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VOLUME II

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PART I

CONTRIBUTIONS TO PROBABILITY THEORY

EDITED BY LUCIEN M. LE CAM
AND JERZY NEYMAN

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CONTENTS OF PROCEEDINGS, VOLUMES I, II, III, IV, AND V

Volume I—Theory of Statistics

General Theory

T. W. ANDERSON and S. M. SAMUELS, Some inequalities among binomial and Poisson probabilities. R. R. BAHADUR, An optimal property of the likelihood ratio statistic. GEORGE A. BARNARD, The use of the likelihood function in statistical practice. D. BASU, Problems relating to the existence of maximal and minimal elements in some families of statistics (subfields). YU. K. BELYAEV, On confidence intervals and sets for various statistical models. FRIEDHELM EICKER, Limit theorems for regressions with unequal and dependent errors. R. H. FARRELL, Weak limits of sequences of Bayes procedures in estimation theory. R. H. FARRELL, J. KIEFER, and A. WALBRAN, Optimum multivariate designs. JAROSLAV HÁJEK, On basic concepts of statistics. J. L. HODGES, JR., Efficiency in normal samples and tolerance of extreme values for some estimates of location. J. L. HODGES, JR. and E. L. LEHMANN, Moments of chi and power of t. WASSILY HOEFFDING, On probabilities of large deviations. PETER J. HUBER, The behavior of maximum likelihood estimates under nonstandard conditions. OSCAR KEMPTHORNE, The classical problem of inference—goodness of fit. H. KUDŌ, On partial prior information and the property of parametric sufficiency. YU. K. LINNIK, On the elimination of nuisance parameters in statistical problems. JAMES MAC QUEEN, Some methods for classification and analysis of multivariate observations. KAMEO MATUSITA, Classification based on distance in multivariate Gaussian cases. EMANUEL PARZEN, On empirical multiple time series analysis. YU. V. PROHOROV, Some characterization problems in statistics. ROY RADNER, A note on maximal points of convex sets in ℓ_∞ . C. R. RAO, Least squares theory using an estimated dispersion matrix and its application to measurement of signals. KAROLY SARKADI, On testing for normality. R. A. WIJSMAN, Cross-sections of orbits and their application to densities of maximal invariants.

Sequential Procedures

PETER J. BICKEL and JOSEPH A. YAHAV, Asymptotically pointwise optimal procedures in sequential analysis. DAVID BLACKWELL, Positive dynamic programming. Y. S. CHOW and H. ROBBINS, A class of optimal stopping problems. Y. S. CHOW and H. ROBBINS, On values associated with a stochastic sequence. ARYEH DVORETZKY, Existence and properties of certain optimal stopping rules. THOMAS S. FERGUSON, On discrete evasion games with a two-move information lag. M. V. JOHNS, JR., Two-action compound decision problems. JERZY ŁOŚ, Horizon in dynamic programs.

Information Theory

T. KITAGAWA, Information science and its connection with statistics. ALFRED RÉNYI, On some basic problems of statistics from the point of view of information theory. M. ROSENBLATT-ROTH, Approximations in information theory. J. WOLFOWITZ, Approximation with a fidelity criterion.

Nonparametric Procedures

PETER J. BICKEL, Some contributions to the theory of order statistics. RALPH BRADLEY, Topics in rank-order statistics. Z. GOVINDARAJULU, L. LE CAM, and M.

RAGHAVACHARI, Generalizations of Chernoff-Savage theorems on asymptotic normality of nonparametric test statistics. PRANAB KUMAR SEN, On a class of two sample bivariate nonparametric tests. I. VINCZE, On some questions connected with two sample tests of Smirnov type.

Volume II, Part I—Theory of Probability

Probability on Algebraic Structures

SIMEON M. BERMAN, Sign-invariant random elements in topological groups. R. GANGOLLI, Abstract harmonic analysis and Lévy's Brownian motion of several parameters. LEONARD GROSS, Abstract Wiener spaces. EDITH MOURIER, Random elements in linear spaces. C. RYLL-NARDZEWSKI, On fixed points of semigroups of endomorphisms of linear spaces. A. and C. IONESCU TULCEA, On the existence of a lifting commuting with the left translations of an arbitrary locally compact group.

Distributions in Functional Spaces

HERMANN DINGES, Random shifts of stationary processes. T. HIDA and N. IKEDA, Analysis on Hilbert space with reproducing Kernel arising from multiple Weiner integral. K. ITÔ, Generalized uniform complex measures in the Hilbertian metric space with their application to the Feynman integral. A. V. SKOROHOD, On the densities of probability measures in functional spaces. LESTER DUBINS and DAVID A. FREEDMAN, Random distribution functions.

Stochastic Processes and Prediction

HARALD CRAMÉR, A contribution to the multiplicity theory of stochastic processes. R. M. DUDLEY, On prediction theory for nonstationary sequences. K. URBANIK, Some prediction problems for strictly stationary processes. A. M. YAGLOM, Outline of some topics in linear extrapolation of stationary random processes.

Martingales

M. BRELOT, Capacity and balayage for decreasing sets. LESTER DUBINS and GIDEON SCHWARZ, On extremal martingale distributions. STEVEN OREY, F -processes. VOLKER STRASSEN, Almost sure behavior of sums of independent random variables and martingales.

Special Problems

D. A. DARLING, Some limit theorems associated with multinomial trials. GERARD DEBREU, Integration of correspondences. WILLIAM FELLER, On regular variation and local limit theorems. MILOSLAV JIŘINA, General branching processes with continuous time parameter. EUGENE LUKACS, On the arithmetical properties of certain entire characteristic functions. HERMAN RUBIN, Supports of convolutions of identical distributions. E. SPARRE ANDERSEN, An algebraic treatment of fluctuations of sums of random variables. LAJOS TAKÁCS, On combinatorial methods in the theory of stochastic processes.

Volume II, Part II—Theory of Probability

Markov Processes

R. M. BLUMENTHAL and R. K. GETOOR, Accessible terminal times. LEO BREIMAN, First exit times from a square root boundary. E. B. DYNKIN, General lateral conditions for some diffusion processes. H. KESTEN, The Martin boundary of recurrent random walks on countable groups. M. MOTOO, Application of additive functionals to the boundary problem of Markov processes (Lévy system of U -processes). TADASHI UENO,

A survey on the Markov process on the boundary of multidimensional diffusion. H. KUNITA and T. WATANABE, Some theorems concerning resolvents over locally compact spaces. DAVID G. KENDALL, On Markov groups. J. M. O. SPEAKMAN, Some problems relating to Markov groups. DAVID WILLIAMS, Uniform ergodicity in Markov chains. J. G. BASTERFIELD, On quasi-compact pseudo-resolvents. J. M. O. SPEAKMAN, A note on Markov semigroups which are compact for some but not *all* $t > 0$. DANIEL RAY, Some local properties of Markov processes. DONALD ORNSTEIN, A limit theorem for independent random variables. CHARLES STONE, On local and ratio limit theorems. JOHN LAMPERTI, Limiting distributions for branching processes. SAMUEL KARLIN and JAMES MCGREGOR, Uniqueness of stationary measures for branching processes and applications. W. L. SMITH, Some peculiar semi-Markov processes. W. L. SMITH, A theorem on functions of characteristic functions and its application to some renewal theoretic random walk problems. F. SPITZER, Renewal theorems for Markov chains.

Ergodic Theory

ROBERT J. AUMANN, Random measure preserving transformations. J. R. BLUM, H. D. BRUNK, and D. L. HANSON, Roots of the one-sided N -shift. R. V. CHACON, A geometric construction of measure preserving transformations. A. G. HAJIAN and YUJI ITO, Conservative positive contractions in L^1 . KONRAD JACOBS, On Poincaré's recurrence theorem. SHIZUO KAKUTANI, Ergodic theory of shift transformations. ULRICH KRENGEL, Classification of states for operators. KLAUS KRICKEBERG, Strong mixing properties of Markov chains with infinite invariant measure. CALVIN C. MOORE, Invariant measures on product spaces. JACQUES NEVEU, Existence of bounded invariant measures in ergodic theory. MURRAY ROSENBLATT, Transition probability operators.

Volume III—Physical Sciences

Astronomy

E. M. BURBIDGE and G. R. BURBIDGE, Evolution of galaxies. W. H. MC CREA, Age distribution of galaxies. THORNTON PAGE, Masses of galaxies: singles and members of multiple systems. BEVERLY LYND, Space distribution of small dark nebulae. W. C. LIVINGSTON, On correlations between brightness velocity and magnetic fields in the solar photosphere.

Physics

R. L. DOBRUSHIN, Existence of phase transitions in models of a lattice gas. J. M. HAMMERSLEY, Harnesses. HERBERT SOLOMON, Random packing density.

Spectral Analysis

M. S. BARTLETT, The spectral analysis of line processes. BENOIT MANDELBRÖT, Sporadic random functions and conditional spectral analysis; self-similar examples and limits.

Control Processes

JOHN BATHER and HERMAN CHERNOFF, Sequential decisions in the control of a spaceship. RICHARD BELLMAN, On the construction of a mathematical theory of the identification of systems. P. WHITTLE, The deterministic-stochastic transition in control processes and the use of maximum and integral transforms.

Reliability

RICHARD E. BARLOW and ALBERT W. MARSHALL, Bounds on interval probabilities for restricted families of distributions. YU. K. BELYAEV, B. V. GNEDENKO, and A. D. SOLOVIEV, On some stochastic problems of reliability theory. Z. W. BIRNBAUM

and J. D. ESARY, Some inequalities for reliability functions. B. V. GNEDENKO, Some theorems on standbys. FRANK PROSCHAN and RONALD PYKE, Tests for monotone failure rate. A. D. SOLOVIEV, Theory of aging elements.

Volume IV—Biology and Problems of Health

Information, Processing, and Cognition

MARY A. B. BRAZIER, The challenge of biological organization to mathematical description. RICHARD BELLMAN, Adaptive processes and intelligent machines. H. J. BREMERMAN, Quantum noise and information. VIOLET R. CANE, Mathematical models for neural networks. EDWARD A. FEIGENBAUM, Information processing and memory. JULIAN FELDMAN, Recognition of pattern in periodic binary sequences. WALTER REITMAN, Modeling the formation and use of concepts, percepts, and rules.

Demography

NATHAN KEYFITZ, Estimating the trajectory of a population. MINDEL C. SHEPS, Uses of stochastic models in the evaluation of population policies. I. Theory and approaches to data analysis. EDWARD B. PERRIN, Uses of stochastic models for the evaluation of population policies. II. Results of a Monte Carlo program.

Ecology

DOUGLAS G. CHAPMAN, Stochastic models in animal population ecology. E. C. PIELOU, The use of information theory in the study of the diversity of biological populations. J. G. SKELLAM, Seasonal periodicity in theoretical population ecology.

Epidemiology

C. C. SPICER, Some empirical studies in epidemiology. D. E. BARTON, F. N. DAVID, EVELYN FIX, MAXINE MERRINGTON, and P. MUSTACCHI, Tests for space time interaction and a power function. F. N. DAVID, EVELYN FIX, and P. MUSTACCHI, Three tests for space time interaction: a comparative evaluation. NORMAN T. J. BAILEY, The simulation of stochastic epidemics in two dimensions. ROBERT BARTOSZYNSKI, Branching processes and the theory of epidemics. J. GANI, On the general stochastic epidemic. H. E. DANIELS, The distribution of the total size of an epidemic.

Genetics

THEODOSIUS DOBZHANSKY, Genetic diversity and diversity of environments. HOWARD LEVENE, Genetic diversity and diversity of environment: mathematical aspects. G. MALECOT, Identical loci and relationship. OSCAR KEMPTHORNE, The concept identity of genes by descent. D. E. BARTON, F. N. DAVID, EVELYN FIX, and MAXINE MERRINGTON, A review of analysis of karyographs of the human cell in mitosis. J. O. IRWIN, A theory of the association of chromosomes in karyotypes, illustrated by Dr. Patricia Jacobs' data. WALTER F. BODMER, Models for DNA mediated bacterial transformations. WALTER F. BODMER, SAMUEL KARLIN, and JAMES MC GREGOR, The rate of production of recombinants between linked genes in finite populations. SAMUEL KARLIN and JAMES MC GREGOR, The number of mutants maintained in a population. R. C. LEWONTIN, The genetics of complex systems. P. A. P. MORAN, Unsolved problems in evolutionary theory.

Chance Mechanisms in Living Organisms

S. R. BERNARD, L. R. SHENTON, and V. R. RAO UPPULURI, Stochastic models for the distribution of radioactive materials in a connected system of compartments. PREM

S. PURI, A class of stochastic models for response after infection of the absence of defense mechanism. J. GANI, Models for antibody attachment to virus and bacteriophage.

Cellular Phenomena

HERBERT E. KUBITSCHKEK, Cell generation times: ancestral and internal controls. H. RUBIN, Cell growth as a function of cell density. WALTER R. STAHL, Measures of organization in a model of cellular self-reproduction based on Turing machines. DAVID BURNETT-HALL and W. A. O'N. WAUGH, Sensitivity of a birth process to changes in the generation time distribution.

Carcinogenesis

DAVID LINDER and STANLEY M. GARTLER, Problem of single cell vs. multicell origin of a tumor. WOLFGANG J. BÜHLER, Single cell vs. multicell hypotheses of tumor formation. T. TIMOTHY CROCKER and BERYL J. NIELSEN, Chemical carcinogens and respiratory epithelium. K. B. DE OME, The mouse mammary tumor system. DAVID W. WEISS, Immunology of spontaneous tumors. M. B. SHIMKIN, R. WIEDER, D. MARZI, N. GUBAREFF, and V. SUNTZEFF, Lung tumors in mice receiving different schedules of urethane. A. GRENDON, H. B. JONES, and M. WHITE, Effects of urethane dose and time pattern on tumor formation. JERZY NEYMAN and ELIZABETH I. SCOTT, Statistical aspect of the problem of carcinogenesis.

Experimentation

F. YATES, A fresh look at the basic principles of the design and analysis of experiments. P. ARMITAGE, Some developments in the theory and practice of sequential medical trials. HERMAN CHERNOFF, Sequential models for clinical trials. JEROME CORNFELD and SAMUEL W. GREENHOUSE, On certain aspects of sequential clinical trials. BRADLEY EFRON, The two-sample problem with censored data. MARVIN A. SCHNEIDERMAN, Mouse to man: statistical problems in bringing a drug to clinical trial.

Decision Theory in Medical Diagnosis

LEONARD RUBIN, MORRIS F. COLLEN, and GEORGE E. GOLDMAN, Frequency decision theoretical approach to automated medical diagnosis. CHARLES D. FLAGLE, A decision theoretical comparison of three procedures of screening for a single disease. LEE B. LUSTED, Logical analysis in medical diagnosis. J. T. CHU, Some decision making methods applicable to the medical sciences.

Volume V—Weather Modification

Physical Background

M. NEIBURGER, Physical factors in precipitation processes and their influence on the effectiveness of cloud seeding.

Large Randomized Experiments

LOUIS J. BATTAN and A. RICHARD KASSANDER, JR., Summary of results of a randomized cloud seeding project in Arizona. J. BERNIER, On the design and evaluation of cloud seeding experiments performed by Electricité de France. WAYNE L. DECKER and PAUL T. SCHICKENDANZ, The evaluation of rainfall records from a five year cloud seeding experiment in Missouri. DONALD L. EBERLY and LEWIS H. ROBINSON, Design and evaluation of randomized wintertime cloud seeding at high elevation. K. R. GABRIEL, The Israeli artificial rainfall stimulation experiment evaluation for the period 1961-65. LEWIS O. GRANT and PAUL W. MIELKE, JR., A randomized cloud seeding experiment at

Climax, Colorado, 1960–65. EMILIO PEREZ, Sixteen years of experimentation on artificial stimulation of rain in the Necaxa and Lerma watersheds, Mexico. PAUL SCHMID, An experiment of hail suppression in Switzerland. E. J. SMITH, Cloud seeding experiments in Australia. S. A. CHANGNON, JR. and F. A. HUFF, The effect of natural rainfall variability in verification of rain modification experiments. THOMAS J. HENDERSON, Tracking silver iodide nuclei under orographic influence.

Nonrandomized Operations

GLENN W. BRIER, THOMAS H. CARPENTER, and DWIGHT B. KLINE, Some problems in evaluating cloud seeding effects over extensive areas. HANS GERHARD MÜLLER, Weather modification experiments in Bavaria.

Methodological Discussion

ARNOLD COURT, Randomized cloud seeding in the United States. L. G. DAVIS and C. L. HOSLER, The design, execution and evaluation of a weather modification experiment. ARCHIE KAHAN, The Bureau of Reclamation's Atmospheric Water Resources Research Program. VUJICA M. YEVDJEVICH, Evaluation of weather modification as expressed in stream-flow response. JERZY NEYMAN and ELIZABETH L. SCOTT, Some outstanding problems relating to rain modification. JERZY NEYMAN and ELIZABETH L. SCOTT, Appendix. Planning an experiment with cloud seeding. JERZY NEYMAN and ELIZABETH L. SCOTT, Note on the Weather Bureau ACN Project. J. M. WELLS and M. A. WELLS, Note on Project SCUD. JERZY NEYMAN and ELIZABETH L. SCOTT, Note on techniques of evaluation of single rain stimulation experiments. ROBERT B. DAVIES and PREM S. PURI, Some techniques of summary evaluations of several independent experiments. BARRY R. JAMES, On Pitman efficiency of some tests of scale for the Gamma distribution. FRANK YATES, Discussion of reports on cloud seeding experiments.

Observational Data

A collection of data from cloud seeding experiments in five countries.

PREFACE

THE PURPOSE OF THE Berkeley Statistical Symposia, held every five years, is to stimulate research through lectures by carefully selected speakers and through prolonged personal contacts of scholars brought together from distant centers. Accordingly, particular Symposia last from four to seven weeks. On occasion, and this was the case with the Fifth Symposium, they are conducted in two parts, one in June–July, emphasizing theory, and the other in December–January, emphasizing applications. The winter part of the Fifth Symposium was held in conjunction with the 132nd Annual Meeting of the American Association for the Advancement of Science.

The Proceedings of the Symposia are intended to present a comprehensive cross-section of contemporary thinking on problems of probability and statistics and on selected fields of application. The rapid growth of research in statistics and especially in probability makes it increasingly difficult to achieve a complete coverage of the field, but sincere efforts are made to invite to the Symposia representatives of all the existing schools of thought, each individual having complete freedom of expression.

The organization of the theoretical part of the Fifth Berkeley Symposium was carried out, and the contributors were selected, with the participation of an Advisory Committee composed of Professors J. L. Doob, S. Karlin, and H. Robbins, delegated for this purpose by the American Mathematical Society and by the Institute of Mathematical Statistics. In addition, we had the assistance of Professor D. L. Burkholder, the Editor of the *Annals of Mathematical Statistics*. The interest of the American Mathematical Society and of the Institute of Mathematical Statistics and their help are deeply appreciated.

While a broad coverage of contemporary work in the theory of probability and statistics is difficult, the field of applications of these disciplines is currently so wide that the program of a single symposium can include no more than a few particular domains. The domains covered at the Fifth Symposium were selected on two principles. First, some applied problems appeared as subjects of studies by outstanding probabilists and statisticians invited to the Symposium on account of their work in theory. Second, an effort was made to delineate a few fields of substantive studies that appear particularly promising for probabilistic and statistical treatment. One of the most fruitful fields of this category is undoubtedly biology and problems of health. Here we profited greatly by the advice of Drs. LaMont Cole, Jerome Cornfield, F. N. David, Louis Hellman, Samuel Greenhouse, Hardin Jones, Samuel Karlin, David Krech, Lincoln Moses, Curt Stern, Michael B. Shimkin, and Cornelius Tobias. Quite a few of these colleagues are connected with the broad research activity of the National Institutes of Health and helped to bring to our attention many novel and important subfields of research.

In the field of astronomy we are deeply indebted to Drs. N. U. Mayall, Rudolph Minkowski, and Thornton L. Page.

For advice in the field of meteorology we are grateful to Drs. Earl Droessler, James Hughes, Dwight B. Kline, Morris Neiburger, Jerome Spar, Edward P. Todd, and P. H. Wyckoff. Special thanks are due to Dr. Kenneth B. Spengler, Secretary of the American Meteorological Society.

Following the established tradition, Volume I of the present *Proceedings* is given to the theory of statistics. Volume II is devoted to the theory of probability. Because of the large amount of material, about 1000 pages in print, this volume had to be divided into two parts formed through a somewhat arbitrary classification of papers. Volume III includes papers related to physical sciences: astronomy, theory of control, physics, and the theory of reliability. Volume IV, on biology and problems of health, includes papers on information and brain phenomena, on chance mechanisms in live organisms, on epidemiology, on genetics, on medical diagnosis, on clinical trials, on carcinogenesis and cellular phenomena, on demography, and on ecology. Some of these subdomains are already subjects of well developed statistical treatment. Others appear to offer interesting and important possibilities.

Compared to the *Proceedings* of the earlier Symposia, Volume V, being entirely given to the problem of artificial weather modification, is an innovation. With the classification adopted for the first four volumes, weather modification would fit Volume III. It is assigned a special volume because of the specificity of the domain and because of its separateness from all the other fields dealt with in Volume III. Also, the novelty of the problem of weather modification, considered by itself and as a field for statistical research, indicated the desirability of producing a comprehensive coverage of the more extensive experiments. Finally, it appears probable that the readership of the material being published in Volume V will be essentially different from that expected to be interested in Volume III.

The Fifth Symposium would not have been possible without very substantial financial support from various sources. Hearty thanks are due to Dr. Clark Kerr, President of the University of California, for a special grant made several years in advance of the Symposium. Without this grant, no planning and no initial steps for the organization of the Symposium would have been possible. This initial triggering grant of the University of California was later supplemented by the subsidy of the University Editorial Committee, without which the publication of the *Proceedings*, to be sold at a reasonable price, would have been a very difficult problem. To a very considerable extent, the theoretical part of the Symposium and the part concerned with physical sciences, were financed by The Program in Mathematics of the National Science Foundation, by the Air Force Office of Scientific Research, by the Army Research Office, and by the Office of Naval Research. The large program on biology and problems of health was made possible by a grant of the National Institutes of Health. The

program on weather modification was organized using a grant of the Atmospheric Sciences Section of the National Science Foundation. Finally, we wish to record special help from the Office of Naval Research, in the form of air transportation for a number of foreign participants in the Symposium.

It is our pleasure to acknowledge gratefully the generosity of the governmental institutions enumerated. The vitality of our Symposia and the growth of the *Proceedings*, from 500 pages in 1945 to about 3,000 in 1965, seem to indicate that the funds provided are being spent to fill a real need.

The problems connected with the publication of such an amount of technical material are very substantial, especially since some of the material was originally written in languages other than English and required translation. All efforts were made toward speedy publication at a reasonable price, and we are pleased to acknowledge the excellent cooperation and assistance we received from the University of California Press.

For the translation of manuscripts, we are indebted to Drs. Amiel Feinstein, Morris Friedman, and Mrs. C. Stein. We are also indebted to several of our colleagues in the Department for work connected with the preparation of manuscripts for the printer. Special thanks are due to Professors E. L. Scott, M. Loève, E. W. Barankin, to Drs. Carlos-Barbosa Dantas, W. Bühler, Nora Smiriga, Grace Yang, to Mr. Steve Stigler, and Mrs. M. Darland. We are pleased to acknowledge the technical help of Mrs. Sharlee Guise and Mrs. Carol Rule Roth.

For taking care of the many complexities of editing technical manuscripts we are deeply indebted to Miss Susan Jenkins whose patience and skill deserve superlative praise. Thanks are also due to Mrs. Virginia Thompson for her greatly appreciated assistance in the same process.

To Mr. August Frugé, the Director of the University of California Press, we extend heartfelt thanks for financial, technical, and moral support in publishing so much difficult material. Special thanks are due also to Joel Walters, Editor of the University of California Press. In spite of all our efforts, we found ourselves unable to keep up with the schedule of publication proposed by the Press, but we must thank them for helping us to keep the delays at a minimum and for producing a publication in accordance with the usual excellent standards of the University of California Press.

Many thanks are due to our Administrative Assistant, Miss M. Genelly for taking care of many financial and organizational difficulties. For transportation, housing, and other logistic problems connected with the organization of the meeting itself, very valuable assistance was received from the staff of the Laboratory and in particular from Miss June Haynes and Mrs. J. Lovasich.

As was the case on many earlier similar occasions, for supervising and taking care of the innumerable intricacies of local organization we are deeply indebted to our colleague Professor Elizabeth L. Scott. It is a pleasure to express here our deepest appreciation.

Last but not least we wish to thank the Department of Statistics of the

University of California, Berkeley, and all our colleagues therein, for their sympathetic attitude and help. Particular thanks are due to David Blackwell.

During the winter part of the Fifth Symposium, the Statistical Laboratory lost one of its organizers as well as one of its most active members. Our colleague and cordial friend, Professor Evelyn Fix died of a heart attack on December 30, only a few hours after she acted as one of the hostesses at the banquet of the Symposium. *Sit ei terra levis!*

LUCIEN LE CAM

JERZY NEYMAN
Director, Statistical Laboratory

January, 1967

CONTENTS

Probability on Algebraic Structures

✓ SIMEON M. BERMAN—Sign-Invariant Random Elements in Topological Groups	1
✓ R. GANGOLLI—Abstract Harmonic Analysis and Lévy's Brownian Motion of Several Parameters	13
✓ LEONARD GROSS—Abstract Wiener Spaces	31
✓ EDITH MOURIER—Random Elements in Linear Spaces	43
✓ CZESŁAW RYLL-NARDZEWSKI—On Fixed Points of Semigroups of Endomorphisms of Linear Spaces	55
✓ A. IONESCU TULCEA and C. IONESCU TULCEA—On the Existence of a Lifting Commuting with the Left Translations of an Arbitrary Locally Compact Group	63

Distributions in Functional Spaces

HERMANN DINGES—Random Shifts of Stationary Processes	99
TAKEYUKI HIDA and NOBUYUKI IKEDA—Analysis on Hilbert Space with Reproducing Kernel Arising from Multiple Wiener Integral	117
KIYOSI ITÔ—Generalized Uniform Complex Measures in the Hil- bertian Metric Space with Their Application to the Feynman Integral	145
A. V. SKOROHOD—On the Densities of Probability Measures in Functional Spaces	163
✓ LESTER E. DUBINS and DAVID A. FREEDMAN—Random Distri- bution Functions	183

Stochastic Processes and Prediction

HARALD CRAMÉR—A Contribution to the Multiplicity Theory of Stochastic Processes	215
--	-----

R. M. DUDLEY—On Prediction Theory for Nonstationary Sequences	223
K. URBANIK—Some Prediction Problems for Strictly Stationary Processes	235
A. M. YAGLOM—Outline of Some Topics in Linear Extrapolation of Stationary Random Processes	259
 <i>Martingales</i>	
M. BRELOT—Capacity and Balayage for Decreasing Sets . . .	279
LESTER E. DUBINS and GIDEON SCHWARZ—On Extremal Martingale Distributions	295
STEVEN OREY— F -Processes	301
VOLKER STRASSEN—Almost Sure Behavior of Sums of Independent Random Variables and Martingales	315
 <i>Special Problems</i>	
D. A. DARLING—Some Limit Theorems Associated with Multinomial Trials	345
GERARD DEBREU—Integration of Correspondences	351
WILLIAM FELLER—On Regular Variation and Local Limit Theorems	373
MILOSLAV JIŘINA—General Branching Processes with Continuous Time Parameter	389
EUGENE LUKACS—On the Arithmetical Properties of Certain Entire Characteristic Functions	401
HERMAN RUBIN—Supports of Convolutions of Identical Distributions	415
E. SPARRE ANDERSEN—An Algebraic Treatment of Fluctuations of Sums of Random Variables.	423
LAJOS TAKÁCS—On Combinatorial Methods in the Theory of Stochastic Processes	431