# THE LUMINOSITY FUNCTION OF EXTRAGALACTIC RADIO SOURCES

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Except for the neighborhood of the galactic plane, radio sources are uniformly distributed over the sky. The isotropic distribution suggests that these sources are external galaxies which are indeed among the first objects that were identified as radio sources. Positions of relatively moderate accuracy pointed clearly to some nearby bright galaxies that were long known as peculiar. But the second strongest small source in the sky, Cygnus A, turned out to be a faint extragalactic object. Since sources of the intrinsic strength of Cygnus A may be expected to be observable at distances beyond the reach of the 200-inch telescope, it was clear that not all the sources can be expected to be optically observable.

Precise positions have now been obtained for a large number of sources, but all attempts to identify them have led to the conclusion that, even with positions of an accuracy of the order of a few minutes of arc, only a fraction of all sources can be identified [1]. The main reason for the difficulty of finding identifications always has been sought in the assumption that most sources are intrinsically strong sources of the Cygnus A type at distances where the galaxies are too faint to permit identifications. Sufficient progress has now been made to put this assumption to a quantitative test by a determination of the luminosity function of radio sources.

A preliminary quantitative study of the problem was first made by Ryle [2]. Essentially the argument was that the small sizes found for many of the unidentified sources indicate large distance and therefore small space density. The number-intensity relation of radio sources then requires that most of the sources have high luminosity, comparable to that of the strongest sources known.

Since radio methods do not yet permit the determination of distances, the luminosity function can be determined only with the aid of optical data, and therefore only for identified sources. For the comparison of optical and radio data it is convenient to use radio magnitudes instead of flux density. The radio magnitude is defined as

$$m_r = -53.4 - 2.5 \log S_{158}, \tag{1}$$

where $S_{158}$ is the flux density in $\mathrm{Wm^{-2}(c/sec)^{-1}}$ at a frequency of 158 Mc/sec. All logarithms are to the base 10. The spectrum of the extragalactic sources follows a simple power law of the frequency $\nu$. The flux density is proportional to $\nu^x$; the spectral index $x$ of extragalactic sources is in the range between about $-0.6$ and $-0.9$.

The first attempt at a determination of the luminosity function was made recently by Mills [3]. Photographic magnitudes $m_p$ of identified objects were determined by eye estimates. The difference $m_r - m_p$, which is independent of distance, can now be used as a measure of the strength of a source. The number $n_i\Delta m$ of identifications with galaxies brighter than $m_p$ having $m_r - m_p$ in an interval $\Delta m$ then can be obtained from the list of identified sources. By combining this result with Hubble's [4] relation for the number of galaxies brighter than $m_p$, the probability then can be derived that a galaxy chosen at random has a value of $m_r - m_p$ in the interval $\Delta m$. Since the difference $m_r - m_p$ is equal to the difference of the absolute magnitudes $M_r - M_p$, and since there is reason, as will be shown below, to assume that $M_p$ has a well-determined mean value $\overline{M}_p$, the probability of a value of $m_r - m_p$ can be considered as the probability of the absolute radio magnitude $M_r = m_r - m_p + \overline{M}_p$.

No better procedure is feasible as long as the photographic magnitudes are the only optical data available. But it is unsatisfactory that the counts do not refer to a fixed volume. They extend to a given limiting magnitude, and therefore refer to a volume that depends on the absolute magnitude of the objects and sources.

Investigation with the 200-inch telescope of objects suspected as possible identifications has now reached a stage in which for about half of the objects considered as true identifications the nebular redshift has been determined. Sufficient absolute magnitudes are therefore available to permit a new approach to a determination of the luminosity function.

For those objects for which the redshift $z = \delta\lambda/\lambda_0$ is known, the distance modulus is obtained from

$$m - M = 5 \log cz - 5 \log H - 5, \tag{2}$$

as long as cosmological effects can be neglected. $H$ is the Hubble constant, the value 75 km/sec per $10^6$ psc will be adopted. Absolute magnitudes $M_p$ and $M_r$ can then be obtained for the sources with known redshift. If the values of $M_p$ are found to have a well-determined mean $\overline{M}_p$, the assumption can be made that this value is valid for all sources. The value $m_p - \overline{M}_p$ is then assumed for the distance modulus and values of $M_r$ can thus be derived for sources with unknown $z$. The number $k(M)\Delta m$ of sources with absolute magnitude between $M - \Delta m/2$ and $M + \Delta m/2$ can now be counted.

If $m_{r0}$ is the limiting radio magnitude to which the search for identifications is complete, sources with $M_r$ are contained in the volume whose radius $r$ in parsec is given by

$$\log r = 0.2(m_{r0} - M_r) + 1. \tag{3}$$

If $q$ is the area of the sky in steradians that has been searched completely for identifications, the volume $v_M$ in Mpc$^3$ that contains the sources with absolute radio magnitude $M_r$ is then given by

$$\log v_M = -15.5 + 0.6(m_{r0} - M_r) + \log q. \tag{4}$$

The number $n_M$ of sources with absolute radio magnitude $M_r$ per Mpc$^3$ thus becomes, per interval $\Delta M = 1$,

$$\log n_M = \log k(M) + 15.5 - 0.6(m_{r0} - M_r) - \log q. \tag{5}$$

There are two surveys from which lists of identified sources can be compiled that fulfill the necessary condition of at least approximate completeness to a limiting radio magnitude in a defined area of the sky: the survey by Mills, Slee, and Hill [5] in Sydney at 86.5 Mc/sec and the new survey (3C survey) by Ryle's group [6] at 159 Mc/sec. Precise positions for 84 sources of the 3C survey have been determined by Elsmore, Ryle, and Leslie [7]. Precise right ascensions determined by J. Bolton and his group are now available for a large part of the 3C sources. Sources were identified by inspection of the plates of the National Geographic Society–Palomar Observatory Sky Survey. The search for identifications can consider only galaxies brighter than a certain limit determined by the accuracy of the radio positions. The presence of a galaxy as bright as $m_p$ within the error area $\epsilon^2$ of a radio position can be considered as significant only if $\epsilon^2 n(m_p) \ll 1$, where $n(m_p)$ is the number of galaxies brighter than $m_p$ per unit area. Since multiple galaxies are less frequent than single galaxies the limit for their acceptance as identification is 2 to 3 magnitudes fainter than for single galaxies.

The survey of Mills, Slee, and Hill covers the zone between declination $-20°$ to $+10°$, an area of roughly 3 steradians if the area within 12° from the galactic equator is excluded. This area has been searched completely to a flux density of $20 \times 10^{-26}$ or $m_r = 9.2$ by the author and by Mills. A list of identifications has been given by Mills [3]. Investigation of all suspected objects with the 200-inch telescope showed, however, that some of these identifications are to be rejected because the objects in question were actually not double galaxies, as judged on the 48-inch plates, but a star in the foreground and a single galaxy too faint to be acceptable as identification. On the other hand, some new identifications could be added with the aid of the 200-inch plates. The relevant data for the revised list of identifications are given in table I, which contains the identifications of all sources above $m_r = 9.2$ in the area.

The great variety of objects with which sources have been identified is evident. Obviously, it would be desirable to determine the luminosity function separately for different types of objects, but the number of identifications is still too small to permit a division into separate classes. In thus grouping together single and double galaxies, a choice has to be made of which magnitude to use for doubles. Since the components are not clearly separated in all doubles, the combined magnitude of the components has been used throughout. With this choice the mean value of $M_p$ for the sources with measured redshift is $-20^{m}.3 \pm 0.8$. This

## TABLE I

### IDENTIFIED EXTRAGALACTIC SOURCES OF THE SYDNEY SURVEY WITH $m_r < 9.2$

| Source | $m_p$ | Type | $cz$ | $m - M$ | $M_p$ | $m_r$ | $M_r$ | Remarks |
|---|---|---|---|---|---|---|---|---|
| $00 - 1\underline{0}$ | 18 | *d* | | 38.3 | | 8.8 | −29.5 | |
| $00 - 1\underline{8}$ | 18 | *d* | | 38.3 | | 9.0 | −29.3 | |
| $00 - 0\underline{6}$ | 19 | *d* | | 39.3 | | 9.0 | −30.3 | |
| $00 - 0\underline{15}$ | 17 + 17 | *SO* + *Sb* | +13800 | 36.3 | −20.1 | 9.0 | −27.3 | |
| $00 - 0\underline{17}$ | 15 | *EO* | | 35.3 | | 7.6 | −27.7 | |
| $01 - 0\underline{5}$ | 13 + 13 | *SO* + *EO* | +3000 | 33.0 | −20.8 | 7.6 | −25.4 | NGC 545/547 |
| $02 - 1\underline{10}$ | 18.5 + 18.5 | *EO* + ?*E5* | | 38.0 | | 8.4 | −29.6 | |
| $02 - 0\underline{14}$ | 9.9 | *Sc* | +1030 | 30.6 | −20.7 | 8.6 | −22.0 | NGC 1068 |
| $02 + 0\underline{10}$ | 13 + 13 | *SO* + *EO* | +2300 | 32.4 | −20.2 | 7.2 | −25.2 | |
| $03 + 0\underline{3}$ | 13 | *E1* | | 33.3 | | 8.6 | −24.7 | |
| $05 - 1\underline{1}$ | 15 | *Sbp* | | 35.3 | | 9.2 | −26.1 | |
| $05 + 0\underline{2}$ | 15 | *Sap* | | 35.3 | | 8.5 | −26.8 | |
| $09 - 1\underline{4}$ | 17 | *d* | +15900 | 36.6 | −19.6 | 5.5 | −31.1 | Hyd A |
| $10 - 0\underline{18}$ | 17 | *Sbcp* | | 37.3 | | 9.3 | −28.0 | |
| $11 - 0\underline{8}$ | 17 + 17 | *EO* + *EO* | | 36.5 | | 8.8 | −27.7 | |
| $12 + 0\underline{4}$ | 12.3 | *Sb*? | Virgo Cl. | 30.6 | −18.3 | 8.8 | −21.8 | NGC 4234 |
| $12 + 0\underline{5}$ | 11.2 | *E3* | Virgo Cl. | 30.6 | −19.4 | 7.5 | −23.1 | NGC 4261 |
| $12 - 1\underline{18}$ | 12.8 + 12.8 | *SO* + *EO* | +4500 | 33.8 | −21.8 | 8.2 | −25.6 | NGC 4782/4783 |
| $14 - 0\underline{19}$ | 11.7 | *Sc* | | 32.0 | | 9.3 | −24.6 | |
| $15 + 0\underline{5}$ | 13.5 | *SO* | | 33.8 | | 7.2 | −26.6 | |
| $22 - 0\underline{9}$ | 18 | *d* | | 38.3 | | 8.0 | −30.3 | |
| $23 + 0\underline{3}$ | 15.5 | *Sbc* | | 35.8 | | 8.2 | −27.6 | |
| $23 - 1\underline{12}$ | 18 | *d* | | 38.3 | | 8.8 | −29.5 | |

value has been adopted for the rest of the objects to obtain distance moduli and absolute radio magnitudes. The small number of identifications made it necessary to choose a counting interval of 2 magnitudes for $k(M_r)$. The values of $n(M_r)$ were then obtained from (5). They are plotted as open circles in figure 1

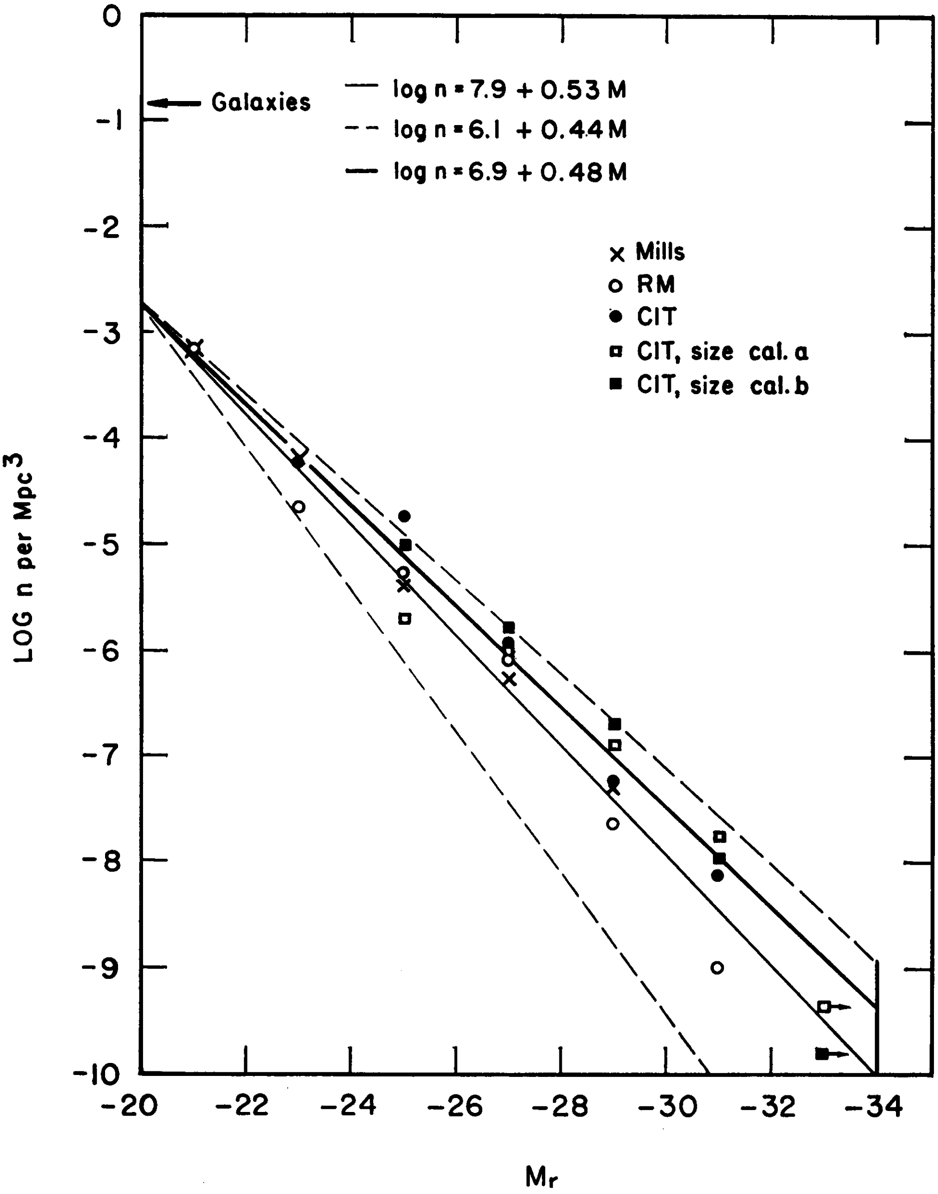

FIGURE 1

The luminosity function of extragalactic radio sources.

## TABLE II

### IDENTIFIED EXTRAGALACTIC SOURCES OF THE 3C SURVEY WITH $m_r \leqq 8.5$

| Source | $m_p$ | Type | $cz$ | $m - M$ | $M_p$ | $m_r$ | $M_r$ | Size (min. of arc) | Remarks |
|---|---|---|---|---|---|---|---|---|---|
| 3C26 | 17 + 17 | *SO* + *Sb* | | 37.2 | | 8.5 | −28.7 | <1′ | |
| 3C33 | 18.5 | *EO* | | 39.5 | | 7.1 | −32.4 | 1.3 | |
| 3C40 | 13 + 13 | *SO* + *EO* | 3000 | 33.0 | −20.8 | 8.2 | −24.8 | 6 *d* | NGC 545/547 |
| 3C66 | 13 | *E* | 6300 | 34.6 | −21.6 | 8.3 | −26.3 | 4 | |
| 3C71 | 10 | *Scp* | 1030 | 30.6 | −20.6 | 8.2 | −22.4 | <0.4 | NGC 1068 |
| 3C75 | 13 + 13 | *EO* + *SO* | 2300 | 32.4 | −20.2 | 8.1 | −24.3 | 3 | |
| 3C78 | 13 | *E1* | | 34.0 | | 8.4 | −25.6 | 0.8 | |
| 3C84 | 13.3 | *Sc* + *Sc* | 5430 | 34.2 | −20.9 | 7.0 | −27.2 | 0.8(5) | NGC 1275 |
| 3C98 | 16 | *E2* | | 37.0 | | 7.3 | −29.7 | 2 | |
| 3C171 | 20 | | | 41.0 | | 8.5 | −32.5 | 1 | |
| 3C196 | 19.5 | | | 40.5 | | 7.0 | −33.5 | 0.2 | |
| 3C219 | 18 + 18.5 | | | 38.8 | | 7.6 | −31.2 | 1.6 | |
| 3C234 | 19 | | | 40.0 | | 8.1 | −31.9 | 1.0 | |
| 3C270 | 11 | *E3* | * | 30.6 | −19.6 | 6.0 | −24.6 | 7 | NGC 4261 |
| 3C274 | 10 | *EO* | * | 30.6 | −20.6 | 4.0 | −26.6 | 0.7(10) | NGC 4486 (M87) |
| 3C278 | 13 + 13 | *SO* + *EO* | 4500 | 33.8 | −21.6 | 7.9 | −25.9 | 2.2 | NGC 4782/4783 |
| 3C280 | 18.5 | | | 39.5 | | 8.2 | −31.3 | 0.5 | |
| 3C295 | 21.5 | *p* | | 41.4† | −19.9† | 6.4 | −35† | 0.2 | |
| 3C298 | 18 | | | 39.0 | | 7.9 | −30.1 | 0.5 | |
| 3C310 | 18.5 + 18.5 | | | 38.7 | | 7.6 | −31.1 | 1.8 | |
| 3C315 | 19 + 19 | | | 39.2 | | 8.3 | −30.9 | 1 | |
| 3C317 | 12.5 | *SO* | | 33.5 | | 7.8 | −25.7 | 0.7 | |
| 3C327 | 17 | | | 38.0 | | 7.6 | −30.4 | 2.2 | |
| 3C338 | 14 | *SO* | 8800 | 35.3 | −21.3 | 8.0 | −27.3 | 1.5 | NGC 6166 |
| 3C353 | 13‡ | | | 34: | | 5.4 | −28.6 | 3.5 | |
| 3C405 | 15.5 | *d* | 17100 | 36.8 | −21.3 | 1.9 | −34.9 | 1.2 | CYG A |
| 3C433 | 17 | *E* + *E* | | 38.0 | | 8.3 | −29.7 | 0.5 | |

* Virgo Cluster.

† Not corrected for redshift.

‡ $m_p = 17$, but in area of heavy obscuration. Estimated absorption $4^m$.

as a function of $M_r$. The original list of identifications given by Mills has been treated in the same way. The results, plotted as crosses in figure 1, do not differ essentially from those obtained from table I, an indication that the result is not affected critically by small defects of the list of identifications.

The 3C survey covers the sky between declinations $-10°$ and $+54°$. The area has been searched by Dewhirst [8] and, after more positions of high precision became available, by Bolton and his group who have added a considerable number of identifications to those noted by Dewhirst and to the few long-established identifications. The list in table II gives the identifications complete to $m_r = 8.5$ in an area of about 3 steradians. Taking again the magnitude of the combined components as that of a double galaxy, we find $-21^m\!.0 \pm 0.7$ as mean value of $M_p$ for the sources with measured redshift. This value is not significantly different from that found for the objects in table I. Since the magnitudes in tables I and II have been estimated by different observers, it seems most proper to use individual values for each table. With $\overline{M}_p = -21.0$, distance moduli and $M_r$ were obtained for the rest of the objects in table II. Values of $k(M)$ were again counted in intervals of 2 magnitudes. The values of $n(M)$ are plotted in figure 1 as filled circles.

Considering the approximate nature of the data, the agreement between the results from the Sydney survey in table I and the 3C survey in table II is quite satisfactory. The agreement becomes poorer toward the bright end, where relatively low values of $n(M)$ are found. It is obvious that incompleteness of the lists of identification may be involved. At the limit for $m_r$, sources with $M_r = -23$ and $M_r = -25$ are galaxies with $m_p = 11$ and $m_p = 13$, respectively; not many galaxies of such brightness can have been missed. But at the bright end of the luminosity function very faint galaxies are involved; here it is very unlikely that the identifications really are complete.

The only way to test for completeness is through the use of the unidentified sources. Their sizes are the only quantity that permits us to obtain distances that can be expected to be statistically correct even if the individual values deserve no confidence. A close correlation between apparent size and distance is not to be expected. Moreover, since many sources are highly elongated and since the sizes have been measured only in right ascension, projection effects will be added to the intrinsic scatter. Nevertheless, the plot of apparent sizes against distance moduli in figure 2 shows a sufficient degree of correlation to permit the use of the apparent size as distance indicator. A difficulty arises from an apparent systematic difference between the sources with distance moduli derived from observed redshifts, plotted as large filled circles, and the sources with distance moduli derived from $\overline{M}_p$, plotted as small filled circles. This difference is most likely due to observational selection. The apparently brighter objects were favored in the observations of the redshift; this means also a preference for the absolutely bright galaxies, so that the value of $\overline{M}_p$ may be too bright. To obtain some judgment on the effect of this systematic difference, two calibration lines were determined, both assuming proportionality of apparent size and distance.

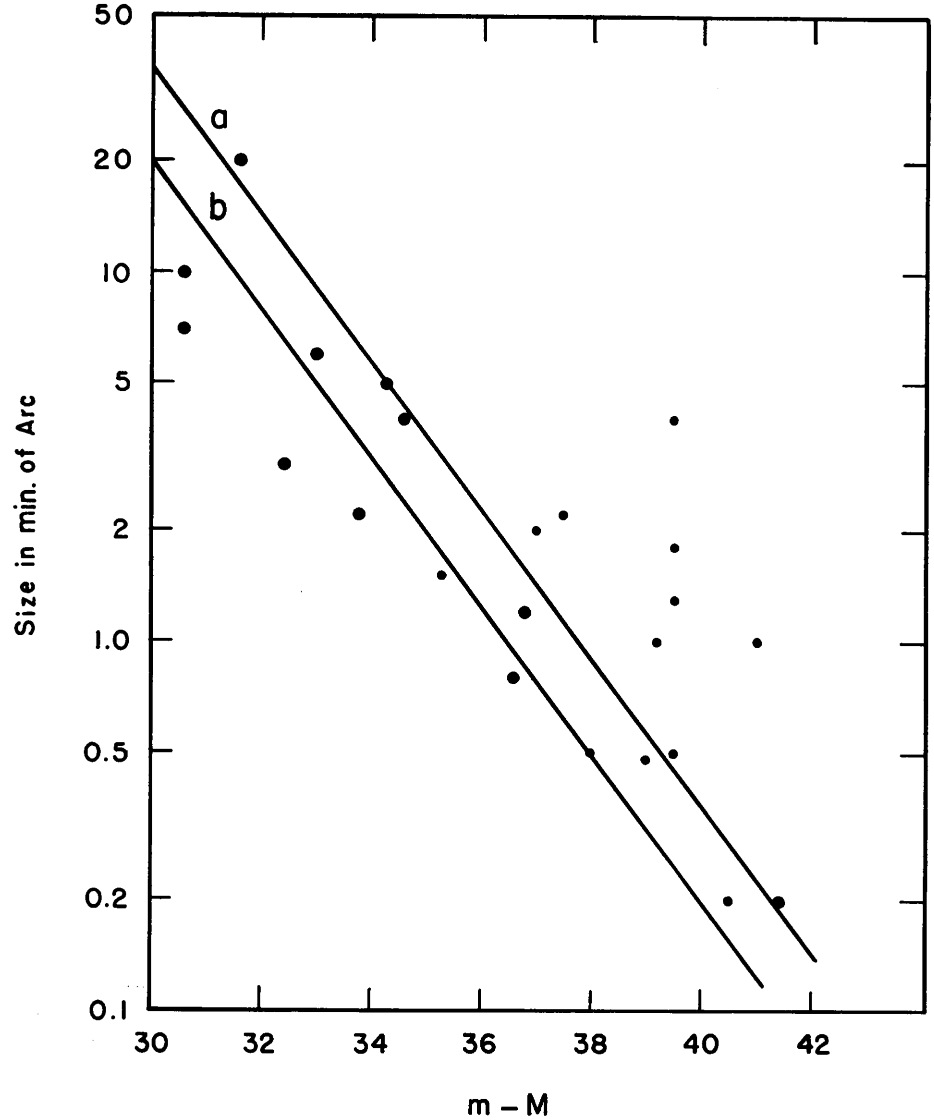

FIGURE 2

Apparent size and distance modulus.

Line a is fitted to all points; line b to those points only for which the redshift has been measured. An unpublished list of sizes by Bolton was then used to derive distance moduli of unidentified sources. The values of $n(M)$ could then be determined. They are plotted in figure 1 as open and as filled squares for calibration by lines a and b in figure 2, respectively. The fit with the values obtained from the identified sources in tables I and II is quite good at the fainter end. At the brighter end the values from the unidentified sources are systematically high and indicate the expected incompleteness of the lists of identified sources. It should be noted that the points at $M_r = -34$ are lower limits because only an upper limit for the size of these sources was available.

The data plotted in figure 1 show that the luminosity function must be between the thin broken line at the upper edge of all points and the thin solid line near the lower edge. The heavy line

$$\log n_M = 6.9 + 0.48\, M_r \tag{6}$$

has been adopted as the best representation of the luminosity function. A gradient of 0.6 is indicated by a thin broken line. It is evident that the gradient cannot be as steep as this. Therefore, the sources of bright $M_r$ will be most frequent among sources of given $m_r$.

The heavy arrow in the upper left corner of figure 1 indicates the mean space density of all galaxies obtained from Oort's [9] value of $3.1 \times 10^{-31} g$ cm$^{-3}$ for the total mass density of the universe, with the assumption that $2 \times 10^{10}$ solar masses is the mean mass of galaxies. Since all sources stronger than $M_r - 21$ together are only a small fraction of all galaxies, most galaxies are expected to be weak sources, in good agreement with the observations of bright galaxies [10], [11].

We may now use the luminosity function to compute the number of sources brighter than a given flux density as a function of the flux density. For this computation we assume that the luminosity function breaks off at $M_r = -34$. The effect of this artificial assumption will be discussed later. As long as the redshift is small enough to be neglected, the number of sources is easily computed. Sources with $M_r$ that appear with $m_r$ come from a shell with the volume $4\pi r^2\, dr$, where $r$ is obtained from (3). The number $n(m_r, M_r)$ of sources with $m_r$ and $M_r$ thus becomes from (6), for the whole sky and per interval $dm = 1$, $dM = 1$,

$$\log n(m_r, M_r) = -7.34 + 0.6\, m_r - 0.12\, M_r, \tag{7}$$

and the number of all sources brighter than $m_r$ is then

$$\log N(m_r, M_r) = -7.22 + 0.6\, m_r - 0.12\, M_r. \tag{8}$$

By summing for $M_r$ from $-20$ to $-34$, we thus obtain the total number of sources brighter than $m_r$,

$$\log N(m_r) = -2.54 + 0.6\, m_r, \tag{9}$$

or, introducing the flux density from (1), the total number of sources brighter than $S$,

$$\log N_S = -32.04 - 1.5 \log S_{158}, \tag{9'}$$

which is the relation where $N_S \sim S^{-3/2}$, the "3/2 law" of a uniform space distribution. Effects of the redshift are not entirely negligible, however, even for $N_m$ little larger than one, so that the simple relations (9) and (9′) have little practical value.

If the effects of redshifts are to be taken into account, a cosmological model has to be selected. The Einstein–de Sitter model leads to relatively simple relations that have been given and used by Mills [3]. But this model, in which space is infinite and Euclidean, leads to a redshift-magnitude relation that is not in good agreement with the best observational data for clusters of galaxies. These data show the linear relation between the bolometric magnitude and the logarithm of the redshift which follows [12], [13], [14] from a model with a space of positive curvature with a negative acceleration parameter determined by the value of the constant $q_0 = +1$. It seems preferable to use this model for the discussion of the number-intensity relation for the radio sources.

For this model, the number of sources stronger than a given flux-density can be given in a parametric representation (McVittie, private communication). For the sake of simplicity, the spectral index here will be assumed to be $x = -1$. If $S_s$ is the flux-density of a source at the standard distance of 10 parsecs, to be obtained from (1) with $m_r = M_r$, and $z_s = 10^{-5}H/c$, the redshift at the standard distance, that is, $z_s = 2.5 \times 10^{-9}$ with the value 75 km/sec/Mpc for the Hubble constant, the flux-density $S$ of a source is

$$S = \frac{S_s z_s^2}{z^2}. \tag{10}$$

If $\xi$ is defined by

$$\frac{1}{2}\operatorname{cosec}^2\left(\frac{\pi}{4} - \xi\right) = 1 + z, \tag{11}$$

where $0 < \xi < \pi/4$, the number of sources with $S_s$ corresponding to $M_r$ that are stronger than $S$ is

$$N_{S,M} = \pi\alpha(4\xi - \sin 4\xi). \tag{12}$$

For very small redshifts, (12) must go over into (8). For $z \ll 1$, (11) becomes

$$\xi = \frac{1}{2} z, \tag{11'}$$

and, from (12) and (10),

$$N_{S,M} = \frac{4\pi}{3}\alpha\, z_s^3 S_s^{3/2} S^{-3/2}. \tag{12'}$$

This relation for small $z$ is indeed identical to (8). From (12′), (1), and (8),

$$\log \alpha = 17.96 + 0.48\, M_r. \tag{13}$$

To obtain the total number of sources stronger than a given flux-density, the number of sources for a given value of $M_r$ has to be computed from (10), (11), (12), and (13). The total number then is obtained by adding the numbers for all values of $M_r$.

The results of this computation are shown in figure 3; the use of $x = -1$ instead of the more realistic value $-0.7$, exaggerates the cosmological effects only to a slight degree that is irrelevant in view of the present uncertainty of

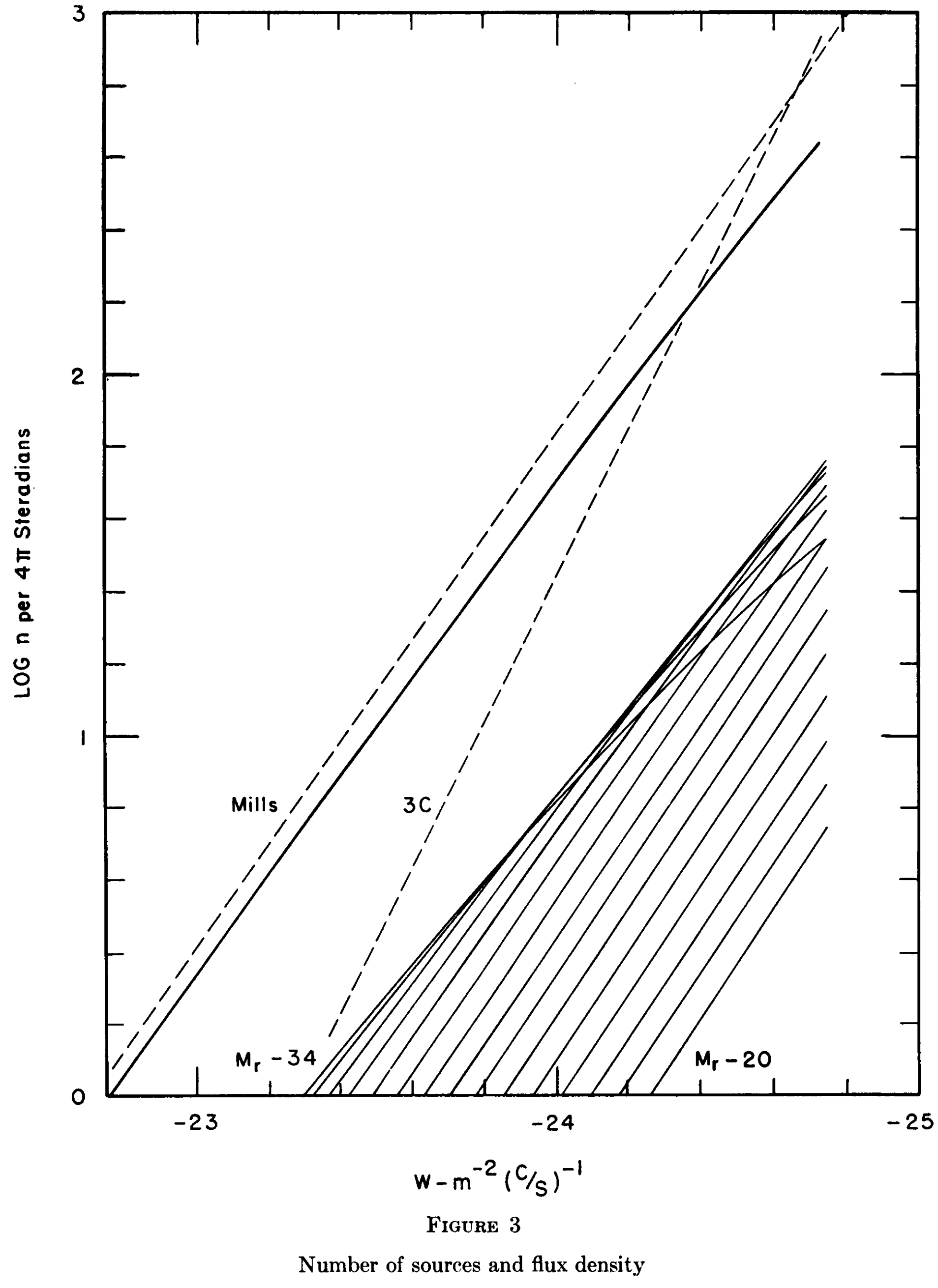

FIGURE 3

Number of sources and flux density

the observed source counts. The array of thin lines in the lower right part of the figure shows the individual values of $N_{S,M}$ as function of $S$. The total expected source count is given by the heavy line. It is obvious that for sources with $M = -35$ and stronger the cosmological effect becomes so large that they would contribute very little to the source count; even sources with $M = -34$ contribute not more than a few per cent of the total source count above 50. The cut-off of the luminosity function made at $M_r = -34$ thus cannot cause any appreciable error.

The source counts obtained from different surveys disagree as to the slope of the $\log n - \log S$ relation. The line with the slope 1.5 in figure 3 is the best representation of the source counts according to Mills' [3] most recent data. The line with the slope 2 has been given as the representation of the source counts from the 3C survey [6]. Unpublished results by Bolton and his group show that the discordance is largely due to the fact that a number of strong extended sources in the Sydney survey are spurious and to the fact that some strong sources with large diameters have been missed by the interferometer used for the 3C survey. The true source counts are therefore expected to lie between the Sydney and the 3C results. Inasmuch as the computed curve lies between the Sydney and the 3C line it is satisfactory, but the observations contradict the downward curvature at low flux densities and large numbers which is a direct consequence of cosmological effects.

If the observational results are correct in showing a constant slope of the $\log n - \log S$ relation, and if the luminosity function (6) is correct, the assumption is unavoidable, as McVittie [11] has shown, that there are strong evolutionary effects and that sources were stronger or strong sources more numerous at the time, billons of years ago, when the radiation from the most distant sources was emitted. The presence of evolutionary effects of this kind is not implausible.

Evolutionary effects have to be assumed only if the slope of the $\log n - \log S$ relation is larger than 1.5. A slope of 1.5 could result, as can be seen from figure 3, if the sources with $M_r = -33$ and $M_r = -34$ would be only about 1/3 as abundant as the luminosity function (6) indicates. Sources with $M_r = -34$ would now be very rare objects. Taking the data as they are in figure 3, there should be in the whole sky one source with $M_r = -34$ at $S_{85} = 5 \times 10^{-24}$, three such sources at $S_{85} = 2.5 \times 10^{-24}$. Actually observed are Cygnus A at $1.2 \times 10^{-22}$ and 3C 295, the most distant object known at this time, at $2.5 \times 10^{-24}$. Cygnus A is in any case an unlikely, but fortunate, accident, being much nearer to us than statistical expectation. In figure 3, 3C 295 is one of three sources with $M_r = -34$ that are expected in the whole sky with a flux density of $2.5 \times 10^{-24}$; if the luminosity function drops off faster toward its bright end, so that sources with $M_r = -34$ are three times less frequent, it would be the only source in the sky to be expected at its flux density.

Finally, we may derive from the luminosity function the fraction of sources that can be identified with a given positional accuracy. Cosmological effects have

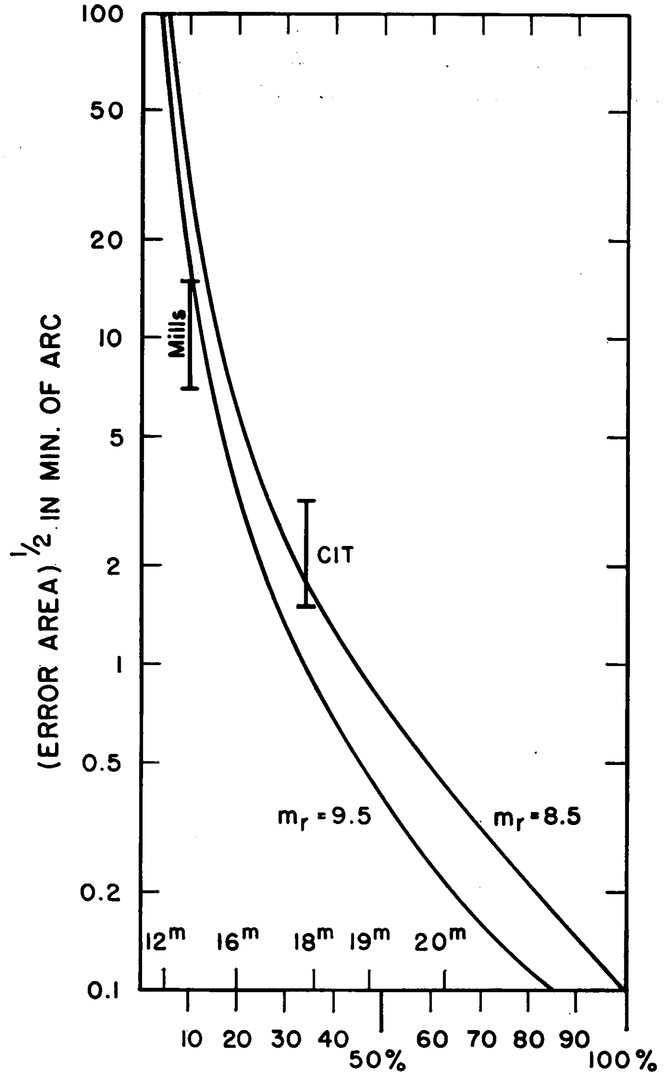

FIGURE 4

Fraction of sources that can be identified with a given positional accuracy.

been neglected; they do not affect values in the range of present accuracy. Assuming $-20.5$ for $\overline{M}_p$, we have

$$m_r = M_r + m_p + 20.5. \tag{14}$$

If this is introduced into (7), we obtain the number of sources with $m_p$ and $M_r$

$$\log n(m_p, M_r) = 5.0 + 0.6\, m_p + 0.48\, M_r. \tag{15}$$

By summing up for all values of $M_r$, we can, for a survey with a given limiting $m_r$, obtain the number of sources with photographic magnitude $m_p$, and from these numbers the fraction of sources that have photographic magnitudes brighter than $m_p$. Disregarding the fact that about half of the sources are double galaxies, we use Hubble's [4] counts of galaxies, corrected by Sandage. We have for the number of galaxies per square degree brighter than $m_p$

$$\log n(m_p) = +0.6\, m_p - 8.87. \tag{16}$$

The number of galaxies that are accidentally in the error area $\epsilon^2$ is $\epsilon^2 n(m_p)$; if we require that this number is 0.1 to establish an identification, we have

$$\log \epsilon = 4.4 - 0.3\, m_p \tag{17}$$

for the error area in square degrees needed to identify a source with a galaxy with $m_p$.

The fraction of sources with photographic magnitude brighter than $m_p$ is plotted as a function of the required error area in figure 4 for limiting values $m_r = 8.5$ and $m_r = 8.2$, corresponding respectively to the limits of table I and table II. A second scale on the abscissa gives the values of $m_p$ corresponding to each fraction. Taking as error the geometric mean of twice the probable errors of the source positions in right ascension and declination, the errors for table I range about from 7′ to 15′; the objects in table I represent an identified fraction of 10 per cent. The corresponding values for table II are an error of 1′.5 to 3′ and an identified fraction of 28 per cent. These values, plotted as vertical lines in figure 4, fit fairly well to the computed curves. If the double galaxies had been taken into account, the computed curves would shift to the right, leaving room for an incompleteness of the identification. But it is obvious that the luminosity function (6) substantiates the interpretation of the difficulty encountered in identifying sources as due to the faintness of the objects.

The author is indebted to B. Y. Mills, to J. G. Bolton, and to T. A. Matthews for making material available in advance of publication, and for advice and discussion.

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