 91.	virial theorem	Table IV	250.	280.
Mean clusters		100	7. to 67.	virial theorem	Table IV	130.	600.

all been converted to $H = 100$ km/sec Mpc (corresponding to cosmological age 10^{10} years), and very rough averages have been listed for the various types. These are plotted on figure 1.

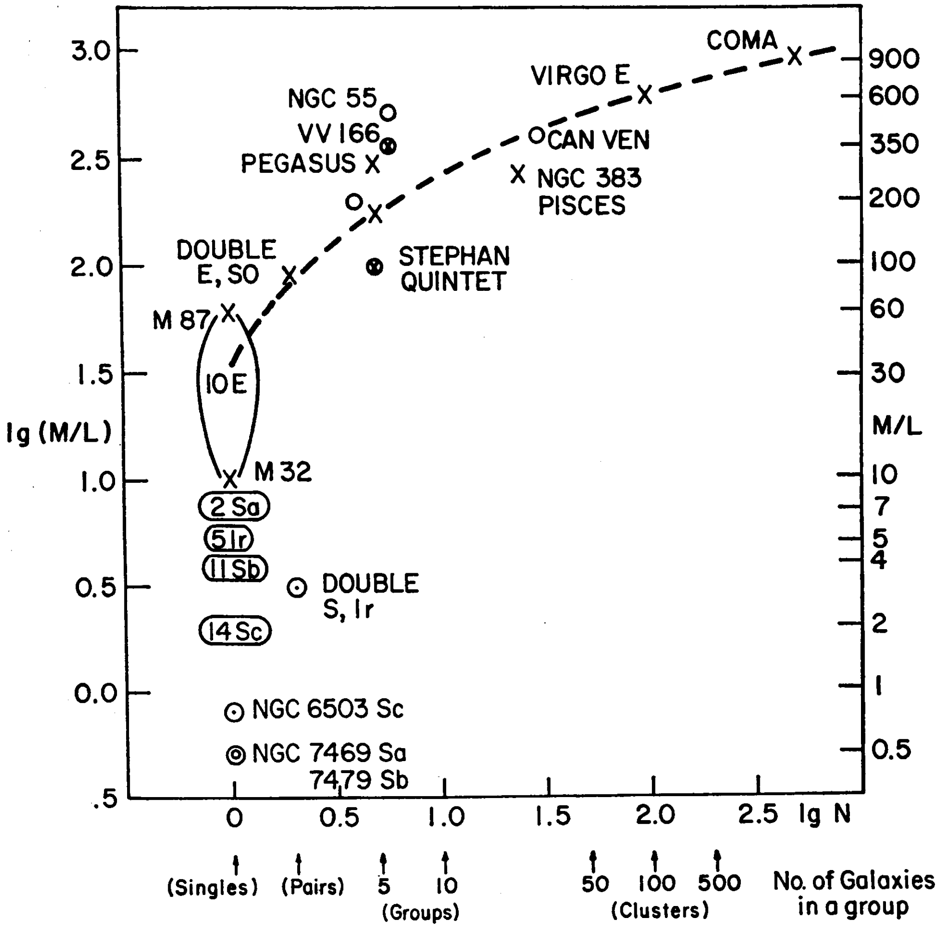

FIGURE 1

Average values of M/L for galaxies.

3. Mean masses in pairs of galaxies

The masses of individual galaxies listed in table I are subject to systematic errors on the low side because the circular (rotation) velocities of stars or luminous (H II) gas clouds cannot be measured near the outer edge of a galaxy. In effect, this ignores the mass in an outer rim of a spiral, where the luminosity is too low for optical velocity measurements to be made. However, most authors correct for this by extrapolating the mass distribution, assuming that the density drops off smoothly to zero, thus adding an amount roughly equal to 0.3 M. Errors of measurement are estimated to be of the order 0.5 M (r.m.s.).

Masses of pairs of galaxies must be derived statistically on assumption (3) in the introduction, and most of the observational data were combined in an analysis by Page [51] which can be briefly summarized as follows. The observations consist of separate radial velocities for each galaxy in a pair or pair like group. The mean of these two velocities for one pair V is used as the distance indicator. From Hubble's law of redshifts,

$$V = h \times 10^{-4} D, \tag{3.1}$$

where V is in km/sec, D is the distance in parsecs, and $h \approx 1$ from all the recent studies of distances of galaxies. A detailed study of measurement errors in V showed a standard deviation of 90 km/sec, with weights of single observations ranging from 0.05 to 20. There is a further dispersion in equation (3.1) due to errors in D, and values of h ranging from 0.75 to over 1.5 have been used in the literature.

The difference between the two radial velocities in a pair ΔV is assumed to be the projection of a circular orbital velocity v and the observed angular separation S (in minutes of arc) is the projection of the line of centers (of length r) divided by the distance D and by the number of minutes in a radian. By Kepler's harmonic law of two body gravitational orbits, rv^2 is proportional to the sum of the masses; hence, for masses measured in solar units (1 sun = 2×10^{33} gm),

$$M_1 + M_2 = 675 \frac{SV(\Delta V)^2}{h} \cos^3 \varphi \cos^2 \psi \geqq 675 \frac{SV(\Delta V)^2}{h}, \tag{3.2}$$

where φ is the projection angle of r, having an unknown value between 0 and $\pi/2$, v is assumed to be perpendicular to r (circular orbit), and ψ is another angle involved in the projection of v, having some value between 0 and 2π. When $\varphi = \pi/2$ one galaxy is behind the other, and the pair would not be recognized as a double. At the other extreme, very wide pairs (large S) were not selected for observation. Holmberg [41] had found from an analysis of the projected separations of many pairs that the distribution of r is

$$p(r) = K\left[1 + \left(\frac{r}{r_m}\right)^3\right] \tag{3.3}$$

for $0.03\, r_m < r \leqq r_m$, where K is a normalizing constant, and r_m is determined from approximate distance estimates to be about $(2.3/h) \times 10^5$ psc, apparently the largest possible distance between two galaxies in stable orbit around each other. Equation (3.3) also applies to double stars with a much smaller value of r_m; it probably represents a statistical result of the condensation of stars (and galaxies) from selfgravitating gas clouds, and the later perturbations of a pair by encounters with single stars (or galaxies).

On the assumptions that φ, ψ, M, and r are independent of each other in a sample of many double galaxies, and that the errors in ΔV are normally distributed (so that the mean square of measured ΔV must be reduced by the variance σ^2/W)

$$(\Delta V)^2 - \frac{\sigma^2}{W} = 5.92 \times 10^{-8} h\overline{M} \left(0.19 + \frac{10^4}{SV}\right), \tag{3.4}$$

where σ is the standard deviation and W the weight of measurements of ΔV, $\overline{M}$ is the mean mass of a single galaxy in all the pairs, and the relative errors in S and V are negligible compared with $\sigma/\Delta V$. Equation (3.4) is a regression between observed $(\Delta V)^2$ and observed $(0.19 + 10^4/SV)$, and a least squares solution for $h\overline{M}$ was made from the observations of 33 pairs of galaxies, yielding a value $h\overline{M} = 2.6 \times 10^{11} \pm 1.4 \times 10^{11}$ solar masses. In another 19 cases, observations referred to groups of N galaxies approximating a pair. The simplest of these ($N = 3$) consisted of a close pair of galaxies with a more distant satellite galaxy; the most complex ($N = 5$) consisted of a close group of four with a satellite. These were included with the factor $N/2$ on the right of equation (3.4) yielding $h\overline{M} = 3.1 \times 10^{11} \pm 1.1 \times 10^{11}$.

Least squares solutions of equation (3.4) were also made for subsets of the data, as shown in table II, from which it is clear that the mean mass of an elliptical $\overline{M}_E = 30\overline{M}_S$, where $\overline{M}_S$ is the mean mass of spirals in these pairs and groups. The mixed systems confirm this fact, which is of importance in the theory of evolution of galaxies. It is also indicated in the individual mass determinations of table I, although these vary widely in the case of elliptical (E) galaxies.

The total luminosity of a large group of stars was at first expected to be proportional to the total mass, even though any one star may be 10000 times more luminous or 1000 times less luminous than the Sun. However, all theories of stellar evolution show that massive stars of very high luminosity are short lived, so that an old population of stars should have lower luminosity for a given total mass. The ratio M/L in solar units is as small as 10^{-3} for young giant stars and as large as 1000 for long lived dwarf stars. The luminosity of a galaxy is defined in these solar units as

$$\begin{aligned} L &= D^2\, 10^{0.104-0.4m} \\ &= \left(\frac{V}{h}\right)^2 10^{8.104-0.4m}, \end{aligned} \tag{3.5}$$

where m is the measured apparent photographic magnitude of the galaxy. Introducing the sum of N luminosities into equation (3.4), we get another regression involving the same left side, the desired mean M/hL, and the observables V/S, V^2, and the sum $\sum 10^{8.104-0.4m}$ on the right. Least squares solutions for $\overline{M/hL}$ yield the values given in table II and show that the mean M/L for massive E galaxies is 30 to 60 times the value for spirals (S), somewhat more than would be expected if the E galaxies consist simply of older stars. This may indicate an admixture of nonluminous matter in E galaxies, although optical evidence of obscuring dust clouds and radio evidence of nonluminous hydrogen

TABLE II

AVERAGE MASS AND M/L, DOUBLE GALAXIES

Notes. Each system includes N_i galaxies in two groups treated as mass points. For pure pairs, $N_i = 2$, and no other galaxy is nearby. High weight observations include only those systems for which observed relative velocities have weight greater than 0.5. h is the Hubble constant in units of 10^{-4} km/sec psc ($h \cong 1$). $\overline{M}$ is the mean mass of one galaxy, in suns. L is the total photographic luminosity of a galaxy, in suns. $\overline{M}_E$ is the means mass of E and S0 galaxies. $\overline{M}_S$ is the mean mass of S, SB, Irr galaxies. Each value of $h\overline{M}$ and $\overline{M/hL}$ results from a least squares solution from which r.m.s. errors of the mean were also determined. The values of $\overline{M/hL}$ for S and Irr galaxies were incorrectly listed in the first publication [51].

No. of Systems n	No. of Galaxies, ΣN_i (by type) Irr	S	SO	E	Mean Mass $h\overline{M}_{\cancel{E}}/10^{10}$ (suns)	Mean M/L $\overline{M/hL}$ (solar units)	Notes
52	2	52	17	43	31.2 ± 10.6	38.0 ± 19.9	all systems
33	1	29	10	26	26.0 ± 13.9	31.2 ± 26.0	pure pairs only
41	1	44	13	33	28.7 ± 9.0	43.8 ± 15.2	high weight obs. only
16	2	32	0	0	4.0 ± 4.2	3.2 ± 4.2	S and Irr only
10	1	19	0	0	1.6 ± 2.3	1.4 ± 1.8	pure pairs only
13	1	27	0	0	1.5 ± 1.7	1.3 ± 1.5	high weight obs. only
18	0	0	11	26	66.2 ± 29.	98.0 ± 68.0	E and S0 only
13	0	0	8	18	63.6 ± 38.	92.0 ± 92.0	pure pairs only
13	0	0	8	19	59.4 ± 15.	90.0 ± 37.0	high weight obs. only
18	0	20	6	17	31.4 ± 17.	46.0 ± 23.0	mixed systems
10	0	10	2	8	27.7 ± 23.	41.0 ± 34.0	pure pairs only
15	0	17	5	14	31.4 ± 18.	46.0 ± 26.0	high weight obs. only
					$h\overline{M}_E/10^{10}$		assuming $\overline{M}_E = 30\overline{M}_S$
15	0	17	5	14	60.7 ± 36.		mixed only
13	0	0	8	19	59.4 ± 15.		E and S0 only
28	0	17	13	33	60.0 ± 19.		E, S0, and mixed
13	1	27	0	0	43.4 ± 53.		S and Irr only
41	1	44	13	33	59.6 ± 16.		all high weight obs.

are limited to spirals. It is possible that other forms of matter are involved, such as collapsed masses or very low temperature stars.

The validity of these results has been discussed [51] and it is shown that the assumption of circular orbits and the tidal effects neglected in equation (3.2) are not likely to have affected the results significantly. If M is positively correlated with r, so that more massive pairs are systematically of wider separation than less massive ones (a possible result of the mechanics of galaxy formation or of later perturbations by intruders), then the values of $h\overline{M}$ in table II are *underestimated.* If the observed pairs are all embedded in an intergalactic medium of uniform density ρ, the mass involved in equations (3.2) and (3.4) would be $2M + 4\pi\rho r^3/3$ and this dependence on r or SV again results in an underestimate.

Motions in clusters of galaxies and cosmological models fitted to the Hubble Law of redshifts imply values of ρ as high as 10^{-28} gm/cm^3. The resulting increase in $\overline{M}$ is approximately $5 \times 10^{35}\ \rho/h^3$ or about 10^7 solar masses, which is insignificant (only one part in 10^3 or 10^4).

Although the selection in S has been accounted for, other effects of selection might influence the means in table II. Selection of the higher luminosity pairs is to be expected, although small diameter galaxies and ones of low surface brightness are apt to be overlooked on photographs; high surface brightness is selected for velocity measurements. Because the more luminous galaxies in a class are expected to be the more massive ones, the estimated average masses, $\overline{M}_E$ and $\overline{M}_S$ are undoubtedly biased toward higher values. However, the large ratio $\overline{M}_E/\overline{M}_S$ cannot be explained as a result of this selection, and for three reasons: (1) the E galaxies included in the set of pairs ([51], pp. 293–294) are somewhat fainter than the S galaxies included; (2) in the mixed pairs, E galaxies are as often brighter than S galaxies as they are fainter; and (3) the results for mixed pairs confirm $\overline{M}_E/\overline{M}_S = 30$. Note, also, that for the pairs selected, the mean luminosity $\overline{L}_E \cong 0.67\ \overline{L}_S$ if the spread is not extreme.

It has been suggested that galaxies in pairs differ systematically from single galaxies, but this is not supported by the mass estimates for single and double spirals in table I. Moreover, the morphological types E0 to E7, S0, Sa, Sb, Sc, SBa, SBb, SBc, and Irr I all appear normal in pairs, although the rare dwarf elliptical and dwarf irregular types are not represented in this sample. These dwarfs probably are much less massive.

The set of observations may include "optical pairs"—chance lineups of two galaxies, one far beyond the other. The number of such chance pairs, as distinguished from dynamical pairs with $r < r_m$, clearly depends on the number of galaxy images per square degree and on the maximum separation S_m used to define a pair. Pólya [56] derived the probability that, if n points are distributed at random on a sphere, none of them will fall within angle S from an $(n + 1)$th point

$$p(S, n) = (\cos S/2)^{2n} \cong \frac{e^{-nS^2}}{4.78 \times 10^7}, \tag{3.6}$$

and this was used by Holmberg [40] to estimate N_2, the number of chance pairs in a square degree of the sky where N_1 single galaxies are randomly distributed

$$N_2 \cong \frac{\pi N_1^2 S^2}{7200}. \tag{3.7}$$

For separations S less than six minutes of arc and $N_1 \cong 1.3$ galaxies per square degree brighter than $m \cong 15$, equation (3.7) yields $N_2 = 0.027$ per square degree, or less than six per cent of the pairs counted by Holmberg in photographs covering 15000 square degrees. Thus, it is argued that few or none of the 33 pairs studied [51] are chance lineups.

The best available data on pairs can be found in a catalog of galaxy redshift measurements by Humason, Mayall, and Sandage [44], where the effects of selection are expected to be extreme because of the difficulty of photographing spectra (in addition to the selection of the galaxies from photographs). Of 920 galaxies listed by HMS, 188 satisfy the pair requirement, $S \leqq 3(a_1 + a_2)$ in 94 separate pairs of which 26 were selected by the observers *because* they were pairs. Galaxies in the other 68 pairs were observed singly. Twenty six are single, isolated pairs, nineteen of them included in the mean mass determinations by Page [52].

The linear dimensions A of all these and many other galaxies in the HMS [44] catalog can be calculated from the angular diameters in minutes of arc a given by de Vaucouleurs [38] and the redshift velocities V using Hubble's Law, equation (3.1), with $h = 1$

$$A = 0.00292\, a\, V \text{ kpc}. \tag{3.8}$$

The mean values of A in table III and the distribution of magnitudes shown in figure 2, show that galaxies in pairs differ only slightly from single ones of the same type, and that the spiral types have a larger spread in dimensions than ellipticals. (Measurement errors in V are relatively small; $\sigma_V \cong 100$. Some of

TABLE III

MEAN DIMENSIONS OF GALAXIES BY TYPES
Diameter A in kpc ± mean deviation.

	128 Single Field Galaxies		98 Galaxies in Groups		51 Galaxies in Isolated Tight Pairs					
					All Pairs		Similar Types		Mixed Types	
Type	n	A	n	A	n	A	n	A	n	A
E0 to E7	29	10.8 ± 4.	41	10.1 ± 4.	21	8.5 ± 4.				
S0, SB0	20	11.3 ± 4.	27	15.6 ± 4.	9	7.0 ± 3.				
E, S0, SB0	49	11.0 ± 4.	68	12.3 ± 4.	30	8.1 ± 4.	20	8.8 ± 4.	9	6.7 ± 3.
Sa	12	17.6 ± 7.	8	12.1 ± 3.	2	12.6 ± 5.				
SBa	8	14.5 ± 7.	1	17.8	0					
Sa, SBa	20	16.4 ± 7.	9	12.7 ± 3.	2	12.6 ± 5.				
Sb	13	18.9 ± 5.	4	25.7 ± 12.	5	15.9 ± 7.				
SBb	6	24.6 ± 7.	7	27.0 ± 4.	1	7.1				
Sb, SBb	19	20.7 ± 6.	11	26.6 ± 7.	6	14.5 ± 7.				
Sc	18	16.6 ± 4.	4	22.0 ± 7.	10	16.0 ± 6.				
SBc	18	16.4 ± 5.	0		2	13.0 ± 4.				
Sc, SBc	36	16.5 ± 5.	4	22.0 ± 7.	12	14.4 ± 6.				
Sa, Sb, Sc	43	17.6 ± 5.	16	18.0 ± 7.	17	14.8 ± 6.				
SBa, SBb, SBc	32	17.5 ± 6.	8	25.9 ± 4.	3	11.1 ± 4.				
S, SB	75	17.6 ± 6.	24	20.6 ± 6.	20	14.3 ± 6.	9	18.4 ± 7.	9	13.2 ± 4.
Ir	4	5.2	6	12.1 ± 7.	1	12.3				
All types	128	14.6	98	14.4	51	10.8				

the deviations in A may be due to errors in measuring a.) Although the sample by no means represents all the data [38], and although the dispersions are large, the mean absolute dimensions in table III imply that elliptical and lenticular galaxies (E, S0, and SB0) are less than two thirds of the size of spirals (Sa, Sb, Sc, SBa, SBb, and SBc). Moreover, galaxies of different types in a tight pair ($S < 3a_1 + 3a_2$) are smaller yet. The average masses of elliptical galaxies in a similar sample of tight pairs (table II) is 30 times the average for spirals; hence the density of matter in the former must be over 100 times larger than the density in spirals.

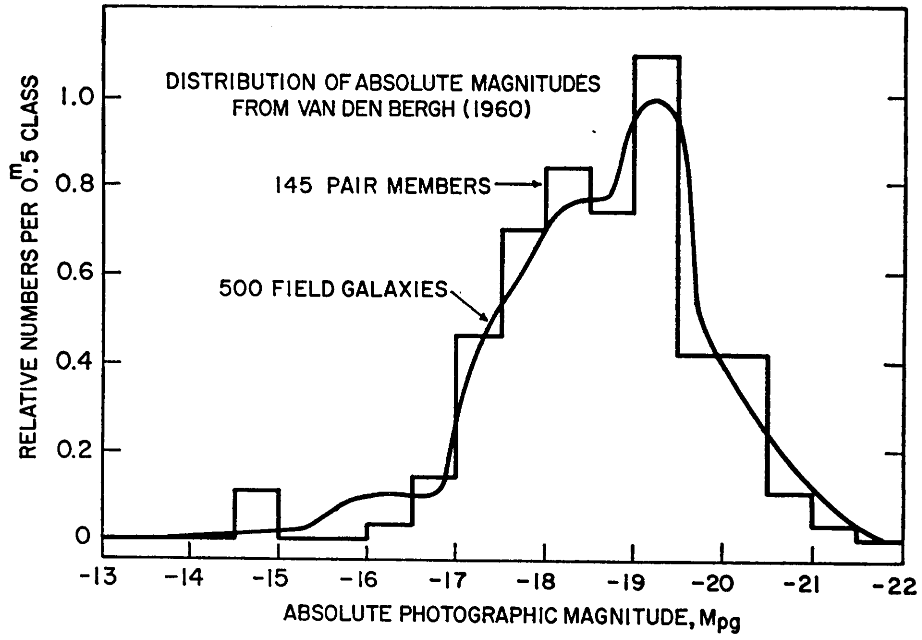

FIGURE 2

Distribution of absolute magnitude from van den Bergh [66].

4. Masses and stability of clusters of galaxies

The average masses of galaxies were first estimated by Zwicky [72] and Smith [64] from velocity dispersions in clusters of galaxies. Deviations from the mean of all measured radial velocities of galaxies in a cluster are interpreted as projections of randomly oriented individual velocities with respect to the center of mass. On the assumption that the measured velocities are a fair sample of all the velocities of member galaxies, and that the cluster is stable, the virial theorem can be applied [72], or the largest relative velocities can be equated to the velocity of escape [67]. If the distance is known and if symmetry can be assumed so that a distribution of galaxies around the center of mass can be inferred, the mass of the cluster can be determined by either method. This total mass, divided by the number of galaxy images counted on photographs of the cluster, gives an average galaxy mass which is generally 10 to 50 times larger than masses of individual galaxies determined from rotations or orbital motions in pairs, as shown in tables I and IV. Table IV is taken primarily from the papers discussed in the 1961 Conference, with values of $\overline{M}$ and $\overline{M/L}$ converted to a Hubble constant $H = 100$ km/sec Mpc where necessary.

TABLE IV

MASSES OF GROUPS AND CLUSTERS OF GALAXIES

From de Vaucouleurs [28], Burbidge [4], [6], and Page, Dahn and Morrison [55].
$H = 100$ km/sec Mpc.

Group or Cluster	Angular Diameter	Total m_{pg}	$\overline{V}/100$	R (Mpc)	N_b	$N_b\overline{M}/10^{10}$	$\overline{M}/10^{10}$	$\overline{M/L}$
VV115 (Seyfert)	1′.9		44.	0.01	5	24.	5.	
VV116		12.7	64.		2E, 3S	100.	20.	
VV150	1′.2		73.	0.02	S			
VV166 (NGC 67-72)			67.9		3E, 3S			350.
VV288 (Stephan)	3′.7	11.8	67.	0.04	5(E, S)	500.	100.	100.
NGC 55	500′	8.7	5.5	0.4	6S	600.	100.	500.
NGC 383 (Pisces)					25E	125000.	500.	260.
NGC 3031-77 (M81)		6.	2.		>4S		120.	200.
NGC 6027 (Serpens)	1′.6	14.	45.	0.01	3E, 3S			
NGC 7619 (Pegasus)	120′	11.	40.	0.3	5E	2500.	500.	300.
Local Group					2I, 2S, 2E		400.	
Sculptor	950′		3.	0.37	6	1700.	280.	
NGC 3561			87.					
NGC 6166	2′.5	13.0	9.1	0.03	5E	1400.	280.	175.
Abell 2199	12′		90.	0.15	>19			
Mean group				0.15	8	1000	150	280
Can Ven Cluster	19°.	6.6	6.8	1.1	30S	4500.	150.	400.
Fornax	5.7		15.	0.75	30	4700.	157.	
Pegasus	2.0		39.	0.67	50	4200.	84.	
U Ma	10.		20.	1.8	50	2800.	56.	
Hercules	1.4		108.	1.3	50S, 30E	5600.	70.	
Virgo E	11.5	6.3	11.	1.1	100E	24000.	240.	600.
Virgo S	11.0		19.	1.8	100S	<45000.	<450.	
Coma	9.0	9.4	67.	5.2	500E	75000.	150.	900.
NGC 541					500S	5000.	10.	
Mean cluster				1.7	100	10000	130	600

Some of the groups and clusters are identified in the first column by numbers in the catalog of Vorontsov-Velyaminov [70], some by the NGC number of bright galaxies in them and some by the constellation where they appear. Angular diameters are given in minutes of arc for the smaller groups and in degrees for larger clusters. The total photographic magnitude of the whole group or cluster of galaxies and mean radial velocity in km/sec are given as before,

$\bar{V}/100$ being equal to the distance in Mpc. The radius, R of the cluster is in Mpc. The number of bright galaxies N_b (no fainter than one fifth to one tenth of the brightest) in the cluster is used to obtain the mean mass of a galaxy $\bar{M}$ from the total mass estimate $N_b\bar{M}$. (There is a large uncertainty in N_b due to foreground and background galaxies.) The total mass (listed under $N_b\bar{M}$) is obtained from the virial theorem applied to the deviations $V_i - \bar{V}$, assuming that each group or cluster is stable. The ratio $\overline{M/L} = N_b\bar{M}/\sum L_b$ and is less affected by the uncertainty in N_b but may still be wrong by a factor of two [68]. The upward trend of M/L with N_b shown in figure 1 is as yet unexplained.

Three reasons have been proposed for these excessive cluster masses: (1) the galaxies in large, compact clusters differ systematically from others (in fact, it has been claimed that such cluster members are predominantly or entirely E galaxies); (2) there are other forms of mass in clusters, generally called intergalactic matter; and (3) the clusters are *not* stable, so that the cluster mass estimate is unfounded. The conference organized by Neyman, Page, and Scott [49] met primarily to consider this third possibility and the hypothesis proposed by Ambartsumian [1], [2]. In effect, Ambartsumian assumed sudden release of vast amounts of energy to account for the large dispersion in observed radial velocities of galaxies in some groups and clusters. Discussion revealed two further difficulties in any statistical analysis of motions in a cluster of galaxies: the unwitting inclusion of foreground or background galaxies as cluster members (uncertainty in N_b), and peculiar patterns of motion (contraction and subclustering) that invalidate the conventional application of the virial theorem.

It appeared from this discussion that there are at least four categories of systems with different degrees of stability:

(a) close pairs of galaxies are probably stable;

(b) small groups like Stephan's Quintet are most likely to be unstable, often explosive;

(c) loose irregular clusters such as the Virgo Cluster are suspected to be unstable, but not violently so;

(d) compact regular clusters such as the Coma Cluster are probably stable.

Six stages of instability stability were recognized:

(a) explosive expansion, as assumed by Ambartsumian;

(b) mild expansion;

(c) contraction;

(d) dynamical stability to which the virial theorem applies;

(e) stability of form involving a continuous exchange of galaxies between a cluster and the field, to which the virial theorem does *not* apply;

(f) subclustering, or clusters of clusters, for which the virial theorem must be modified.

The most serious observational difficulty was recognized to be the identification of the members of a cluster or group, excluding foreground and background galaxies, yet including faint members. One of the major theoretical difficulties is that the calculated time for unstable groups and clusters to disperse is generally 10^8 years or so—much less than the estimated ages of individual member galaxies, and inconsistent with the idea that member galaxies were all formed in the cluster where they now appear. So short a cluster life raises the question of cluster formation and is probably inconsistent with the observed velocity dispersion among field galaxies.

The conference report ends with four more questions:

"What is the evidence that members of a cluster had a common origin?

"Are nongravitational forces involved in the dynamics of small groups of galaxies?

"In what way are the extragalactic radio sources associated with individual galaxies or with clusters?

"What is the mechanism by which the galaxies were formed, and how does it account for clustering?"

In the four years since this was written, direct evidence (both radio and optical) has been obtained of explosive energy release in galaxies. At the same time astronomers have developed greater acceptance of an intergalactic medium and a greater interest in the mechanism of the formation and evolution of galaxies. Lynds and Sandage [45] discovered clouds of ionized gas apparently "splashed" out of the center of the nearby spiral M82 about 1.5×10^6 years ago, and Schmidt [62] discovered the superluminous quasistellar objects (QSO's or "quasars"). Their strong radio emission led to this discovery, and other means of identifying them are now under study. Theoretical studies by several authors have been discussed at special symposia [58], [53], generally starting from a protogalaxy gas cloud assumed to have a density much higher than the present mean density of galaxy matter (product of the number of galaxies per unit volume and the average mass of a galaxy, about 3×10^{-30} gm/cm^3). In fact, Sciama [63] assumes an intergalactic density of 10^{-28} gm/cm^3 in the form of ionized hydrogen at 100,000°K which would be unobservable in both optical and radio frequencies, and would have thermal instabilities leading to condensing masses of about 10^{11} suns.

The enormous energy output of the QSO's may be due to gravitational collapse [58] in the few cases where initial conditions were right (zero angular momentum), and other conditions may have led to condensation of pairs, groups, or clusters of galaxies.

5. The evolution of galaxies

It is now virtually certain that galaxies slowly change in appearance over periods of billions of years, due primarily to the formation of stars from interstellar gas and the aging of the stars (a process first studied statistically 30 years ago, and now the subject of detailed calculations based on nuclear reactions in individual stars). The generally accepted concept is that stars condensed from primordial gas clouds or regions of higher density in a universal gaseous medium. As they age, the stars become redder and less luminous, although their masses remain nearly constant. Since E and S0 galaxies have low luminosity for their large masses, it was at first natural to assume that evolution carried a blue spiral galaxy into the redder E type. However, it is difficult to account in this way for the larger mass of the E galaxies, and for tight pairs consisting of one E and one spiral galaxy.

The evolutionary development of stars in the Milky Way has been worked out by Schmidt [61] and others on the assumption that the rate of star formation depends on the density of the gas from which they form. Holmberg [42] then collected mass estimates like those in tables I and II, and size estimates like those in table III, and showed that the resulting average densities of galaxies are correlated with color and morphological type in the sense that high density implies red, E type galaxies. He argues that the small scatter on a plot of density versus color of galaxies proves that (1) galaxies are all of about the same age and (2) the initial density of each primordial gas cloud determines the morphological type of the galaxy evolved. Dense gas clouds formed stars quickly; these stars aged, reddened, and now have the low luminosity (high M/L) of an E galaxy. In gas clouds of lower density, stars formed later and have not yet aged; hence, we see them as blue, highly luminous spirals of low M/L.

These ideas were discussed at the Congress of the International Astronomical Union [53] and it was noted that the initial sizes of the primordial gas clouds, their angular momenta, and possibly their turbulence and chemical content may also affect the morphological types of the galaxies that evolved. In addition to the average densities, colors, and morphological types of galaxies that have been studied so far, it is possible to derive for a large sample of galaxies:

(a) density distribution (from accurate Doppler shifts in many spectra of each galaxy—as reported by the Burbidges in papers cited 1960–65 primarily for spirals);

(b) approximate central density, from inclinations of lines in individual spectra, now measured for over 100 galaxies by Mayall [46], Lindblad and Page (as yet unpublished);

(c) total angular momentum (from the above measures);

(d) color and luminosity distribution, including central region colors (as measured by Holmberg [42] and others);

(e) mean M/L and the differences between M/L near the center and in outer regions (from the above measures);

(f) gas content (from the hydrogen 21 cm radio emission flux);

(g) distribution of interstellar gas and stars of various types (from the intensities of lines in spectra).

Preliminary results indicate the expected correlation between central densities from (a) and (b) and central colors from (d), and between angular momenta (c), gas content (f), and morphological types. The most serious discrepancy remains in the large values of mean M/L for galaxies (particularly E and S0 types), which are not consistent with means of M/L for individual stars with a distribution of masses similar to stars near the sun. It seems likely [53] that this may be explained either by large numbers of very small, faint stars in E galaxies, or by large collapsed masses with low or zero luminosity. The formation of small stars and the lower cutoff in frequency distribution of stellar masses probably depend on the turbulence in the primordial gas cloud from which a galaxy condenses. The formation of large nonluminous masses by collapse is possibly a later stage in the evolution of some contracting galaxies with low angular momentum that for a brief period are highly luminous quasistellar objects [58].

Zwicky [73] finds evidence of a sequence of "compact galaxies" that may be earlier stages in the collapse; he estimates that there are two of these, on the average, in every square degree of the sky as photographed by the large telescopes on Mount Wilson and Palomar. As reliable methods are developed for identifying these small images on photographs [49] and after their distances have been reliably estimated, it will be possible to calculate the relative numbers in a volume of space and provide a statistical basis for theories of evolution of galaxies.

The existence of pairs, groups and clusters of galaxies is undoubtedly related to the early stages of evolution, and the trend toward larger M/L in larger groups and clusters shown in figure 1 may provide an important clue to the mechanism involved. Since large M/L is expected for small mass stars, and since the size of stars formed in a large gas cloud depends on the scale of early density fluctuations, it may be that the patterns of turbulence in primordial gas clouds nearly 10^{10} years ago can account for the types and clustering of galaxies observed today.

REFERENCES

[1] V. A. AMBARTSUMIAN, "Über Mehrfachgalaxien," *Izv. Akad. Nauk Armjan. Ser. Fiz.-Mat. SSR*, Vol. 9 (1956), pp. 23–43.

[2] ———, "Instability phenomena in systems of galaxies," *Astronom. J.*, Vol. 66 (1961), pp. 536–540.

[3] E. M. BURBIDGE, "The strange extragalactic systems: Mayall's object and IC 883," *Astrophys. J.*, Vol. 140 (1964), pp. 1617–1620.

[4] E. M. BURBIDGE and G. R. BURBIDGE, "Rotation and internal motions in NGC 5128," *Astrophys. J.*, Vol. 129 (1959), pp. 271–281.

[5] ———, "Motions in barred spiral galaxies. I. The nuclei of NGC 1097 and NGC 1365," *Astrophys. J.*, Vol. 132 (1960), pp. 30–36.

[6] ———, "A further investigation of Stephan's Quintet," *Astrophys. J.*, Vol. 134 (1961), pp. 244–247.

[7] ———, "The mass of a Vorontsov-Velyaminov object," *Astrophys. J.*, Vol. 138 (1963), pp. 1306–1307.

[8] ———, "V-V 144, an exploding galaxy?," *Astrophys. J.*, Vol. 140 (1964), pp. 1307–1309.

[9] ———, "The velocity field in M51," *Astrophys. J.*, Vol. 140 (1964), pp. 1445–1461.

[10] E. M. BURBIDGE, G. R. BURBIDGE, D. J. CRAMPIN, V. C. RUBIN, and K. H. PRENDERGAST, "The rotation and mass of NGC 6503," *Astrophys. J.*, Vol. 139 (1964), pp. 539–544.

[11] ———, "The rotation and mass of NGC 3521," *Astrophys. J.*, Vol. 139 (1964), pp. 1058–1065.

[12] E. M. Burbidge, G. R. Burbidge, and R. A. Fish, "The masses of elliptical galaxies. A redetermination of the mass of M32," *Astrophys. J.*, Vol. 133 (1961), pp. 393–404.

[13] ———, "The masses of elliptical galaxies. II. The mass of NGC 3379," *Astrophys. J.*, Vol. 134 (1961), pp. 251–256.

[14] E. M. Burbidge, G. R. Burbidge, and K. H. Prendergast, "Mass distribution and physical conditions in the inner region of NGC 1068," *Astrophys. J.*, Vol. 130 (1959), pp. 26–37.

[15] ———, "The rotation and mass of NGC 2146," *Astrophys. J.*, Vol. 130 (1959), pp. 739–748.

[16] ———, "The rotation, mass distribution, and mass of NGC 5055," *Astrophys. J.*, Vol. 131 (1960), pp. 282–292.

[17] ———, "The rotation and approximate mass of NGC 3556," *Astrophys. J.*, Vol. 131 (1960), pp. 549–552.

[18] E. M. Burbidge, G. R. Burbidge, and K. H. Prendergast, "The rotation, mass distribution, and mass of NGC 2903," *Astrophys. J.*, Vol. 132 (1960), pp. 640–653.

[19] ———, "Motions in barred spiral galaxies. II. The rotation of NGC 7479," *Astrophys. J.*, Vol. 132 (1960), pp. 654–660.

[20] ———, "Motions in barred spiral galaxies. III. The rotation and approximate mass of NGC 3504," *Astrophys. J.*, Vol. 132 (1960), pp. 661–666.

[21] ———, "The rotation and mass of NGC 5005," *Astrophys. J.*, Vol. 133 (1961), pp. 814–820.

[22] ———, "The rotation and approximate mass of NGC 3623," *Astrophys. J.*, Vol. 134 (1961), pp. 232–236.

[23] ———, "Motions in NGC 3646, a strange spiral galaxy," *Astrophys. J.*, Vol. 134 (1961), pp. 237–243.

[24] ———, "The rotation and mass of NGC 157," *Astrophys. J.*, Vol. 134 (1961), pp. 874–879.

[25] ———, "The rotation and approximate mass of NGC 5248," *Astrophys. J.*, Vol. 136 (1962), pp. 128–132.

[26] ———, "The rotation and velocity field of NGC 253," *Astrophys. J.*, Vol. 136 (1962), pp. 339–391.

[27] ———, "Motions in barred spiral galaxies. V. The velocity field in NGC 5383," *Astrophys. J.*, Vol. 136 (1962), pp. 704–712.

[28] ———, "The rotation and mass of NGC 1084," *Astrophys. J.*, Vol. 137 (1963), pp. 376–380.

[29] ———, "The rotation and physical conditions in the Seyfert galaxy NGC 7469," *Astrophys. J.*, Vol. 137 (1963), pp. 1022–1032.

[30] ———, "The velocity field, rotation, and mass of NGC 4258," *Astrophys. J.*, Vol. 138 (1963), pp. 375–384.

[31] E. M. Burbidge, G. R. Burbidge, and V. C. Rubin, "A study of the velocity field in M82 and its bearing on explosive phenomena in that galaxy," *Astrophys. J.*, Vol. 140 (1964), pp. 942–968.

[32] E. M. Burbidge, G. R. Burbidge, V. C. Rubin, and K. H. Prendergast, "Motions in barred spirals. VI. The rotation and velocity field of NGC 613," *Astrophys. J.*, Vol. 140 (1964), pp. 85–93.

[33] N. H. Dieter, "Neutral hydrogen in M33," *Astronom. J.*, Vol. 67 (1962), pp. 217–221.

[34] R. Duflot-Augard, "Vitesse de rotation et masse de la partie antrale de la galaxie NGC 2782," *Publ. Obs. Haute Provence,* Vol. 5 (1960), pp. 1–3.

[35] G. de Vaucouleurs, "Rotation and mass of the large Magellanic cloud," *Astrophys. J.*, Vol. 131 (1960), pp. 265–281.

[36] ———, "Southern galaxies. I. Luminosity, rotation, and mass of the Magellanic system NGC 55," *Astrophys. J.*, Vol. 133 (1961), pp. 405–412.

[37] G. de Vaucouleurs, and A. de Vaucouleurs, "Rotation and mass of the Magellanic-type galaxy NGC 4631," *Astrophys. J.*, Vol. 137 (1963), pp. 363–375.

[38] ———, *Reference Catalog of Bright Galaxies*, Austin, University of Texas Press, 1964.

[39] R. A. Fish, "A mass-potential-energy relationship in elliptical galaxies and some inferences concerning the formation and evolution of galaxies," *Astrophys. J.*, Vol. 139 (1964), pp. 284–305.

[40] E. Holmberg, "A study of double and multiple galaxies," *Lund Annalen*, No. 6 (1937), 158 pp.

[41] ———, "On the masses of double galaxies," *Medd. Lunds Astrom. Obs. Ser. I*, No. 186 (1954), pp. 1–20.

[42] ———, "Colors, luminosities, and mass densities of galaxies," *Ark. Astronom.*, Vol. 3 (1964), pp. 387–460.

[43] H. C. van de Hulst, E. Raimond, and H. van Woerden, "Rotation and density distributions of the Andromeda nebula derived from observations of the 21-cm line," *Bull. Astronom. Inst. Neth.*, Vol. 15 (1957), pp. 1–16.

[44] M. Humason, N. Mayall, and A. Sandage, "Redshifts and magnitudes of extragalactic nebulae," *Astronom. J.*, Vol. 61 (1956), pp. 97–162.

[45] C. Lynds and A. Sandage, "Evidence for an explosion in the center of the galaxy M82," *Astrophys. J.*, Vol. 137 (1963), pp. 1005–1021.

[46] N. U. Mayall, "Advantages of electronic photography for extragalactic spectroscopy," *Ann. Astrophys.*, Vol. 23 (1960), pp. 344–359.

[47] R. Minkowski, "NGC 6166 and the cluster Abell 2199," *Astronom. J.*, Vol. 66 (1961), pp. 558–561.

[48] G. Münch, "Analysis of composite radiation. III. The mass-luminosity ratio in stellar systems," *Publ. Astronom. Soc. Pacific*, Vol. 71 (1959), pp. 101–105.

[49] J. Neyman, T. Page, and E. Scott, "Conference on the instability of systems of galaxies," *Astronom. J.*, Vol. 66 (1961), pp. 533–636.

[50] J. Neyman and E. Scott, "Problems of selection bias in the statistics of galaxies," *Bull. Inst. Internat. Statist.*, Vol. 40 (1963), pp. 1026–1050.

[51] T. Page, "Average masses of the double galaxies," *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability*, Berkeley and Los Angeles, University of California Press, 1961, Vol. 3, pp. 277–306.

[52] ———, "M/L for double galaxies, a correction," *Astrophys. J.*, Vol. 136 (1962), pp. 685–686.

[53] ———, "The evolution of galaxies," *Science*, Vol. 146 (1964), pp. 804–809.

[54] ———, "Radiation spikes in H II regions," *Smithsonian Astrophysical Observatory Special Report*, No. 196, 1965, 15 pp.

[55] T. Page, C. C. Dahn, and F. F. Morrison, "Statistics on double galaxies and their formations," *Astrophys. J.*, Vol. 66 (1961), pp. 614–619.

[56] G. Pólya, "Zur Statistik der Spherischen Verteilung der Fixsterne," *Astronom. Nachr.*, Vol. 208 (1919), pp. 175–180.

[57] A. Poveda, "A mass-luminosity relation for dust-poor stellar systems," *Astrophys. J.*, Vol. 134 (1961), pp. 910–915.

[58] I. Robinson (ed.), *Proceedings of the First Texas Symposium on Relativistic Astrophysics*, Chicago, University of Chicago Press, 1965.

[59] A. Sandage, "The existence of a major new constituent of the universe: the quasi-stellar galaxies," *Astrophys. J.*, Vol. 141 (1965), pp. 1560–1578.

[60] M. Schmidt, "A model of the distribution of mass in the galactic system," *Bull. Astronom. Inst. Neth.*, Vol. 13 (1956), pp. 15–41.

[61] M. Schmidt, "The distribution of mass in M31," *Bull. Astronom. Inst. Neth.*, Vol. 14 (1957), pp. 17–19.

[62] ———, "A star-like object with large red-shift," *Nature*, Vol. 197 (1963), p. 1040.

[63] D. W. Sciama, "On the formation of galaxies and their magnetic fields in a steady state universe," *Quart. J. Roy. Astronom. Soc.*, Vol. 5 (1964), pp. 196–211.

[64] S. Smith, "The mass of the Virgo cluster," *Astrophys. J.*, Vol. 83 (1936), pp. 23–30.

[65] S. van den Bergh, "The evolution of galaxies," *Publ. Astronom. Soc. Pacific*, Vol. 71 (1959), pp. 5–11.

[66] ———, "A reclassification of the northern Shapley-Ames galaxies," *Publ. David Dunlap Obs.*, Vol. 2 (1960), pp. 159–199.

[67] ———, "Stability of clusters of galaxies," *Astronom. J.*, Vol. 66 (1961), pp. 566–571.

[68] L. Volders, "Neutral hydrogen in M33 and M101," *Bull. Astronom. Inst. Neth.*, Vol. 14 (1959), pp. 323–335.

[69] L. Volders and J. A. Högbom, "Observations of neutral hydrogen in IC 1613, NGC 6822 and M82," *Bull. Astronom. Inst. Neth.*, Vol. 15 (1961), pp. 307–314.

[70] B. A. Vorontsov-Velyaminov, *Atlas of Peculiar Galaxies*, Moscow, Moscow University, 1960.

[71] A. Wyse, and N. Mayall, "Distribution of mass in the spiral nebulae Messier 31 and Messier 33," *Astrophys. J.*, Vol. 95 (1942), pp. 24–47.

[72] F. Zwicky, "Redshifts of extragalactic nebulae," *Helv. Phys. Acta.*, Vol. 6 (1933), pp. 110–127.

[73] ———, "Compact galaxies and compact parts of galaxies. I," *Astrophys. J.*, Vol. 140 (1964), pp. 1467–1471.