CONTINUOUS SAMPLING PLANS

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1. Introduction

The purpose of the present paper is to review the subject of continuous sampling plans. These plans are used where production is continuous and the formation of inspection lots for lot-by-lot acceptance may be impractical or artificial, often the case for conveyor line production. The inspection is carried out by alternate sequences of consecutive item inspection (often called the 100% inspection) and sequences of production which are not inspected or from which sample items are inspected. In the plans discussed in this paper, each item inspected is classified as defective or nondefective. The theory has not yet been extended to permit continuous sampling for items that are measured on a continuous scale.

2. The Dodge plan

Perhaps the simplest continuous sampling plan is the one proposed by Dodge [1] in his pioneer paper in 1943. This procedure (called CSP-1) follows. At the outset of inspection, inspect 100% of the units consecutively as produced and continue such inspec-

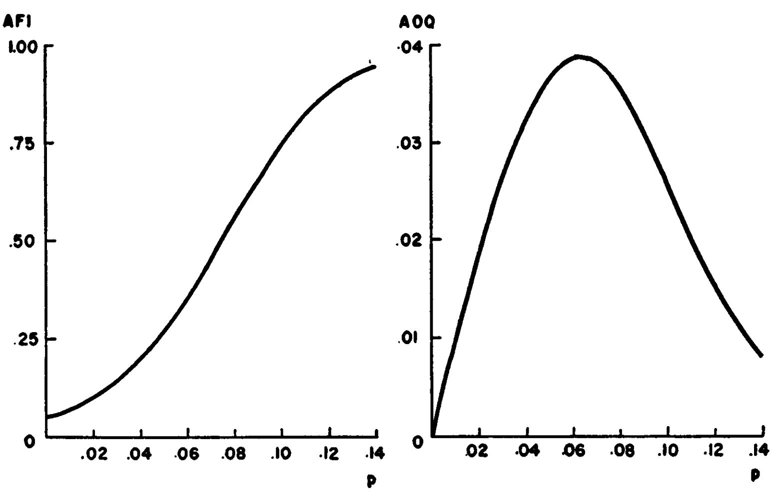

FIGURE 1
Properties of CSP-1 plan; $f = .05$, $i = 38$

tion until i units in succession are found clear of defects. When this happens discontinue 100% inspection and inspect only a fraction f of the units, selecting one unit at random from each segment of $1/f$ items. If a single defective is found, revert immediately to 100% inspection of succeeding units and continue until again i units in succession are found clear of defects. In other plans the rules for partial and 100% inspection are more complicated but the basic notion of continuous inspection may be illustrated with this simple plan. The objective of this plan is to provide assurance that the long run percentage of defective units in the accepted product will be held down to a prescribed limit, the average outgoing quality limit or AOQL. Evaluation of the statistical properties of the plan has been made under the assumption of control—qualities of the items are mutually independent binomial random variables with constant parameter p. Its statistical properties may be described by an average outgoing quality (AOQ) curve and by an average fraction inspected (AFI) curve on a sampling basis.

To obtain these curves, we represent the production process as an infinite sequence of items each of which has a probability p of being defective. A sequence $x_1, x_2, \cdots, x_m, \cdots$ represents results on successive inspection trials and can be considered a point in sample space, where $x_m = 0$ if the mth item is nondefective, and $x_m = 1$ if it is defective. Let a segment be a group of one or $1/f$ successive production items from which one item is to be chosen at random for inspection. After the inspection of any item, the size of the segment from which the next item is to be chosen for inspection is determined by past history according to the given rule. The particular sampling attaches either the integer 1 or $1/f$ to each coordinate (x_m) of the sample point. The integer attached to x_m is the number of production items in the segment from which a number is inspected with result x_m. If $y_m (y_m = 1$ or $y_m = 1/f)$ is the integer attached to x_m in the sequence $(x_1, x_2, \cdots, x_m, \cdots)$, then the reciprocal of the average fraction inspected for that sequence is

$$\lim_{n \to \infty} \frac{1}{n} \sum_{m=1}^{n} y_m \tag{1}$$

provided the limit exists.

Now define the reciprocal of the average fraction inspected (AFI) as

$$(\text{AFI})^{-1} = P_1 + f^{-1} P_{f^{-1}}, \tag{2}$$

where P_1 and $P_{f^{-1}}$ are the probabilities that for a randomly chosen m, x_m is the result of inspection of an item selected from a segment of size one and f^{-1} respectively. It can be shown that expressions (1) and (2) are equal with probability one. Thus, the AFI can be represented in terms of the steady state probabilities, P_1 and $P_{f^{-1}}$. We can easily show that

$$\text{AFI} = \frac{f}{f + (1-f)(1-p)^i}. \tag{3}$$

The average outgoing quality is

$$\text{AOQ} = p(1 - \text{AFI}). \tag{4}$$

This formula holds for any plan under the assumption of control, as the outgoing quality is unchanged (p) for uninspected sequences, and is 0 for inspected ones. The AOQL for a controlled process is defined as

$$\sup_p \text{AOQ}. \tag{5}$$

The AOQL for this plan is not easy to obtain analytically, and to help in picking particular plans, Dodge gives constant AOQL contours in the f, i plane. In addition, he gives the values of p for which the probability that 1,000 consecutive units will be accepted by partial inspection is .10 as a function of f.

There are several rather striking features of the Dodge plan which have stimulated further research in the field.

(1) Its heavy dependence on the occurrence of a single defective which may be isolated.

(2) The assumption of control.

(3) Its failure to provide a specific criterion for shutting down production if quality deteriorates sufficiently.

3. Modifications of the Dodge plan

Clearly an abrupt change between 100% inspection and partial inspection may sometimes be unnecessary. In the first place, even a production process, which is at a satisfactory quality level, will produce a certain number of defectives and eventually one of the sampled items will be defective. Further, this abrupt change may lead to hardships in personnel assignments and, hence, in the administration of an inspection program. In a very complicated and expensive item, such as an aircraft engine, this transition may require major readjustments. In a later paper published in 1951, Dodge and Torrey [2] propose two modifications of the plan (CSP-2 and CSP-3) which delay the beginning of 100% inspection and also add some protection against spotty quality. In the CSP-2 plan we start out as in the CSP-1 plan by inspecting 100% until i successive good items are found, after which partial inspection is introduced; it reverts to 100% inspection not on the basis of a single defective, but whenever two defectives occur spaced less than k units apart.

It is evident that CSP-2 differs from CSP-1 in that it delays invoking 100% inspection. Dodge and Torrey present curves for determining values of f and i for a given value of the AOQL when $k = i$, and when the process is in control. The AOQ function is always larger than the AOQ function for CSP-1 for a given f and i, and is equal to

$$\text{AOQ} = p\left[\frac{(1-f)\,q^i\,(2-q^k)}{f\,(1-q^i)\,(1-q^k)+q^i\,(2-q^k)}\right]. \tag{6}$$

Clearly the plan does carry a higher risk of accepting a short run of abnormally poor quality than the CSP-1 plan with the same AOQL. Another modification has been proposed to correct this. The CSP-3 plan is the same as the CSP-2 except that when a defective is found, the next four units are inspected. Hence, it provides protection against surges of highly defective product. The AOQ function is given by

$$\text{AOQ} = p\left[\frac{(1-f)\,q^i\,(1+q^4-q^{k+4})}{f\,(1-q^i)\,(1-q^{k+4})+q^i\,(1+q^4-q^{k+4})+4fpq^i}\right]. \tag{7}$$

A plan which allows for smooth transition between sampling and 100% inspection, which is rather conservative in requiring 100% inspection only when quality is quite inferior, and which allows for the inspection to continue to reduce when quality is definitely good, is a multilevel sampling plan. This plan allows for any number of sampling levels subject to the provisions that transitions can occur only between adjacent levels. The origin of this plan is somewhat obscure. It seems to have been used by the Air Force for the inspection of aircraft engines and was proposed by several people including Joseph Greenwood [3]. A particular multilevel plan has recently been discussed in considerable detail by Lieberman and Solomon [4]. Their plan is as follows.

As with the Dodge plan, inspect 100% of the units consecutively as produced and continue until i units in succession are found clear of defects. When i units are found clear of defects, discontinue 100% inspection and inspect only a fraction f. If the next i units inspected are nondefective, then proceed to sampling at rate f^2. If a defective is found, revert to inspection at the lowest level. This plan can be used with any number of levels but from two to six are the numbers which seem to be of practical interest. The first Dodge plan, CSP-1, is easily recognized as a special case containing only one sampling level.

Curves of constant AOQL are developed for a two-level and an infinite-level plan under the assumption of control. An interpolation method for other k's is also given.

The AOQ function can be written as

$$\text{AOQ} = p\,z\,\frac{1-z^k}{1-z^{k+1}} - \frac{f-fz}{1-fz}\,\frac{1-(fz)^k}{1-z^{k+1}} \tag{8}$$

where

$$z = \frac{1}{f}\,\frac{q^i}{1-q^i}. \tag{9}$$

In choosing a value of k to use, the authors show that if the incoming quality p is less than AOQL, the larger the number of levels used, the smaller the average fraction inspected. If p is greater than the AOQL, a single-level plan (CSP-1) minimizes the average fraction inspected.

A plan similar to this is proposed by Greenwood [3]. In his plan the only departure from the multilevel plan is that all items in the subset must be inspected before returning to the preceding sampling level when a defective occurs. Greenwood derives the AOQ function for this plan.

4. Plans which guarantee an AOQL without the assumption of control

The formulas for AOQ in the preceding section were derived on the assumption that the process is in control—a reasonable mathematical model for many processes which is rarely realized exactly. Several workers in the field have turned their attention to this problem. One approach is to examine the common procedures to see how they are affected by relaxing this condition.

The concept of AOQL must be defined for an arbitrary process. The AOQL is the smallest number L such that for every sample point (sequences of defectives or non-defectives) the probability is zero that

$$\limsup_{N\to\infty} \frac{A(N)}{N} > L \tag{10}$$

where $A(N)$ is the number of defects remaining in the segment of N items after inspection.

In 1953, Lieberman showed that the Dodge procedure guarantees an AOQL whether or not the process is in a state of statistical control. In fact, without the assumption of control, and for a given f and i,

$$\text{AOQL} = \frac{1/f-1}{1/f+i}. \tag{11}$$

Naturally this value of the AOQL is higher than the value given by the Dodge result for a fixed f and i. This is not to imply that the Dodge result is not useful. The AOQL is itself an upper bound, rarely achieved. The AOQ may be much less than the AOQL. Consequently, the AOQ for a process which is not in control may be less than the Dodge AOQL. In fact, this AOQL is achieved only for the pathological process which produces all defective items during partial inspection, and produces all nondefective items during 100% inspection. However, the result does point out that if the process behaves irregularly, the standard plans can lead to trouble.

Another approach has been to derive plans specifically designed to prescribe an AOQL. A major result in the field was obtained by Wald and Wolfowitz [6] in 1945 when they proposed a class of what might be called one-sided sequential plans. The statistical principles on which these plans are based are as follows:

Begin with partial inspection, choosing one at random from each successive group of $1/f$ items. Let $x_1, x_2, \cdots$ be the sequence of observations obtained from the first, second, etc., segment partially inspected from the beginning. ($x_i = 0$ if the item inspected in the ith segment is nondefective and $x_i = 1$ if the item is defective.) After the nth stage of partial inspection, calculate an estimated $\hat{p}$ of fraction defective in the product which has passed through the beginning of the inspection operation. This is defined as

$$\hat{p} = (1 - f)\frac{\sum_{i=1}^{n} x_i}{n}. \tag{12}$$

Continue partial inspection as long as this estimate satisfies the inequality

$$\hat{p} \leqq U + \phi(n) \tag{13}$$

where U is the desired AOQL and $\phi(n)$ is a nonnegative function of n which approaches zero as n approaches infinity. When the inequality no longer holds, inspect 100% and replace defective items by good ones until (13) begins to hold again. Eventually this is bound to occur because, in the process described, n will be increasing, but not Σx_i.

Using the strong law of large numbers for dependent random variables, it can be shown that any plan which keeps the estimate bounded also guarantees that the AOQL will not be exceeded in the long run regardless of whether or not the process is in control. Wald and Wolfowitz [6] also show that if the process is in control, plans of this class provide the AOQL with a minimum amount of inspection.

It is instructive at this point to consider a particular type of this general class of plans. Continue partial inspection as long as

$$\sum_{i=1}^{n} x_i < h + sn \tag{14}$$

and if for some n, $\sum_{i=1}^{n} x_i > h + sn$, terminate partial inspection and inspect h/s segments 100%. Repeat the procedure. Except for a slight approximation the AOQL for this plan is equal to $(1 - f)s$.

When the production process is in statistical control and if $p \leqq s$, we know from sequential theory that there exists a positive probability that partial inspection once begun will go on indefinitely. Thus, if $p \leqq s$, the probability is 1 that eventually a sequence of items will be found in which only partial inspection will be employed. On the other hand, if $p > s$, partial inspection will always terminate with probability 1. But, in this case, the average outgoing quality AOQ is given by

$$\text{AOQ} = \frac{(1-f)\, pE(n)}{E(n) + h_2/s} \tag{15}$$

and since

$$E(n) = \frac{h}{p-s} \tag{16}$$

we get

$$\text{AOQ} = (1-f)\, s = \text{AOQL}\,. \tag{17}$$

Thus, for all $p > s$, the average fraction inspected is given by

$$F(p) = 1 - \frac{\text{AOQL}}{p} \tag{18}$$

which is minimum.

While the one-sided sequential inspection plan is optimum from the point of view of cost of inspection (at least in the case of controlled production), it is not a plan which can be recommended in cases where the excessive variability of the outgoing quality in finite batches of the product is an important factor. This becomes apparent when we consider the fact that in order for any material to be inspected 100%, the point

$$\left(\sum_{i=1}^{j} x_i,\ j\right) \tag{19}$$

must reach or exceed the line $y = h + sj$ for some sample size j. But if a few defective items are found during a long stretch of partial inspection, the point can wander so far away from the line that a great deal of unacceptable material can pass by before this fact is noted and the quality of the product improved by the 100% inspection. This situation can be remedied to some extent by employing a two-sided sequential inspection plan.

A two-sided sequential inspection plan is defined by two lines

$$y = h_2 + sj \tag{20}$$

and

$$y = -h_1 + sj \tag{21}$$

where h_1 and h_2 are positive constants and j represents the number of items inspected at the jth stage of partial inspection. In this plan, partial inspection continues as long as the point $\left(\sum_{i=1}^{j} x_i,\ j\right)$ lies between the above two lines. Partial inspection terminates when, for some $j = n$, either

$$\sum_{i=1}^{n} x_i \leqq -h_1 + sn \tag{22}$$

or

$$\sum_{i=1}^{n} x_i \geqq h_2 + sn\,. \tag{23}$$

In the former case no 100% inspection is called for and the inspection procedure is simply repeated on new material. In the latter case, the inspection is resumed on new material only after h_2/s segments, that is, h_2/fs items, have been inspected 100%.

Except for the action taken, the inspection procedure described above is identical with the sequential method of lot-by-lot acceptance inspection. Consequently, all the sequential theory can be employed in studying the plan. Thus, for example, if the production is in statistical control, the AOQ curve can be computed from the formula

$$\text{AOQ} = \frac{p(1-f)E(n)}{E(n) + [1 - L(p)]\, h_2/s} \tag{24}$$

where $L(p)$ is the operating characteristic of the sequential probability ratio test and $E(n)$ is the average sample number.

In the two-sided sequential plan, the AOQ is always less than AOQL even if $p > s$ but approaches it asymptotically as p approaches 1. Thus, introducing the lower line increases somewhat the cost of inspection but usually not to an appreciable extent unless h_1 is made exceedingly small.

One other type of sequential plan was presented recently by M. A. Girshick [7]. It is defined by the three integers m, N, and f. The plan operates as follows. The units of product in the production sequence are divided into segments of size f^{-1}. Inspection begins by selecting at random one item from each consecutive segment of f^{-1} items. The items are inspected in sequence and the number of defectives found, as well as the number of items examined, are cumulated. This procedure is continued until the cumulative number of defectives reaches m. At this point, the size of the sample n is compared with the integer N. If $n \geqq N$, the product which has passed through inspection is considered acceptable and the inspection procedure is repeated on the new incoming product. If, on the other hand, $n < N$, the following actions are taken: (a) the next $N - n$ segments [$(N - n)f^{-1}$ units] are inspected 100%; and (b) after that, the inspection procedure is repeated.

This procedure always guarantees, whether the process is in control or not, that the AOQL cannot exceed $(1 - f)m/N$.

Girshick also presented various operating characteristics of this plan, under control, such as the probability of inspection terminating with acceptance ($n \geqq N$),

$$L(p) = \sum_{j=0}^{m-1} \binom{N-1}{j} p^j q^{N-1-j}, \tag{25}$$

and discussed the biased and unbiased estimates of the process average.

The AOQ curves plotted against p for this plan and the Dodge plan are presented in figure 2. Thus the Dodge plan requires more inspection than necessary for large proportion defective just to achieve the AOQL. This is one of the achievements of minimum inspection plans.

Girschick also modified this plan by introducing two sampling rates, f_1 and f_2; f_2 is used initially and f_1 is used if $n \geqq N$. If 100% inspection is ever necessary, f_2 is used again.

5. Plans which provide for termination of production

A criticism of all these continuous sampling plans, particularly those mentioned in the last section, is that they emphasize doing enough inspection to bring quality down to the AOQL but do not provide automatic penalties for poor quality. If the inspection is performed by a consumer or purchaser, he may do a very large amount of inspection to insure quality. Sometimes this may be avoided by administrative action. Dodge's [1] original paper says:

"The inspection plan is most effective in practice if it is administered in such a way as to provide an incentive to clear up causes of trouble promptly. Such an incentive may be had by imposing a penalty on the operating or manufacturing department when defects are encountered. Normally no such penalty is imposed if both the sampling inspection and the 100% inspection are performed by this same person or group of persons; then the two costs merge. The inspector then merely serves as an agency for screening defects when quality goes bad. It is accordingly recommended that sampling inspection and 100% inspection operations be treated as two separate functions."

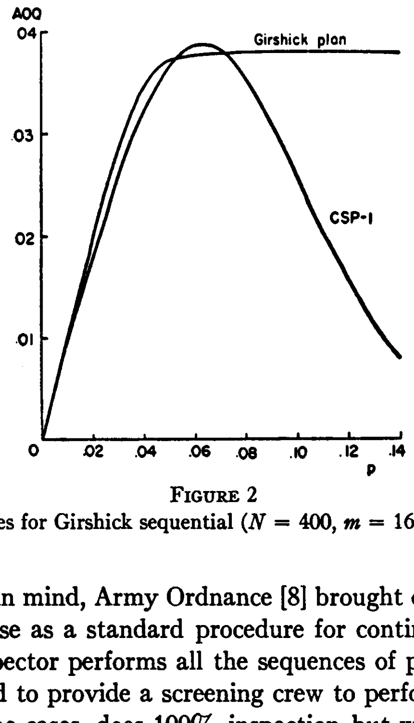

FIGURE 2

Comparison of AOQ curves for Girshick sequential ($N = 400$, $m = 16$, $f = .05$) and CSP-1 ($i = 38$, $f = .05$).

With this possibility in mind, Army Ordnance [8] brought out a very comprehensive set of Dodge plans to use as a standard procedure for continuous sampling. In these plans, the sampling inspector performs all the sequences of partial inspection and the manufacturer is required to provide a screening crew to perform the 100% inspection. The government, in some cases, does 100% inspection but will charge the contractors for it.

The sampling inspector may verify this 100% inspection by inspecting the screened material and, of course, take fairly drastic action if any errors are found.

Another attack on this problem has been made by Rosenblatt and Weingarten of Navy Ordnance [9] who modify the Dodge plan by limiting the number of 100% inspection sequences an inspector is allowed to perform. In addition to f and i, their plan specifies a maximum allowable number of sequences of consecutive item inspection beyond the first. At any time during a day's production if this number is exceeded, all inspection is stopped and all the items on the line are not accepted (although product that has passed the inspector is accepted). The inspector informs the manufacturer as to which defects have occurred and that he will not begin to reinspect product until the manufacturer locates the source of difficulty and gives him assurance that he has removed the cause.

This provision for shutting down the acceptance line seems to be desirable in a continuous sampling plan. That is, we want the plan to check on product, to do enough inspection to make quality good if there are any minor fluctuations, and to provide a basis for shutting down the line and looking for corrective action if they are major. The original Dodge plan and the Army Ordnance plan rely largely on the fact that if product is generally bad or if it deteriorates, a large amount of product will have to be reinspected. However, the Navy Ordnance plan has the advantage that specific criteria for making this decision are provided.

This same feature is in a plan proposed by Girshick and Rubin [10] who also pioneered in trying to establish a specific model for the process. According to their model, a machine which is producing items may be in one of three states:

(1) Satisfactory production: in control with fraction defective p_1.

(2) Unsatisfactory production: in control with fraction defective $p_2 > p_1$.

(3) Out of production for repair.

When machine is in state (1), there is a constant probability (g) of jumping to state (2); once it achieves state (2), the machine stays there until it is brought to repair. Under a very general cost function, the two authors show that the procedure which maximizes the long run average income per item produced is the following.

Let

$$y_n = \frac{1}{1-g}\frac{p_2}{p_1}$$

if the nth item is inspected and defective,

$$y_n = \frac{1}{1-g}\frac{1-p_2}{1-p_1}$$

if the nth item is inspected and nondefective, and

$$y_n = \frac{1}{1-p}$$

if the nth item is not inspected. Let

$$Z_n = y_n(1+Z_{n-1}), \qquad Z_0 = 0\,. \tag{26}$$

Assume that when the machine leaves the repair shop the first item is not inspected. Then for suitably chosen positive constants a^* and b^* with $b^* < a^*$, the optimum procedure states that items are not inspected as long as $Z_n < b^*$. Inspection begins as soon as $Z_n \geqq b^*$, and inspection continues until either $Z_n < b^*$ or $Z_n \geqq a^*$. In the former case production continues but inspection terminates, in the latter case inspection terminates and the machine is put in the repair shop.

It is to be noted that whenever for some n_0, $Z_{n_0} < b^*$, the number of items to be skipped is completely determined. For if k is the number of items to be skipped, then k must satisfy the equation

$$Z_{n_0+k} = \sum_{j=1}^{k}\left(\frac{1}{1-g}\right)^j + \left(\frac{1}{1-g}\right)^k, \qquad Z_{n_0} \geqq b^*\,. \tag{27}$$

Summing the above equation and solving for k yields

$$k = \left[\log\left(\frac{gb^*+1}{gZ_{n_0}+1}\right)\Big/ -\log(1-g)\right] \tag{28}$$

where the symbol $[t]$ stands for the smallest integer greater than or equal to t. The interesting fact is that the optimum rule prescribes that inspection or noninspection shall occur in batches of items.

To calculate the constants a^* and b^* presents difficult problems mathematically and administratively. They depend on such things as the income from good items, cost of inspection, loss from passing bad items, etc., which are difficult to estimate. However, every plan makes an implicit assumption about these costs and it is important that they be exhibited as clearly as possible. However, given the parameters of the cost functions, the calculation of a^* and b^* still presents technical difficulties.

A computable plan which has some of the properties of the Girshick-Rubin plan is a modified sequential plan recently introduced by I. R. Savage. Under the proposed plan one of three decisions is made after each item is produced. These decisions are

(1) Stop the production process and attempt to improve outgoing quality.

(2) Produce another item and inspect it.

(3) Produce and accept K more items.

The proposed plan is determined by four positive constants, h_1, h_2, K, and $s < 1$.

When production starts, inspect 100% and count the number of defects (d_n) in the first n items.

If $d_n \geqq h_2 + sn$, make decision (1). If $-h_1 + sn < d_n < h_2 + sn$, make decision (2). If $d_n \geqq -h_1 + sn$, make decision (3).

After either actions (1) or (3), start all over with 100% inspection.

The operating characteristics of this sequential procedure are given by Wald's approximate formulas. The probability of accepting a sequence of K is

$$L_p = \frac{X^{h_1+h_2} - X^{h_1}}{X^{h_1+h_2} - X} \quad \text{when} \quad p = \frac{X^s - 1}{X - 1}, \qquad 0 < X < p, \tag{29}$$

and the expected sample size to reach decisions is

$$\bar{n}_p = \frac{L_p(h_1 + h_2) - h_2}{s - p}. \tag{30}$$

The average outgoing quality is

$$\text{AOQ} = \frac{pKL_p}{L_pK + (1 - s)\bar{n}_p + L_p h_1 - (1 - L_p)h_2}. \tag{31}$$

Several alternative procedures to the above have been treated. In particular, the procedure has been modified so that items found to be defective are replaced with good items as in the previous mentioned plans.

Another modification that has been treated is to replace decision (3) by the following:

(3*) Produce and accept the next $K - 1$ items. Then inspect the Kth item. If it is good, accept the next $K - 1$ items and inspect the Kth. Continue until a defective item is found and then start 100% inspection. That is, instead of starting over after deciding quality is good, the plan requires partial inspection.

6. Desirable directions for research

Intuitively, it appears that there are several desirable statistical properties which continuous sampling plans should have:

(1) The plan should not depend heavily on the assumption of control.

(2) The plan should provide for terminating inspection and shutting down the line when the quality deteriorates sufficiently.

(3) The plan should provide some attention against spotty quality, that is, the probability of passing a segment of a given size for unsatisfactory quality should be kept small.

(4) As quality deteriorates, the plan should require only enough inspection to make the AOQ approach the AOQL.

A given plan will not have all these properties and further study is needed to spell out the appropriate area of application of each plan. What is needed most is a well-defined model and information on economic factors.

In the existing procedures the choice of some of the parameters is quite arbitrary, for example, the choice of the sampling rate f for Dodge procedures. The model of control is not very meaningful since this implies that the probability of obtaining a defective item is always a constant value p. If this were true, one would estimate the process average. If the estimate was sufficiently small, no further inspection would be performed. If the estimate of the process average was too large, the process would be rejected. In either case inspections would cease. More meaningful models should be formulated, and optimum procedures derived, or properties of existing procedures determined.

All existing continuous sampling plans are based upon attributes. Just as lot-by-lot sampling inspection of variables can play an important role in lot-by-lot inspection, continuous sampling by variables may be important in the field of continuous sampling.

Even with the existing models there remain many unanswered questions. All the plans presented guarantee an AOQL. Even this concept is subject to criticism. What does long run quality really mean operationally? As John Maynard Keynes comments, "In the long run we are all dead."

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