PROCEEDINGS OF THE
SECOND BERKELEY SYMPOSIUM

PROCEEDINGS *of the* SECOND BERKELEY SYMPOSIUM ON MATHEMATICAL STATISTICS AND PROBABILITY

Held at the Statistical Laboratory
Department of Mathematics
University of California
July 31–August 12, 1950

EDITED BY JERZY NEYMAN

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PREFACE

THE SECOND BERKELEY SYMPOSIUM on Mathematical Statistics and probability was held during the fortnight July 31 to August 12, 1950. The organizational work began early in November, 1949, when the President of the University of California made the basic grant to the Statistical Laboratory to finance the Symposium. At that time only a limited program in mathematical statistics and in probability was contemplated. Later, the generous support of the Office of Naval Research permitted a substantial enlargement of the original plans and also made possible the participation of several scholars from abroad. Gradually, the number of persons and institutions interested in the Symposium grew and the final program included not only sessions of pure mathematical statistics and probability but also several sessions on various fields of application: astronomy, biometry, econometrics, physics, traffic engineering, and wave analysis. Several of these sessions were organized with the financial support of the Dean of the College of Letters and Science. The sessions given to problems of engineering were made possible by the Dean of the College of Engineering through the Institute of Transportation and Traffic Engineering and through the Navy Wave Project under the Institute of Engineering Research.

Sincere thanks are due all these persons and institutions without whose generous support the Second Berkeley Symposium could not have developed into a scholarly event of such imposing magnitude.

The undersigned was away from Berkeley and did not participate in the organizational work during the all important months February through June, 1950, when many of the problems were most pressing. The credit for the successful solution of these problems is due to my colleagues in the Statistical Laboratory, to Professor Michel Loève and to Drs. Edward W. Barankin, Evelyn Fix, Joseph L. Hodges, Jr., Erich L. Lehmann, and Elizabeth L. Scott. Outside of the Statistical Laboratory, substantial organizational work was done by Professors Donald S. Berry and Donald G. Malcolm who organized sessions on statistical problems in engineering.

Hearty thanks are due the authorities of the several learned societies who agreed to support the Symposium by organizing in Berkeley their own Regional West Coast meetings in a program coördinate with that of the Symposium. The societies which met simultaneously with the Symposium are the Biometrics Section of the American Statistical Association, the Biometric Society, Western North American Region, the Econometric Society, and the Institute of Mathematical Statistics.

For a symposium to be successful, it is necessary that its proceedings be published and, furthermore, that they be published soon. It is most sincerely regretted that various unavoidable difficulties delayed the publication of the Proceedings of the First Berkeley Symposium for more than three years. In connection with the present Symposium, strenuous efforts were made to arrange that the voluminous Proceedings would appear within the shortest possible time. On this section of the organizational work, hearty thanks are due the Editorial Committee of the Academic Senate of the University of California for the academic year 1950–1951 for speedy action and for a very substantial sum to help cover the cost of publication.

There was a period in the autumn of 1950 when it was feared that publication of the Proceedings would be seriously delayed or, even, prove to be impossible. The reason was that the estimated cost of printing was so large that it was doubtful if funds could be found to cover it. The Statistical Laboratory, and indeed all the participants of the Symposium are indebted to Mr. August Frugé, Acting Head of the Publishing Department of the University of California Press, for finding ways and means to reduce the estimate substantially, thus making the publication possible without further delay.

Although listed as Editor of the Proceedings, the undersigned does not really deserve this credit. The credit for the editorial work on the Proceedings is due to Drs. Evelyn Fix and Elizabeth L. Scott who, aided by Miss Mildred Genelly, Mrs. Jeanne Lovasich, and Mrs. Eloise Putz spent uncounted hours trying to satisfy, on the one hand, the rigid requirements of uniformity imposed by the University of California Press and, on the other, the natural desire of each author to preserve his own style. Both of these problems had to be solved, although they appeared contradictory. However, my colleagues tried to do their best, remembering the axiom that delay in publication is its worst enemy.

The material printed in this volume consists of papers delivered to the Laboratory before the end of August, 1950, and it is regretted that some of the authors were not able to meet this deadline. Papers delivered later will be published in due course in the Statistical Series of the University of California.

The papers published in the Proceedings are grouped as follows: first, papers on theory of statistics proper, next papers on probability and, finally, papers on various fields of application.

It is a shock to all those close to the Symposium that, during the few months between the time it was held and the reading of the proofs of the Proceedings, two of the participants of the Symposium should have lost their lives in accidents.

In December, 1950, the statistical world suffered the irreparable loss of a brilliant leader and cordial friend. While on a lecture tour in India, Dr. and Mrs. Abraham Wald perished in an airplane accident. Professor Wald's activity in the field of mathematical statistics proper was very brief. His first paper in the *Annals of Mathematical Statistics* appeared in 1939, barely a dozen years ago. Yet he has left behind him an indelible imprint on the thinking of generations that will follow. In fact, the fruits of Wald's work, especially sequential analysis and general theory of multiple decision functions, are with us to stay and it would hardly be an exaggeration to say that they cover the theory of statistics in the modern sense of the word. The influence of Wald on contemporary work may be judged briefly by perusing the statistical papers in this book. In these circumstances it seemed appropriate to make Wald's paper the first in the volume.

The second death occurred in March, 1951, when Dr. H. R. Seiwell perished in an automobile accident. Essentially an oceanographer, Dr. Seiwell was prominent in applying statistical techniques to the most difficult problem of studying ocean waves. His loss will be very painfully felt both among oceanographers and statisticians.

The First Berkeley Symposium took place during the enthusiastic period of re-

turn to peaceful activities after the conclusion of World War II. The atmosphere of the Second Symposium was marred by heavy clouds on the political horizon, threatening a new outburst of hostilities. It is to be hoped that the spirit of international coöperation will prevail and that the gathering storm will subside. With this paramount hope, we look forward to a renewal of the present occasion and to the Third Berkeley Symposium within a few years.

J. NEYMAN
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