Preface

The Seventh MSJ International Research Institute of the Mathematical Society of Japan was held in Tokyo for ten days from June 3rd to 12th, 1998. The theme was 'Class Field Theory – its Centenary and Prospect', which is taken as the title of the book. The program of this international conference is attached at the end of the preface. This volume is a collection of articles contributed by the speakers of the conference. All but a few of them are full scale papers. Some of them are expository on those subjects which are of central issues of algebraic number theory, and are prepared by the leading experts; they contains important and interesting problems with extensive references. Some of them are historical, and vividly explain how number theorists were motivated and exchanged their mathematical ideas.

In 1920 Takagi published the complete version of his class field theory as 'Ueber eine Theorie des relativ Abel'schen Zahlkörpers' in J. Coll. Sci. Tokyo, vol. 41. Chapter V of it is devoted to an affirmative solution to 'Kronecker's youth-dream'. This problem asks, roughly speaking, whether *all* abelian extensions of an imaginary quadratic number field could be obtained by the singular moduli and special values of elliptic functions which have complex multiplication by the elements of the quadratic field; and it was reformulated in a general frame-work by Hilbert as the twelfth problem of his celebrated 23 problems. There is another problem behind class field theory: that is, the principal ideal theorem which was finally proved by Furtwängler in 1930 based on Artin's general reciprocity law. Artin established this significant result in his short paper, 'Beweis des allgemeinen Reziprozitätsgesetzes', in Abh. Math. Sem. Univ. Hamburg, vol. 5 (1927). Like the former problem, the origin of the latter goes back to Kronecker. We may say, however, that the Takagi-Artin class field theory has its direct origins in several papers by Weber and by Hilbert published in 1897–1898:

H. Weber, Ueber Zahlengruppen in algebraischen Körpern I, Math. Ann. 48 (1897), 433–473; II, 49 (1897), 83–100; III, 50 (1898), 1–26;

D. Hilbert, Die Theorie der algebraischen Zahlkörper, Jber. Deutschen Math.-Ver.4 (1897), 175–546; Über die Theorie der relativ-Abelschen Zahlkörper, Nachr. Akad.Wiss. Göttingen 5 (1898), 377–399;

(cf. e.g. K. Miyake, The Establishment of the Takagi-Artin Class Field Theory, in *The Intersection of History and Mathematics* (ed. J. W.

Dauben et al), Birkhäuser Verlag, Basel-Boston-Berlin, 1994, pp.109–128). This is one of the reasons why the editor of this volume chose the above stated theme of the Seventh MSJ International Research Institute in 1998 when he was appointed as organizer.

After the marvelous results of Takagi and Artin, the next act of the drama of class field theory was played by H. Hasse. In this energetic performance, he established his reciprocity law in the form which beautifully embodied the *Local–Global Principle* in class field theory (cf. e.g. G. Frei's article in this volume). Under his influence F. K. Schmidt could demonstrate local class field theory in 1930 based on the global theory. Then Chevalley gave an 'arithmetic proof' to class field theory, and also introduced idèles (cf. e.g. C. Chevalley, La théorie du corps de classes, Ann. Math. 41 (1940), 394–418). And finally in 1950, K. Iwasawa and J. Tate independently developed functional analysis over idele groups of algebraic number fields to present zeta- and L-functions with functional equations in an elegant manner. Here ideles became a natural and basic concept of algebraic number theory. It should also be mentioned that A. Weil gave an important impetus by introducing the ring of adèles in his lecture notes, Adèles and algebraic groups, at Princeton in 1959.

In 1951 and 1952, G. Hochschild, T. Nakayama and J. Tate successfully gave a cohomological description to class field theory. In his lectures at Princeton in 1951–52, E. Artin treated class field theory with this method. It enabled him to give an axiomatic presentation, 'class formation'. Then extensive studies were carried out by Y. Kawada with the partial help of I. Satake.

It would be too much for me to give even a brief sketch of the development of the theory of automorphic functions and forms. I would like, however, to point out just two significant works. Mainly motivated by Hilbert's twelfth problem, E. Hecke developed his works on 'Hilbert modular functions' of two variables in his two papers published in Math. Ann. 71 (1912) and *ibid.* 74 (1913); the title of the second paper is 'Über die Konstruktionen relativ-Abelscher Zahlkörper durch Modulfunktionen von zwei Variabeln'. Here he deals with SL_2 over the ring of integers of a real quadratic number field which discontinuously acts on a direct product of two copies of the complex upper half plane. The other work I point out here is one of C. L. Siegel's works, Symplectic Geometry, Amer. J. Math. 65 (1943). In this paper he worked on Sp_n over the rational number field and established the analytic theory of moduli of (polarized) Abelian varieties. There followed extensive works on automorphic functions and forms in many variables with respect to semi-simple and reductive algebraic groups over algebraic number fields. Weil's lecture notes referred above should also be mentioned here.

Towards the end of the 1960's there appeared three important works by K. Iwasawa, by G. Shimura and by R. P. Langlands. Concerning the first one and the third, we have splendid articles by R. Greenberg and by B. Casselman, respectively, in this book. Let me just point out one of Shimura's works here:

G. Shimura, On canonical models of arithmetic quotients of bounded symmetric domains, Ann. Math. 91 (1970), 144–222; II, *ibid.* 92 (1970), 528–558.

This paper represented a modern summit in the language of adele geometry of which Kronecker once dreamed and then Hilbert anticipated with his twelfth problem.

On the history of Hilbert's twelfth problem, N. Schappacher gave two interesting lectures in our conference as you will see from the program at the end of the preface. However, we could not include his article in this book. For those who are interested in it, I refer to his article, On the history of Hilbert's twelfth problem: a comedy of errors, Matériaux pour l'histoire des mathématiques au XX^{e} siècle (Nice, 1996), 243–273, Sémin. Congr., 3, Soc. Math. France, Paris, 1998. In our conference John Coates gave two inspiring lectures on Iwasawa theory of elliptic curves for graduate students and young mathematicians at the request of the organizers. Those who are interested in the subject are recommended to see his article, Fragments of the GL_2 Iwasawa theory of elliptic curves without complex multiplication, Arithmetic theory of elliptic curves (Cetraro, 1997), 1–50, Lecture Notes in Math., 1716, Springer, Berlin, 1999.

I have mentioned only a strictly limited number of important works on those topics which are closely related with class field theory. It is, of course, apparent that this list is far from sufficient. I certainly failed to mention such important topics as class field theory in positive characteristic, higher dimensional class field theory, and so on. (As for the first theme, we are happy to include an article of P. Roquette in this volume.) I just hope this humble review of mine may help younger number theorists who might be interested in this volume.

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Editor

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