Preface to the Present Volume

The papers in this volume have been submitted by participants of the Symposium on Homotopy theory and related topics, held at Kyoto University from December 4, 1984 until December 8, 1984, and supported by the Ministry of Education. There were twenty five invited speakers and nearly one hundred other participants. The members of the Program Committee would like to thank them for their participation, and the referees of the submitted papers for their kind cooperation.

The papers are divided into the following four parts:

I. Simple homotopy theory and G-actions.

II. Classifying spaces and characteristic classes.

III. Topology of manifolds.

IV. Homotopy problems—unstable and stable cases.

At the top of each part, except the last one, we present expository lectures of S. Araki, H. Toda and M. Kato, rearranged for the convenience of the readers. The top of the last part is a posthumous work of S. Oka, who met a sudden and premature death two months before the Symposium.

Here, we introduce some aspects of the papers.

The topics of Part I are related to so called "Whitehead torsion" and its equivariant version which are indispensable in discussing the topology of non-simply connected manifolds and complexes. S. Araki's lecture is a rearrangement of S. Illman's equivariant Whitehead group $\mathrm{Wh}_G(X)$ of finite G-CW-complexes, where G is a compact Lie group, by making use of restricted G-Whitehead groups with respect to families of closed subgroups of G. One merit of this approach is his Theorem 9.3, which is essential in the paper "Equivariant s-cobordism theorems" mentioned below. The paper of K. Kawakubo is a discussion on G-homotopy equivalences $f: M_1 \to M_2$ between G-manifolds M_1, M_2. He gives necessary and sufficient conditions for f to be a tangential G-homotopy equivalence, and to be a simple G-homotopy equivalence and also counterexamples to an equivariant h-cobordism theorem and an equivariant s-cobordism theorem. Araki and Kawakubo will publish a paper titled "Equivariant s-cobordism theorems" based on these two works, and their works are closely related to the next paper of Matumoto-Shiota. The paper of T. Matumoto and M. Shiota is a solution of the unique G-triangulation of G-manifold. They prove that any differentiable G-manifold M is equivariantly diffeomorphic to a real analytic G-manifold, and then there exist a unique triangulation of the orbit space M/G and a unique G-triangulation of M in an appropriate sense. Their results imply that any compact differentiable G-manifold has a well-defined simple G-homotopy type. M. Masuda gives a fibre homotopy equivalence with the Kervaire invariant one over the complex projective n-space for even n, after his construction of Kervaire invariant.

The topic of Part II is the characteristic classes, or the elements of the cohomology of classifying space, basic structure invariants of fiberings and manifolds. The first half of H. Toda's lecture is a historical note of classifying spaces and their cohomology, and the last half is devoted to determine the cohomology ring of the classifying space for some projective classical groups from a new point of view. The classifying space is defined for the topological groups, the associative H-spaces and for the generalized cohomology theories. The paper of M. Harada and A. Kono is the determination of the cohomology of the classifying spaces for the 3-connective cover of the simple Lie groups. K. Shibata gives a construction of the classifying space for the rational homotopy theory. A. Matsui and H. Sato give a Riemann-Roch formula for an embedding of Euler spaces in terms of the Stiefel-Whitney class of the normal block bundle. S. Morita computes the rational cohomology and the integral cohomology modulo 2 and 3 torsions of $\mathrm{BDiff}_+ T^2$, the classifying space for the group of all orientation preserving diffeomorphisms of the 2-dimensional torus T^2.

Part III consists of topics on manifolds. Kato's lecture is concerned with the classification of locally smooth proper transformation groups (G, M) on a connected paracompact n-manifold M such that $(X, b)=G\backslash M$, where (X, b) is Thurston's n-orbifold, a pair of the orbit space X and a function $b\colon X\to N$, associating with each orbit the order of isotropy subgroup. His basic problem is the converse version of "Uniformization problem": given an (abstract) n-orbifold (X, b), under what condition on the n-orbifold does there exist a transformation group (G, M) such that $G\backslash M=(X, b)$? He introduces reflection orbifolds and rotation orbifolds corresponding to (topological) reflection groups and rotation groups. As an application of a theorem of himself he proves Thurston's conjecture, which states that a bad (not uniformizable) 3-orbifold should contain some bad 2-suborbifolds, for reflection n-orbifolds generalizing the dimension in $n\neq 4, 5$. He also constructs some rotation n-orbifolds for $n\geqq 4$ as counterexamples to the bad orbifold conjecture. T. Yoshida proves that Gromov's 3-dimensional bounded cohomology $H_b^3(S; \boldsymbol{R})$ is infinitely generated for the closed surface S of genus $\geqq 2$. A. Kawauchi considers a compact oriented topological $2m$-manifold W with boundary M. With respect to an infinite cyclic covering of W, he defines signature invariants parametrized by $a \in [-1, 1]$, and computes the signatures of M in terms of that of W. M. Sakuma proves that any amphicheiral knot has no (fix point) free periods (orientation preserving periodic homeomorphism).

Part IV consists of various homotopy problems divided into unstable cases and stable cases. Unstable homotopy theory handles homotopy classes between spaces. Through suspension limit stable homotopy classes are considered, and the stable homotopy theory is closely related to the generalized (co)homology theory.

In his posthumous work, S. Oka generalizes Adams invariant to the homotopy class of a map between spaces which have the same K-groups as spheres. In particular, he considers three cell complexes which are obtained as subquotients of the exceptional group G_2 or H-spaces $G_{2,b}$. He applies the generalized Adams invariant to compute the homotopy classes of several cell complexes, and his final result is that if f: $G_{2,b} \to G_{2,b}$ is an H-map and a homotopy equivalence then f is homotopic to the identity. N. Oda determines the orders of the generators of the 18-stem group $\pi_{n+18}(S^n)$ for $n=10$, 11, 12, which were introduced by Toda but the precise orders were remained in question. H. Ōshima computes several Whitehead products in Stiefel manifold and Samelson products in classical groups. For example, if $n+1$ is not a power of 2, the Whitehead product $[i, i]$ of the class of the natural inclusion i: $S^{d(n+1)-1} \subset O(n+k)/O(n)$ is non-zero for every $k \geqq 1$. M. Mimura and N. Sawashita compute the p-Sylow subgroup of the group $\mathscr{E}(X)$ of self homotopy equivalences for the cases that X is a 2-cell complex or a sphere-bundle over sphere.

G. Nishida determines the stable homotopy group $\{BZ_2^n, BZ_2^m\}$ where BZ_2^k is the classifying space of elementary abelian 2-group Z_2^k. Stably BZ_2^n contains some copies of $M(n)=e_n BZ_2^n$ as a direct summand, where $e_n \in Z_2 GL_n(F_2)$ is the Steinberg idempotent. There is an equivalence of spectra $M(n) \simeq L(n) \vee L(n-1)$ for $L(n)=\Sigma^{-n} Sp^{2^n} S^0 / Sp^{2^{n-1}} S^0$. He shows that the Steinberg idempotent e_n is decomposed as $e_n = a_n + b_n$ in the bigger rings and a_n, b_n are primitive in $\{BZ_2^n, BZ_2^n\}$, and determines the ring structure of $\{M(n), M(m)\}$ and $\{L(n), L(m)\}$. Note that Kuhn showed a split exact sequence $\cdots \to L(n) \to L(n-1) \to \cdots \to L(0) = S^0$ which extends the Kahn-Priddy theorem. J. Mukai gives a characterization of the Kahn-Priddy map ϕ_{2n+1}: $E^{2n+1} P^{2n} \to S^{2n+1}$ by the equality $H(\phi_{2n+1}) = E^{2n+1} p_{2n}$, where H is the Hopf homomorphism and p_m: $P^m \to S^m$ is the canonical map.

T. Kobayashi considers the generalized quaternion group H_m of order 2^{m+2}, $m\geqq 2$, naturally embedded in $S^3=Sp(1)$, the action of H_m on S^{4n+3} and the quotients $N^n(m)=S^{4n+3}/H_m$ and $N_k^{n+k}(m)=N^{n+k}(m)/N^{k-1}(m)$. He proves that if $N_j^{n+j}(m)$ and $N_k^{n+k}(m)$ are of the same stable homotopy type then $j\equiv k \bmod 2^{2n-2}$. M. C. Crabb, K. H. Knapp and K. Morisugi consider the stable Hurewicz map h: $\pi_{4n}^s(HP_m^\infty)\to H_{4n}(HP_m^\infty; Z)$ and the order $\varphi_{n,m}$ of attaching map of the top cell in the space HP_m^n, where $HP_m^n=HP^n/HP^{m-1}$ is the stunted quaternion projective space. They prove that the lower bound given by e-invariants of $\varphi_{n,m}$ coincides with the order $h_{n,m}$ of Coker h. N. Yagita studies $\rho(h, k)=\text{Image}\,\rho(h^*(k)\to k^*(k))$ for maps ρ; $h\to k$ of spectra, in particular $\rho(BP, K(Z.3))$, and applies the results to determine $BP^*(G)$ for the exceptional Lie groups G.

November 16, 1985

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