AN APPROACH TO THE DYNAMICS OF STELLAR SYSTEMS

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The theory of stellar systems begins with the study of our own Galaxy and the approach must at first be mainly of a descriptive kind. The study of the distribution of stars in space and of the distribution of stellar velocities forms a part of astronomy which is named stellar astronomy, or in a more restricted sense, stellar statistics, and serves to describe the properties of our galactic system. The first great pioneer in this field on an empirical basis was William Herschel and, after him, pioneer work was carried out by F. G. W. Struve, Gyldén, Seeliger and others.

A modern era in the study of stellar motions may be said to begin with Kapteyn's discovery of the two star streams, and with the subsequent development of the mathematical statistical methods of describing stellar motions. Schwarzschild introduced the theory of the velocity ellipsoid, which was later developed in a more general way by Charlier. Attempts to develop a dynamical theory of stellar systems on the basis of the ellipsoidal velocity function were made by Eddington, Schwarzschild and Jeans.

A revolution in our ideas concerning the dimensions of our stellar system occurred by Shapley's investigations of the distribution of the globular clusters. These studies definitely expanded the domain of our galactic system wide over the limits of the system devised by Kapteyn, and showed that our Sun is situated very far from the center of the Galaxy. The direction of this center, in the rich region of the Milky Way in Sagittarius, was also clearly indicated.

A corresponding revolution in the domain of stellar motions occurred in the study of large stellar motions relative to the Sun. If we consider physical groups of increasing internal velocity dispersion, the mean motion relative to the Sun increases in a direction which lies in the galactic plane at right angles to the direction towards the center of the Galaxy. This is the phenomenon which has been called the asymmetrical drift of large stellar velocities. The writer showed that this phenomenon, which had been studied in detail by Strömberg, could be interpreted in terms of a general motion of rotation of the system. The stellar system may be divided up into "subsystems" of different angular motion of rotation and different internal velocity dispersion. The maximum angular speed occurs at a vanishing internal velocity dispersion, and is equal to the angular speed of the circular orbits in the galactic plane. This corresponds nearly to the state of motion of the clouds in the Milky Way. With decreasing motion of rotation the internal velocity dispersion increases and the distribution in space assumes a less flattened formation. The globular clusters represent a subsystem of very small angular speed of rotation and of a nearly spherical distribution in space.

Parallel with the exploration of our own Galaxy, the last decades have produced an enormously increased knowledge concerning the external galaxies. In Hubble's system of classification the series of spheroidal galaxies of increasing general flattening ("elliptical nebulae") is followed by a series of types showing spiral structure of increasing width. The spiral types are divided into two parallel series, the "ordinary" spirals in which the central parts have approximately symmetry of rotation, and the "barred" spirals, in which there appears a concentration of matter towards

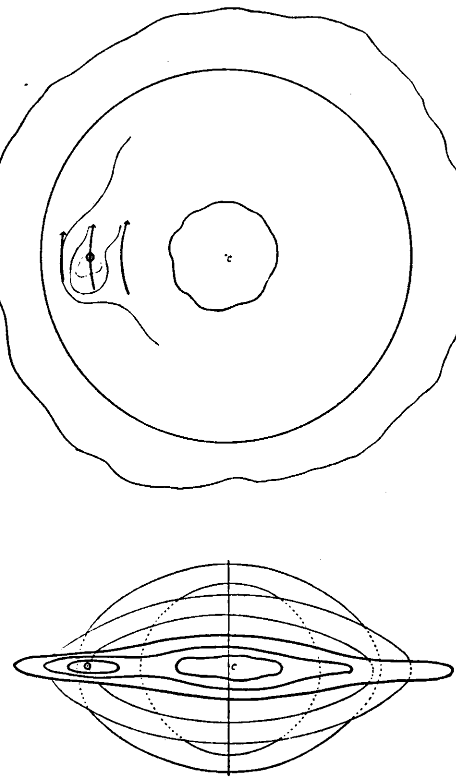

FIGURE 1
The rotating subsystems in the Galaxy

a certain diameter of the system. A theory of the spiral structure must be able to explain the appearance of these two series. Among later developments of great importance for problems concerning the evolution of the galaxies, and for stellar evolution in general, is Baade's segregation of two population groups of different physical characteristics among the stars, type I appearing preferably in the spiral structure, and type II in a more smoothed formation of smaller concentration towards the equatorial plane, but with strong concentration towards the nucleus.

In the study of the orbital motions in a stellar system like our own Galaxy we may start with the assumption that the potential of gravitation has symmetry of rotation. We introduce a rotating coordinate system ξ, η, ζ following the circular motion at a certain distance from the center. The relative orbits in this coordinate system are a kind of epicyclic orbits of the type shown in figure 2.

The character of the orbits depends on six arbitrary parameters, which appear as constants of integration. If we write the frequency distribution in ξ, η, ζ and in the velocities ξ', η', ζ',

$$f(\xi, \eta, \zeta, \xi', \eta', \zeta', t)\, d\xi d\eta d\zeta d\xi' d\eta' d\zeta' ,$$

we know by Liouville's theorem that the function f must be a function of these parameters. Three of these parameters contain the time t explicitly. In a steady

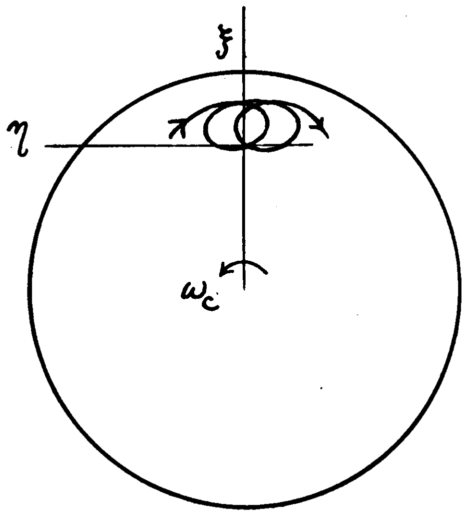

FIGURE 2

Relative orbit in a rotating coordinate system following a circular orbit

state f must therefore be a function of the remaining three parameters. On this basis we may build up a theory of the ellipsoidal distribution of stellar velocities in our neighborhood. In a steady state the velocity ellipse in the galactic plane will have its long axis pointing towards the center of the system and the ratio of the two axes has a certain relation to the properties of the potential field, which connects the velocity ellipsoid in an important way with the phenomenon of differential rotation discovered by Oort.

The spectrographic investigations of galaxies indicate that in the "main body" of the system, including the principal part of the spiral structure, the angular speed of rotation is nearly uniform. I have therefore introduced the term "quasi-spheroidal" stellar system to define a system in which the angular speed ω_c of the circular orbits is nearly constant. In this case the relative "epicyclic" orbits in ξ, η are to the first approximation circles in which the particle moves with the angular speed $2\omega_c$ in the retrograde direction. At a given point, the density, the mean motion, and the dispersion of the velocities, will be determined by the amount of matter carried by the relative orbits intersecting at the point in question. Considering the stars of mass between m and $m + dm$, we may write the frequency along an orbit

$$\psi_m dm d\xi d\eta d\zeta d\xi' d\eta' d\zeta' .$$

Figure 3 gives the projection on the ξ, η-plane of a certain orbit passing through the origin O. C is the center of the stellar system. The center of the projected orbit

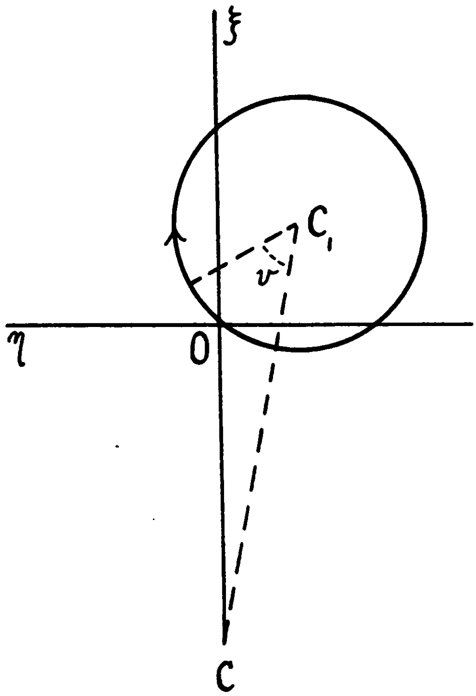

FIGURE 3

Relative orbit in a quasispheroidal system

is C_1, and we define an angle v following the motion about C_1, which has the angular speed $2\omega_c$. The frequency ψ_m may be developed in a Fourier series

$$\psi_m = c_0 + \sum_{n=1}^{\infty} c_n \cos n (v - 2\omega_c t - v_n) ,$$

t being chosen from an arbitrary epoch. In a steady state we assume that the variations cancel out when we consider the sum of matter in all the different orbits passing through O. This is an ideal case, and in a natural system we must of course have a very complicated state of variation, which, as long as we neglect the disturbing potential, will have the frequencies

$$\sigma = n \cdot 2\omega_c , \qquad n = 1, 2, 3, \ldots .$$

When we take into account the disturbing potential of condensations and rarefactions in the medium, we must reconsider the motions of the particles. We may write for the density function

$$\rho = \rho_0 + \rho_1$$

where ρ_0 corresponds to a steady state of motion, and ρ_1 is the deviation from this state. For the gravitation potential φ we assume correspondingly

$$\varphi = \varphi_0 + \varphi_1 .$$

We are in the first place interested in *large condensations*, of dimensions comparable with the system itself, and we assume that the variation in potential may be resolved into a sum of harmonic variations of the type

$$\varphi_1 = \chi\,(\xi, \eta, \zeta)\, e^{i\sigma t} .$$

We assume the condensation to be large compared with the relative orbits of the particles, and that in the vicinity of O we may disregard variations of the phase of the oscillation, so that we may assume χ to be a real function. With a suitable choice of epoch for t we may then write

$$\varphi_1 = \chi\,(\xi, \eta, \zeta) \cos \sigma t .$$

For ξ, η, ζ small we may then develop χ as follows

$$\chi = a_0 + a_1\xi + a_2\eta + a_3\zeta + \tfrac{1}{2}a_4\xi^2 + \tfrac{1}{2}a_5\eta^2 + \tfrac{1}{2}a_6\zeta^2 + a_7\xi\eta + a_8\xi\zeta + a_9\eta\zeta + \dots .$$

By Poisson's equation

$$\nabla^2\varphi_1 = -4\pi G\rho_1$$

we have at the origin O

$$(a_4 + a_5 + a_6) \cos \sigma t = -4\pi G\rho_1 .$$

The equations of motion are to the first order

$$\left.\begin{aligned} \frac{d^2\xi}{dt^2} - 2\omega_c \frac{d\eta}{dt} - 4\omega_c A_c \xi &= \frac{\partial\varphi_1}{\partial\xi} \\ \frac{d^2\eta}{dt^2} + 2\omega_c \frac{d\xi}{dt} &= \frac{\partial\varphi_1}{\partial\eta} \\ \frac{d^2\zeta}{dt^2} + k^2\zeta &= \frac{\partial\varphi_1}{\partial\zeta} \end{aligned}\right\}$$

where

$$\omega_c^2 r = -\frac{\partial\varphi_0}{\partial r}, \qquad A_c = \frac{1}{4\omega_c}\left(\omega_c^2 + \frac{\partial^2\varphi_0}{\partial r^2}\right), \qquad k^2 = -\frac{\partial^2\varphi_0}{\partial\zeta^2},$$

and where r is the projection of the radius vector from the center of the system C on the ξ, η-plane.

In the case of a quasispheroidal system we assume $A_c = 0$, and may then turn the coordinate system so that $a_7 = 0$. We assume $a_8 = 0$, $a_9 = 0$, and further that terms of higher order may be neglected.

We consider a cylinder of unit cross section with its axis parallel to the axis of rotation. The amount of matter in such a cylinder may be ϑ, and we write

$$\vartheta = \vartheta_0 + \vartheta_1 .$$

We have then by the law of continuity

$$\vartheta_1 + \vartheta_0 \left(\frac{\partial \Delta \xi}{\partial \xi} + \frac{\partial \Delta \eta}{\partial \eta} \right) = 0 ,$$

where $\Delta\xi$ and $\Delta\eta$ are the disturbances in ξ and η according to the equations above. After deriving the expressions for $\Delta\xi$ and $\Delta\eta$, this gives as final result

$$\vartheta_1 = -\vartheta_0 \frac{a_4 + a_5}{4\omega_c^2 - \sigma^2} \cos \sigma t .$$

In a steady state the velocity dispersion in the ξ- and η-directions are assumed to be of equal amount α, and the dispersion in the ζ-direction to be γ. In the varied state the dispersions may be α_1, α_2, γ_1. We find

$$\frac{\alpha_1 \alpha_2}{\alpha^2} = 1 - \frac{a_4 + a_5}{4\omega_c^2 - \sigma^2} \cos \sigma t ,$$

and thus we have

$$\frac{\alpha_1 \alpha_2}{\alpha^2} = \frac{\vartheta}{\vartheta_0} .$$

An analogous relation between density variation and velocity dispersion is valid at O for the motion along the ζ-axis, and in the special case when

$$\alpha = \gamma , \qquad \alpha_1 = \alpha_2 = \gamma_1 ,$$

we have

$$\frac{\alpha_1^2}{\alpha^2} = \left(\frac{\rho}{\rho_0} \right)^{2/3} .$$

The relations derived have therefore a certain analogy with the conditions of *adiabatic variations* in a monatomic gas. This is remarkable, because in our case we disregard entirely the effects of encounters or mutual disturbances between individual stars.

By taking into consideration Poisson's equation in the form given above we may derive approximately the frequency σ as follows

$$\sigma^2 = 4\omega_c^2 - 4\pi G \bar{\rho}_0 \frac{a_4 + a_5}{a_4 + a_5 + a_6} .$$

$\bar{\rho}_0$ is a mean of ρ_0 defined by

$$\bar{\rho}_0 = \frac{1}{\vartheta_0} \int_{-\infty}^{+\infty} \rho_0^2 d\zeta .$$

For a deeper analysis of the fluctuations it is necessary to make use of the general equations of mass motion. We employ a rectangular coordinate system x, y, z, rotating with the angular velocity ω about the z-axis. The density function is $\rho\ (x, y, z)$ and the potential of gravitation $\varphi\ (x, y, z)$. The mean motion at the point x, y, z is characterized by the vector ν with the components u, v, w along the axes x, y, z. The components of velocity of individual particles relative to the mean motion are U, V, W. We then have the general equation of mass motion

$$\frac{\partial \nu}{\partial t} + \nu \cdot \nabla \nu = K - \frac{1}{\rho} \operatorname{div} \Pi ,$$

and the equation of continuity

$$\frac{\partial \rho}{\partial t} + \nu \cdot \operatorname{grad} \rho + \rho \operatorname{div} \nu = 0 .$$

K is a vector with the components

$$2\omega v + \omega^2 x + \frac{\partial \varphi}{\partial x}, \qquad -2\omega u + \omega^2 y + \frac{\partial \varphi}{\partial y}, \qquad \frac{\partial \varphi}{\partial z},$$

and Π is the dispersion tensor

$$\begin{matrix} \rho \overline{U^2} & \rho \overline{UV} & \rho \overline{UW} \\ \rho \overline{UV} & \rho \overline{V^2} & \rho \overline{VW} \\ \rho \overline{UW} & \rho \overline{VW} & \rho \overline{W^2} . \end{matrix}$$

We compare the actual system with an ideal system in equilibrium, in which the density and potential functions and the components of mean motion are

$$\rho_0 (x, y, z),\ \varphi_0 (x, y, z),\ u_0,\ v_0,\ w_0 ,$$

and further

$$\Omega_0 = \varphi_0 + \tfrac{1}{2}\omega^2 (x^2 + y^2) .$$

In the actual case we assume $w_0 = 0$, and that u_0, v_0 are small quantities. The dispersion tensor in the steady state is assumed to be characterized by a generalized velocity ellipsoid with the axes

$$\alpha, \beta, \gamma ,$$

thus

$$\overline{U_0^2} = \frac{1}{r^2}(\alpha^2 x^2 + \beta^2 y^2), \qquad \overline{UV}_0 = (\alpha^2 - \beta^2)\frac{xy}{r^2}, \qquad \overline{V_0^2} = \frac{1}{r^2}(\beta^2 x^2 + \alpha^2 y^2),$$

$$\overline{W_0^2} = \gamma^2, \qquad \overline{UW}_0 = 0, \qquad \overline{VW}_0 = 0 .$$

In considering the *interstellar gas* we may take the mean motion and mean density over fairly large regions and include the turbulent velocities in the dispersion tensor. Moreover, as we are considering "large" fluctuations, we must take into account that with increasing size of the fluctuations the mass forces are likely to prevail over the frictional forces.

For the actual system we have

$$\rho = \rho_0 + \rho_1, \qquad \varphi = \varphi_0 + \varphi_1, \qquad \Omega = \Omega_0 + \Omega_1$$

$$u = u_0 + u_1, \qquad v = v_0 + v_1, \qquad w = w_0 + w_1$$

$$\overline{U^2} = \overline{U_0^2} + \Delta\overline{U^2}, \qquad \overline{V^2} = \overline{V_0^2} + \Delta\overline{V^2}, \qquad \overline{UV} = \overline{UV}_0 + \Delta\overline{UV} .$$

We define here u_1, v_1, w_1, ρ_1, φ_1, Ω_1, when following an element in its motion, so that we ascribe to the elements certain displacements Δx, Δy, Δz.

As before we define $\vartheta = \vartheta_0 + \vartheta_1$ to be the mass in a cylinder of unit cross section parallel to the z-axis, and define

$$\eta = \frac{\vartheta_1}{\vartheta_0} .$$

We further define the mean values

$$\bar{u}_1 = \frac{1}{\vartheta_0}\int_{-\infty}^{+\infty} u_1 \rho_0 d z , \qquad \bar{v}_1 = \frac{1}{\vartheta_0}\int_{-\infty}^{+\infty} v_1 \rho_0 d z .$$

We may now apply the "adiabatic" theorem derived above, ignoring the difference between α_1 and α_2, and assuming

$$\Delta \overline{U^2} = \Delta \overline{V^2} = \alpha^2 \eta , \qquad \Delta \overline{U V} = 0 .$$

The equations for the two dimensional motion in x and y may then be written

$$\left.\begin{aligned} \frac{d\bar{u}_1}{dt} - 2\omega \bar{v}_1 &= \frac{\partial \bar{\Omega}_1}{\partial x} - 2\alpha^2 \frac{\partial \eta}{\partial x} - \eta \alpha^2 \frac{\partial \lg \vartheta_0}{\partial x} - \eta \frac{\partial \alpha^2}{\partial x} , \\ \frac{d\bar{v}_1}{dt} + 2\omega \bar{u}_1 &= \frac{\partial \bar{\Omega}_1}{\partial y} - 2\alpha^2 \frac{\partial \eta}{\partial y} - \eta \alpha^2 \frac{\partial \lg \vartheta_0}{\partial y} - \eta \frac{\partial \alpha^2}{\partial y} , \\ \frac{d\eta}{dt} + \frac{\partial \bar{u}_1}{\partial x} + \frac{\partial \bar{v}_1}{\partial y} &= 0 . \end{aligned}\right\}$$

We define a function f such that

$$\nabla_1^2 \Omega_1 = \frac{\partial^2 \Omega_1}{\partial x^2} + \frac{\partial^2 \Omega_1}{\partial y^2} = -4\pi G f \rho_1 .$$

Coutrez has shown that the function f has the property

$$\nabla_1^2 \rho_1 = \nabla^2 (f \rho_1) ,$$

which may serve to estimate f when ρ_1 is given as a function of x, y, z.

We apply the equations above to a "*standard stellar system*," for which we assume

$$\alpha^2 = \alpha_0^2 \left(1 - \frac{r^2}{a^2}\right), \qquad \vartheta_0 = \vartheta_0^{(0)} \left(1 - \frac{r^2}{a^2}\right)^{1/2} .$$

The velocity dispersion is thus assumed to vanish at a certain limiting radius $r = a$, and the quantity ϑ_0 varies in the same way as for a homogeneous spheroid. If the polar coordinates in the equatorial plane are r, θ, and if η is assumed to be a harmonic variation, thus

$$\eta \sim e^{i(\sigma t + s\theta)} ,$$

we find for the amplitude function $\eta(r)$ the equation,

$$\left(1 - \frac{r^2}{a^2}\right)\left(\frac{\partial^2 \eta}{\partial r^2} + \frac{1}{r}\frac{\partial \eta}{\partial r} - \frac{s^2}{r^2}\eta\right) - \frac{7}{2a^2} r \frac{\partial \eta}{\partial r} + \eta \left(\frac{\sigma^2 - 4\omega_1^2}{2\alpha_0^2} - \frac{\omega s}{\sigma a^2} - \frac{3}{a^2}\right) = 0 ,$$

where

$$\omega_1^2 = \omega^2 - 4\pi G \overline{f \rho_0} .$$

Setting

$$\frac{(\sigma^2 - 4\omega_1^2)\, a^2}{2\alpha_0^2} - \frac{s\omega}{\sigma} + s^2 = n^2 - \tfrac{1}{2} n ,$$

a solution which is finite for $r = 0$ is

$$\eta = A_s \left(\frac{r}{a}\right)^s F\left(\alpha, \beta, \gamma, \frac{r^2}{a^2}\right) e^{i(\sigma t + s\theta)} ,$$

where F is the hypergeometric series with

$$\alpha = \tfrac{1}{2}(n + s + \tfrac{3}{2}), \qquad \beta = \tfrac{1}{2}(2 + s - n), \qquad \gamma = s + 1 .$$

F is not convergent, and in order that the series shall terminate, we must have

$$n = s + 2j ,$$

where j is a positive integer. The most important mode for a given s is $j = 1$, in which case

$$\eta = A_s \left(\frac{r}{a}\right)^s e^{i(\sigma t + s\theta)} .$$

The frequency σ is determined by an equation

$$\sigma^3 + q\sigma + r = 0 .$$

This equation will have a pair of complex roots, if

$$\frac{\omega_1^2}{\omega^2} < \tfrac{1}{4}\,[3\, s^{2/3} x^{4/3} - (6 + 7\,s)\, x^2]$$

where

$$x = \frac{a_0}{\omega a} .$$

In this case there will exist a wave of type

$$\eta = A_s \left(\frac{r}{a}\right)^s e^{\gamma t} \cos(\sigma_0 t + s\,\theta + \epsilon)$$

where $\gamma > 0$. The amplitude of the variation will therefore increase exponentially with the time. The angular speed of the wave is

$$\omega - \frac{\sigma_0}{s} .$$

We have $\sigma_0 < 0$, and an important fact is that at the point of instability a wave of this type follows closely the angular speed ω_c of the circular orbits in the system.

From the properties of a "standard stellar system" the condition of instability may be investigated quantitatively for systems of different degrees of flattening. In figure 4 the abscissa is the ratio of the effective axes c and a of the system and the ordinate a quantity $\Delta(\omega_1/\omega)^2$ which is positive in the case of instability and negative in the case of stability. The curves drawn represent the cases $s = 1, 2, 3$. It is evident that with increasing flattening of the system, when we move from right to left in the diagram, instability occurs first for $s = 1$. This represents a simple asymmetry of the system, which may often be traced in nature. Instability occurs next for $s = 2$, which corresponds well to the appearance of a "barred" structure, as it means an increase of density along a certain diameter. For higher values of s instability will occur at higher degrees of flattening. These modes of variation will have a greater number of maxima and minima in θ, and the amplitude will further increase very considerably towards the edge of the system. On account of their greater complication they will be less readily identified in nature. The most important case is thus $s = 2$. The distribution of ϑ at a certain stage of development

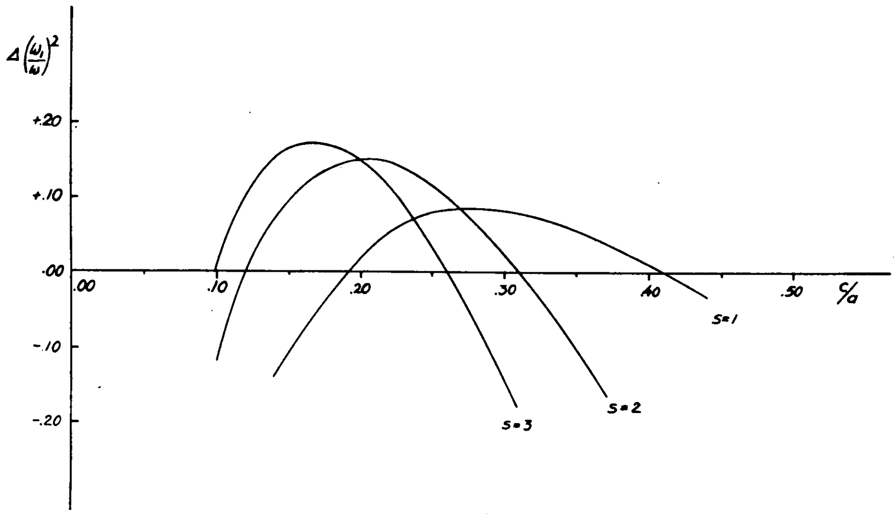

FIGURE 4

The condition of instability for different degrees of flattening and for different values of s

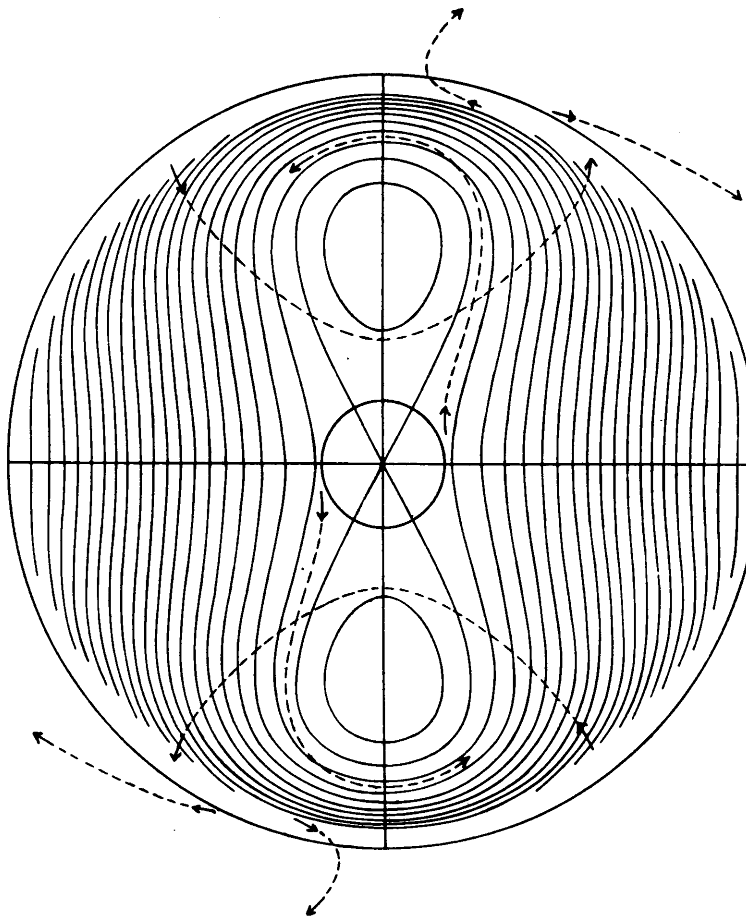

FIGURE 5

The distribution of density in the presence of a wave $s = 2$

of the wave $s = 2$ is shown in figure 5, and illustrates the "theoretical bar."

A deviation from rotational symmetry like that of the theoretical bar, which follows closely the circular orbits in the system, has a very considerable disturbing action on particles which originally follow circular motions. These disturbances are of such a nature that they appear to explain essential features of the spiral structure in the galaxies.

If the bar grows slowly, the resulting displacement of a particle following originally a circular motion will occur at right angles to the disturbing force (figure 6). The direction of displacement is obtained, if we turn the disturbing force 90°

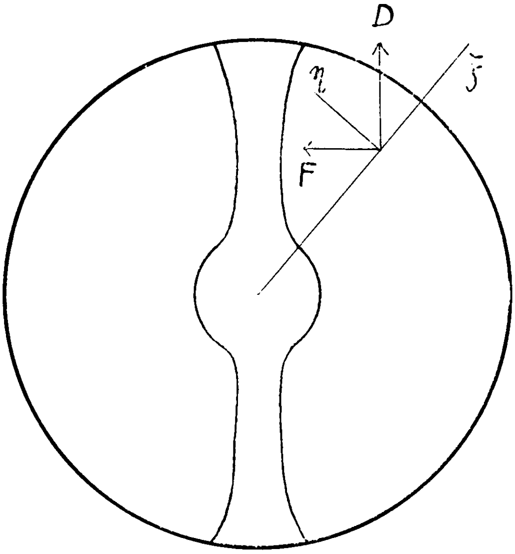

FIGURE 6

The relation between the disturbing force F and the displacement D

against the direction of rotation. In fact, if the disturbing force at a certain point of the ξ, η-system is

$$\frac{\partial \varphi_1}{\partial \xi} = \chi_1 e^{\gamma t}, \qquad \frac{\partial \varphi_1}{\partial \eta} = \chi_2 e^{\gamma t},$$

where χ_1 and χ_2 are considered as constant, we get for $A_c = 0$ and γ small the displacements

$$\Delta \xi = \frac{\chi_2}{2\omega_c \gamma} e^{\gamma t}, \qquad \Delta \eta = -\frac{\chi_1}{2\omega_c \gamma} e^{\gamma t}.$$

We have assumed here $\Delta\xi = 0$, $\Delta\eta = 0$ for $t = -\tau$, where τ is large. In the quadrants following the bar in the rotation the motion will be mainly outwards, and in the preceding quadrants inwards. Some possible tracks of motion are shown by the dotted curves in figure 5.

In a great many cases it is evident that the system has consisted originally of a wide central lens with a ring of matter, presumably containing matter of Baade's type I, at the periphery. When the bar forms and contracts, this ring will be broken up in a very characteristic way. If the bar is pictured vertical, one side of the nebula will move upwards and the other downwards. This "breaking of rings" is a very characteristic feature in the barred spirals, as shown in the typical barred nebula NGC 1300 (figure 7).

The detached matter is likely to proceed out into regions where the density gradient of the remaining matter of the system is very steep, and where, therefore,

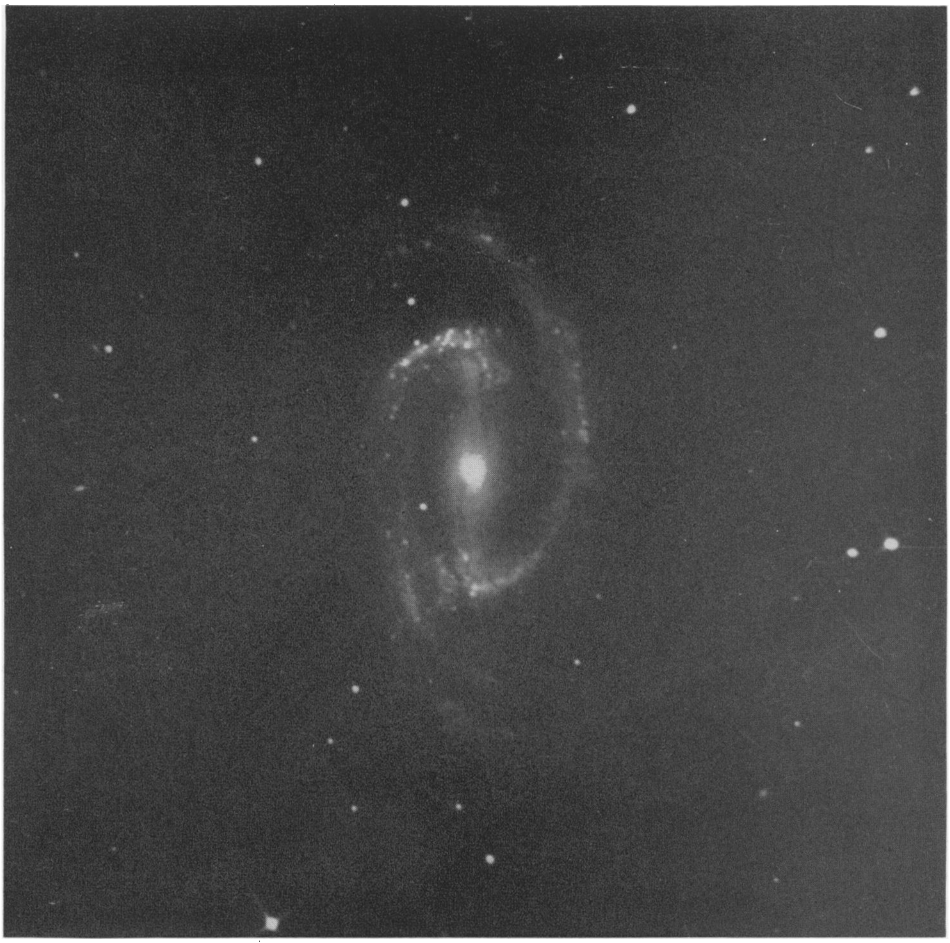

FIGURE 7

The barred spiral nebula NGC 1300. (McDonald Observatory)

circular motions will be unstable. The orbits of individual particles will then be of the "asymptotic" class, extending widely from the original system. The detached matter will, therefore, be dissolved into a fanlike formation, which will propagate a disturbance along the edge of the system against the direction of rotation. The disturbance will "peel off" the outermost matter, and this process will continue, until the peeled off "arm" meets the detached matter on the other side of the nebulae. This disturbance, which will combine with the direct disturbance on the ring, may be called "the edge effect."

In many cases there will be a region about the center in which $A_c > 0$, and the angular velocity of the circular orbits increases towards the center. If our coordinate system ξ, η, ζ follows the angular motion of the outer regions, the circular orbits of the innermost regions will show a certain circular relative motion. We assume this motion to be slow, and we assume further that the bar has contracted to a fairly narrow formation as we often observe in nature. The disturbance will be a downward motion on one side of the bar, and an upward motion on the other side, in both cases in the direction of the relative motion. The result will be a spiral motion which resembles the motion of a charged particle in a cyclotron under the disturbance of the changing electric field. The inner part of a type I population will, therefore, have a tendency to arrange itself in a spiral pattern, which will combine with the spiral pattern of the outer regions. All the three types of

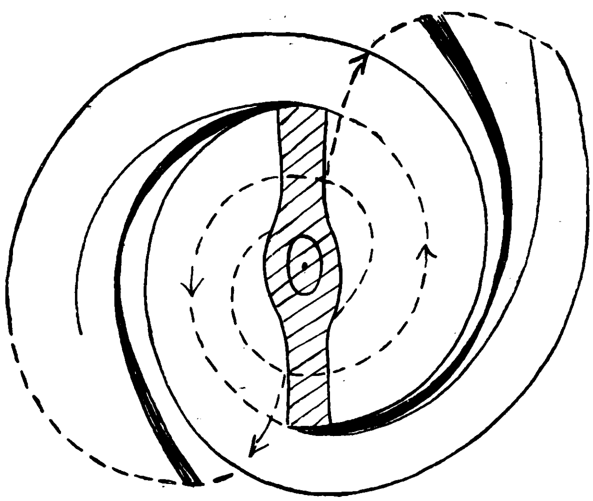

FIGURE 8

Theoretical picture of a barred spiral

disturbance discussed above have been combined in the schematic picture of a barred spiral nebula in figure 8. In the comparison with objects in nature it is of importance that the inner relative motions are in many cases very well revealed by clouds of dark matter, which often form extended dark lanes along the paths of relative motion.

The ordinary type of spiral structure may in many cases be explained by analogy with the barred spirals as systems in which the wave $s = 2$ has produced a dissymmetry of the gravitational field, but without producing a well marked bar. The breaking of rings, the edge effect, and the spiral motion of inner particles will still be present and produce the spiral structure. Especially the "edge effect," causing a wide extension of spiral arms, and proceeding from the outer regions inwards, is likely to be active here. A great number of nebulae show actually a state of transition between the barred and ordinary type. A beautiful example among the large nebulae is Messier 83 (figure 9). The great Andromeda nebula Messier 31 appears to show traces of a bar (figure 10), and the run of the spiral arms, when reconstructed as they would appear if the nebular plane were at right angles to the line of sight, shows many analogies with the barred spirals. Messier 81 is a nebula in which the edge effect prevails, probably produced originally by a density variation of the type $s = 2$, which may still be traced.

In some cases, as for instance the nebula Messier 51, the dissymmetry of the gravitational field is produced by the tidal action of a companion. If the flattening

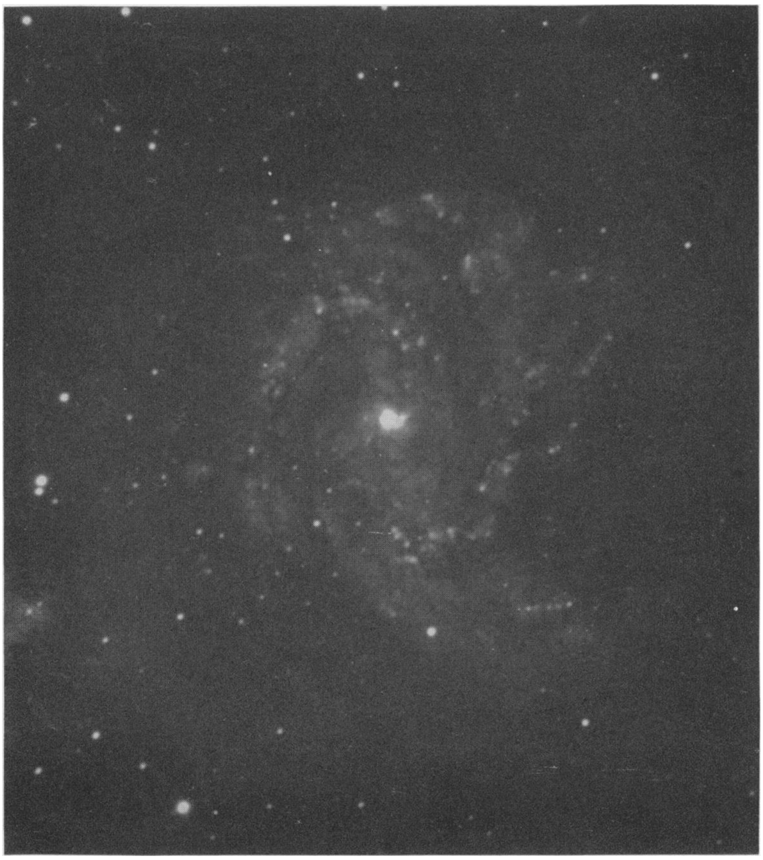

FIGURE 9

The spiral nebula Messier 83. (Mount Wilson Observatory)

of the system increases, the deformation by the tidal effect of the companion may reach a critical stage, after which the disturbing potential field of the deformation itself dominates. We may then expect very strong effects analogous to the breaking of rings and the edge effect in the barred spirals. The spiral structure may ultimately penetrate more and more towards the nucleus.

An investigation of the time scale of the spiral phenomenon shows that the spiral structure is likely to have developed during a few revolutions of the system, so that the present theory appears to be entirely in accordance with the "short" time scale. The part played by a process of increasing flattening in producing the necessary conditions for an instability, leading to the formation of spiral structure, connects the dynamics of stellar systems with the problems of stellar evolution. The process in question may well be a transition of a system from a "prestellar" to a "stellar" state. A condensation of "primitive," highly turbulent gas into clouds of dust, and into stars, will be a process of condensation and dissipation of internal kinetic energy, and must involve an increasing degree of flattening of the system,

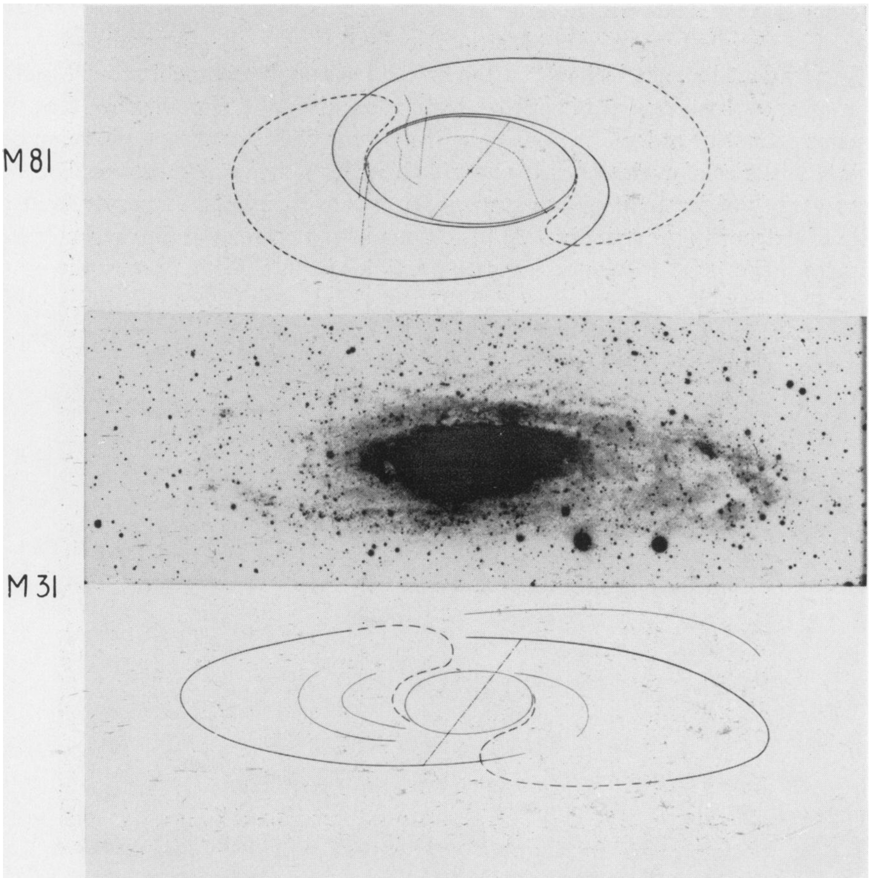

FIGURE 10

Above: Theoretical picture of Messier 81. The fine dotted curves indicate lanes of dark matter.
Below: Diameter of maximum intensity in Messier 31. The dashed curves indicate clouds of dark matter. Only the principal parts of the spiral structure are indicated.

if the system has at least a moderate angular momentum. If the angular momentum surpasses a certain minimum value, the equilibrium form may become quasi-spheroidal in the way assumed here. A state in which the angular velocity ω_c of the circular orbits remains approximately the same out to an effective limit $r = a$, where the velocity dispersion becomes very small, and outside of which the density decreases rapidly, will be much favored by the presence of gas and dust in the outer regions of the system. It is possible that the stars of type II prevailing in the inner regions are considerably older than the stars of type I in the outer regions, which may be largely of a more recent origin.

In the course of the further development the system will enter into a phase in which it is unstable against the density waves of low s, which may lead directly to the formation of a system of the barred type. The bar may not become apparent, if the "edge effect" prevails. This is likely to take place in systems of very considerable angular momentum. The ordinary type of spiral structure should therefore prevail in the late types of Hubble's classification, as actually observed.

It is evident to what high extent the amount of angular momentum will decide the course of development of a stellar system, and it is therefore evident that the systems cannot be ordered into a single line of evolution, but that a great number of parallel series of evolution will exist side by side. This fact tends to explain the great variety of form among the systems. It is of great interest, however, that in spite of extremely great individual variations in structure, certain characteristic features make possible a very general grouping of the objects, and enable us to trace analogies between groups which at first glance seem morphologically quite different. It is on these common features the attention has to be fixed in an approach to a dynamical theory of the systems.