and those obtained by adding to each of them the four classes of the first column. In this way, the following table is obtained:

0	ab	$a'b'$	$ab + a'b'$
ab'	a	b'	$a + b'$
$a'b$	b	a'	$a' + b$
$ab' + a'b$	$a + b$	$a' + b'$	1

By construction, each class of this table is the sum of those at the head of its row and of its column, and, by the data of the problem, it is equal to each of those in the same column. Thus we have 64 different consequences for any equality in the universe of discourse of 2 letters. They comprise 16 identities (obtained by equating each class to itself) and 16 forms of the given equality, obtained by equating the classes which correspond in each row to the classes which are known to be equal to them, namely

$$0 = ab' + a'b, \quad ab = a + b, \quad a'b' = a' + b', \quad ab + a'b' = 1$$

$$a = b, \qquad b' = a', \qquad ab' = a'b, \qquad a + b' = a' + b.$$

Each of these 8 equalities counts for two, according as it is considered as a determination of one or the other of its members.

51. Table of Causes.—The same table may serve to represent all the causes of the same equality in accordance with the following theorem:

When the consequences of an equality $N = 0$ are expressed in the form of determinations of any class U, the causes of this equality are deduced from the consequences of the *opposite* equality, $N = 1$, put in the same form, by changing U to U' in one of the two members.

For we know that the consequences of the equality $N = 0$ have the form

$$U = (N' + X)\ U + N Y U',$$

and that the causes of the same equality have the form

$$U = N' X U + (N + Y)\ U'.$$

Now, if we change U into U' in one of the members o this last formula, it becomes

$$U = (N + X')\,U + N'Y'U',$$

and the accents of X and Y can be suppressed since these letters represent indeterminate classes. But then we have the formula of the consequences of the equality $N' = 0$ or $N = 1$.

This theorem being established, let us construct, for instance, the table of causes of the equality $a = b$. This will be the table of the consequences of the opposite equality $a = b'$, for the first is equivalent to

$$ab' + a'b = 0,$$

and the second to

$$(ab + a'b' = 0) = (ab' + a''b = 1).$$

0	ab'	$a'b$	$ab' + a'b$
ab	a	b	$a + b$
$a'b'$	b'	a'	$a' + b'$
$ab + a'b'$	$a + b'$	$a' + b$	1

To derive the causes of the equality $a = b$ from this table instead of the consequences of the opposite equality $a = b'$, it is sufficient to equate the *negative* of each class to each of the classes in the same column. Examples are:

$$a' + b' = 0, \qquad a' + b' = a'b', \qquad a' + b' = ab + a'b',$$

$$a' + b = a, \qquad a' + b = b', \qquad a' + b = a + b'; \ldots.$$

Among the 64 causes of the equality under consideration there are 16 absurdities (consisting in equating each class of the table to its negative); and 16 forms of the equality (the same, of course, as in the table of consequences, for two equivalent equalities are at the same time both cause and consequence of each other).

It will be noted that the table of causes differs from the table of consequences only in the fact that it is symmetrical to the other table with respect to the principal diagonal

(0, 1); hence they can be made identical by substituting the word "row" for the word "column" in the foregoing statement. And, indeed, since the rule of the consequences concerns only classes of the same column, we are at liberty so to arrange the classes in each column on the rows that the rule of the causes will be verified by the classes in the same row.

It will be noted, moreover, that, by the method of construction adopted for this table, the classes which are the negatives of each other occupy positions symmetrical with respect to the center of the table. For this result, the subclasses of the class N' (the logical whole of the given equality or the logical zero of the opposite equality) must be placed in the first row in their natural order from 0 to N'; then, in each division, must be placed the sum of the classes at the head of its row and column.

With this precaution, we may sum up the two rules in the following practical statement:

To obtain every consequence of the given equality (to which the table relates) it is sufficient to equate each class to every class in the same column; and, to obtain every cause, it is sufficient to equate each class to every class in the row occupied by its symmetrical class.

It is clear that the table relating to the equality $N = 0$ can also serve for the opposite equality $N = 1$, on condition that the words "row" and "column" in the foregoing statement be interchanged.

Of course the construction of the table relating to a given equality is useful and profitable only when we wish to enumerate all the consequences or the causes of this equality. If we desire only one particular consequence or cause relating to this or that class of the discourse, we make use of one of the formulas given above.

52. The Number of Possible Assertions.—If we regard logical functions and equations as developed with respect to *all* the letters, we can calculate the number of assertions or different problems that may be formulated about n simple