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Spaces of Cauchy-Riemann Manifolds

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§1. Introduction.

This work deals with the embeddability problem for three dimensional, compact, strongly pseudoconvex Cauchy-Riemann (CR) manifolds. Such a CR manifold is given by a compact manifold M without boundary, $\dim M = 3$; a rank two subbundle $H \subset TM$; and an endomorphism $J : H \to H$ that satisfies $J^2 = -\mathrm{id}$. (All manifolds, bundles etc. in this paper are C^∞ smooth.) Strong pseudoconvexity means that for any nonzero local section X of H the vector field $[X, JX]$ is transverse to H; or, equivalently, H defines a contact structure on M. By declaring the frame $X, JX, [JX, X]$ positively oriented, M acquires a canonical orientation.

A C^1 function $f : M \to \mathbb{C}$ is CR if it satisfies the tangential Cauchy-Riemann equations

$$Xf + iJXf = 0, \qquad X \in H. \tag{1.1}$$

A central problem of the theory is to understand how many solutions (1.1) has; in particular, if there are sufficiently many C^∞ solutions $f_1, \ldots, f_k$ to give rise to a smooth embedding $f = (f_j) : M \to \mathbb{C}^k$ into some Euclidean space. If this is so, the CR manifold (M, H, J) is called embeddable. In contrast with the higher dimensional case (see [3]) there may be very few CR functions on a three dimensional CR manifold; in fact, typically, the only CR functions are the constants, see [4,8,10,20,21].

We would like to describe the space of all (three dimensional, compact, strongly pseudoconvex) CR manifolds (M, H, J); the subspace of embeddable manifolds; and also to understand how many non isomorphic embeddable CR manifolds there are. Here two CR manifolds

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(M, H, J), (M', H', J') are isomorphic if there is a diffeomorphism $\Phi : M \to M'$ such that $\Phi_* H = H'$ and $\Phi_* J = J'$.

The above classification problems have two components. One is to classify all contact manifolds (M, H). This is a problem of differential topology, and we shall not consider it here. Instead, we shall deal with the other component of classification: given a contact manifold (M, H) describe the space of (embeddable) CR structures J on it. Thus, given a compact three dimensional (M, H), let $\mathcal{S}_0 = \mathcal{S}_0(M, H)$ denote the set of smooth CR structures $J : H \to H$ on it. It is easy to endow this set with the structure of a smooth Fréchet manifold (in the sense of [9,19]). Further, let $\mathcal{B}_0 \subset \mathcal{S}_0$ denote the set of embeddable CR structures, and $\mathcal{M}_0 = \mathcal{B}_0/\sim$ the moduli space of CR structures on M, where $J \sim J'$ if (M, H, J) and (M, H, J') are isomorphic. The Fréchet-Lie group Cont of contact diffeomorphisms of (M, H) acts on $\mathcal{B}_0$, and two CR structures are isomorphic if they are on the same Cont orbit; thus $\mathcal{M}_0 = \mathcal{B}_0/\text{Cont}$.

The problem of describing the moduli space $\mathcal{M}_0$ is complicated by the circumstance that the action of Cont is not free: this is due to the fact that most CR manifolds have no CR automorphisms other than the identity while some, such as the standard sphere in $\mathbb{C}^2$, have. As a result, even if $\mathcal{B}_0$ turns out to be a smooth submanifold of $\mathcal{S}_0$, one will expect $\mathcal{M}_0$ to have complicated singularities. To get around this, we will endow our CR manifolds with a *marking*, a device comparable to passing from the moduli space of Riemann surfaces to the Teichmüller space. It is quite likely that for different CR manifolds different types of marking will be convenient; the markings we will introduce work very well for CR manifolds that are close to the simplest CR manifold, the sphere in $\mathbb{C}^2$. Thus, a marking μ will consist of an ordered pair of distinct points $p_1, p_2 \in M$ and vectors $v_i \in T_{p_i}M$ transverse to H_{p_i}. We will also require that v_i point to the *positive side* of H_{p_i}, i.e., X, JX, v_2 should be a positively oriented frame for nonzero $X \in H_{p_i}$. Given (M, H), we let $\mathcal{S} = \mathcal{S}(M, H)$ denote the Fréchet manifold of pairs (J, μ), where J is a CR structure on (M, H) and μ is a marking; and $\mathcal{B} = \mathcal{B}(M, H) \subset \mathcal{S}$ the subset corresponding to embeddable structures. As contact diffeomorphisms act on markings, we have an action of Cont on $\mathcal{B}$, and we denote $\mathcal{M} = \mathcal{M}(M, H) = \mathcal{B}/\text{Cont}$. The spaces $\mathcal{S}$, $\mathcal{B}$, $\mathcal{M}$ are not very different from $\mathcal{S}_0$, $\mathcal{B}_0$, $\mathcal{M}_0$. Indeed, the mappings $\mathcal{S} \to \mathcal{S}_0$ etc. obtained by forgetting the marking are surjective and have finite (twelve) dimensional fibers. On the other hand, as we shall see, sometimes Cont acts on $\mathcal{B}$ freely, and this means that the structure of $\mathcal{M}$ is easier to describe than that of $\mathcal{M}_0$.

One can in general conjecture that $\mathcal{B}$ (resp $\mathcal{B}_0$) is a closed subset of $\mathcal{S}$ (resp $\mathcal{S}_0$), and even an analytic subset. Further, when $\mathcal{B}$ is a submanifold, $\mathcal{M}$ should also be a manifold. At present there are some results which point in this direction, that we will survey below; but overall, the conjectures are very much open.

Most available results pertain to CR manifolds that are embeddable into $\mathbb{C}^2$. Thus, suppose (M, H, J_0) is embeddable into $\mathbb{C}^2$.

Theorem 1.1. *$J_0 \in \mathcal{S}_0$ has a neighborhood $\mathcal{N}_0$ such that $\mathcal{N}_0 \cap \mathcal{B}_0$ is closed in $\mathcal{N}_0$. Similarly, for any marking μ_0, $(J_0, \mu_0) \in \S$ has a closed neighborhood $\mathcal{N}$ such that $\mathcal{N} \cap \mathcal{B}$ is closed in $\mathcal{S}$.*

This was first proved by Epstein, see [7]. The proof given in Section 2 is based on the stability theorem of [17], also used by Epstein; but our proof avoids the very precise spectral analysis of the tangential Cauchy-Riemann operator from [7]. Applying the more general stability results of H.-L. Li, see [18], the same theorem can be proved e.g. for (M, H, J_0) that is embeddable into the total space of a line bundle over $\mathbb{P}_1$ as the boundary of a neighborhood of the zero section.

For stronger results we will need to assume that (M, H, J_0) is (S^3, H_0, J_0), the CR structure inherited by the unit sphere $\{z \in \mathbb{C}^2 : |z| = 1\}$ from $\mathbb{C}^2$.

Theorem 1.2. *(Bland [1]) If $(M, H, J_0) = (S^3, H_0, J_0)$ then $J_0 \in \mathcal{S}_0$ has a neighborhood $\mathcal{N}_0$ such that $\mathcal{N}_0 \cap \mathcal{B}_0$ is a submanifold of $\mathcal{N}_0$.*

Given a marking $\mu = (p_i, v_i)$ on $(S^3, H_0) \subset \mathbb{C}^2$, look at the complex lines $L_i \subset \mathbb{C}^2$ that pass through p_i and whose tangent space contains v_i. If L_1 and L_2 intersect each other in one point, and this point is an interior point of the unit ball, we say that the marking is elliptic.

Theorem 1.3. *If μ_0 is an elliptic marking of (S^3, H_0) then $(J_0, \mu_0) \in \mathcal{S}$ has a* Cont *invariant neighborhood $\mathcal{N}$ such that*
(a) $\mathcal{U} = \mathcal{N} \cap \mathcal{B}$ is a submanifold of $\mathcal{N}$.
(b) $\mathcal{U} \to \mathcal{U}/\mathrm{Cont}(\subset \mathcal{M})$ is a trivial smooth principal Cont *bundle for some smooth structure on $\mathcal{U}/\mathrm{Cont}$.*

Thus a nonempty open piece of the moduli space $\mathcal{M}$ is an infinite dimensional Fréchet manifold. It is very likely that the neighborhood $\mathcal{N}$ in the above theorem can be chosen to contain all CR structures that admit an embedding into $\mathbb{C}^2$ as a strongly convex hypersurface, with arbitrary "elliptic" markings (ellipticity of a marking in this case can also be defined in terms of the Kobayashi metric, see section 5). What is missing from the proof is an improvement on Bland's Theorem 1.2

to the effect that $\mathcal{N}_0$ can be chosen to consist of all CR manifolds that embed into $\mathbb{C}^2$ as strongly convex hypersurfaces.

As to its form, Theorem 1.3 is related to the slice theorem of Cheng and Lee in [6], but the content is rather different. Indeed, Cheng and Lee construct a local slice for the action of Cont on the space of all CR structures (not just embeddable ones). Also, their approach is more abstract, and the slice is obtained by the application of the implicit function theorem, while in our approach the moduli space is represented in rather concrete terms.

§2. Proof of Theorem 1.1

We first observe that there is a positive integer k such that if a CR manifold (M, H, J) admits a CR embedding into $\mathbb{C}^2$ of class C^k then it also admits a CR embedding of class C^∞, i.e., is embeddable in our terminology. To check this we recall Boutet de Monvel's criterion, see [3], that (M, H, J) is embeddable if the tangential Cauchy-Riemann operator $\bar{\partial}_J : L^2(M) \to L^2_{0,1}(M)$ has closed range (to define the above L^2 spaces we endow M with a continuous Riemann metric). On the other hand if (M, H, J) is C^k embeddable into $\mathbb{C}^2$ then it can be regarded as the C^k boundary of a strongly pseudoconvex domain $D \subset \mathbb{C}^2$, and $\bar{\partial}_J$ becomes the boundary operator $\bar{\partial}_b$. Now Kohn in [12] shows that in this case $\bar{\partial}_b : L^2(\partial D) \to L^2_{0,1}(\partial D)$ has closed range. Strictly speaking Kohn assumes that D has C^∞ boundary, but his proof uses only finitely many derivatives of a defining function of D; whence the theorem is true as soon as ∂D is of class C^k with k sufficiently large. Putting these two results together we obtain our claim.

Next choose a CR embedding $f_0 : (M, H, J_0) \to \mathbb{C}^2$ of class C^∞, a k as in the above observation, and an $\epsilon > 0$ with the property that any C^k mapping $f : M \to \mathbb{C}^2$ whose C^k-distance to f_0 is $\leq \epsilon$ is a (differentiable) embedding. In [17] we proved J_0 has a neighborhood $\mathcal{N}_0 \subset \mathcal{S}_0$ such that for any $J \in \mathcal{N}_0 \cap \mathcal{B}_0$ the CR manifold (M, H, J) admits a CR embedding f into $\mathbb{C}^2$ with $|f - f_0|_{C^{k+1}} < \epsilon$. To verify $\mathcal{N}_0 \cap \mathcal{B}_0$ is closed in $\mathcal{N}_0$, let $J_\nu \in \mathcal{N}_0 \cap \mathcal{B}_0$ be a sequence converging to $J \in \mathcal{N}_0$. Choose CR embeddings $f_\nu : (M, H, J_\nu) \to \mathbb{C}^2$ with $|f_\nu - f_0|_{C^{k+1}} < \epsilon$. In view of the Arzelà-Ascoli theorem there is a subsequence f_{ν_j} that converges to some $f : M \to \mathbb{C}^2$ in the C^k topology; as $|f - f_0|_{C^k} \leq \epsilon$, f is an embedding. It is in fact a CR embedding of (M, H, J) of class C^k. According to our initial observation this implies J is embeddable, $J \in \mathcal{N}_0 \cap \mathcal{B}_0$, and we are done.

The second claim of Theorem 1.1 follows from the first if we note that the mapping $\mathcal{N} \to \mathcal{N}_0$ defined by forgetting the marking is smooth and $\mathcal{B}$ is the preimage of $\mathcal{B}_0$.

§3. Embedding families of CR structures.

In this section we will consider a CR manifold (M, H, J_0) with a CR embedding $f_0 : M \to \mathbb{C}^2$, and a smooth family $j : \mathcal{T}_1 \to \mathcal{S}_0$ of CR structures, parametrized by some neighborhood $\mathcal{T}_1$ of 0 in a Fréchet space $\mathcal{T}$, $j(0) = J_0$. We will also assume that j takes values in the subspace of embeddable CR structures $\mathcal{B}_0 \subset \mathcal{S}_0$. We will prove

Theorem 3.1. *There are a neighborhood $\mathcal{T}_2 \subset \mathcal{T}_1$ of 0 and a smooth family of CR embeddings $f(t) : (M, H, j(t)) \to \mathbb{C}^2$, $t \in \mathcal{T}_2$, such that $f(0) = f_0$.*

Above $f(t)$ being a smooth family means that $\mathcal{T}_2 \ni t \mapsto f(t) \in C^\infty(M, \mathbb{C}^2)$ is a C^∞ mapping. A closely related result is given in [7, part II, Theorem 8.1]. There a real analytic family of embeddable CR structures parametrized by an interval is considered; it is not assumed, though, that the manifolds embed into $\mathbb{C}^2$. The conclusion is then as above (with $\mathbb{C}^2$ replaced by some $\mathbb{C}^n$, and without $f(0) = f_0$). It is also indicated how to extend that theorem to higher dimensional parameter spaces and smooth families if a certain relative index vanishes. This latter condition is known to be satisfied when $(M, H, j(0))$ embeds into $\mathbb{C}^2$.

Observe that Theorem 3.1 is a counterpart of Theorem 1.1 in [17], but while [17] is about embeddings of *perturbations* J_1 of J_0, Theorem 3.1 is about embeddings of *deformation* families. Neither result is a consequence of the other, but their proofs use similar tools. For this reason we start by recalling some results from [17].

Let $\bar{Y}$ be a compact complex manifold with smooth boundary ∂Y and interior Y; we will denote the complex structure of $\bar{Y}$ by $\mathcal{J}_0$. Let $L \to (\bar{Y}, \mathcal{J}_0)$ be a smooth line bundle, holomorphic on Y. The Cauchy-Riemann operator $C^\infty(L) \to C^\infty_{0,1}(L)$ will be denoted $\bar{D}$. In [17] we introduced a scale of anisotropic Sobolev norms $\| \ \|_s$, $s = 1, 2, \dots$ on $C^\infty(L)$ resp. $C^\infty_{0,1}(L)$ whose basic properties we will list below.

Proposition 3.2. *C^k Hölder norms ($k = 1, 2, \dots$) are dominated by $\| \ \|_s$ if $s > 2(n + k)$, $n = \dim_{\mathbb{C}} Y$.*

The norms $\| \ \|_s$ come from inner products $(\ , \ )_s$. It follows that if we denote the completion of $C^\infty(L)$, $C^\infty_{0,1}(L)$ with respect to $\| \ \|_s$ by $\mathcal{H}^s$, $\mathcal{H}^s_{0,1}$, these spaces are Hilbert spaces with inner product (the extension of) $(\ ,\ )_s$, and for $s > 2n+2$ they are continuously embedded into $C^1(L)$ resp. $C^1_{0,1}(L)$. Also, [17, Proposition 2.2] implies that $\bar{D} : \mathcal{H}^s \to \mathcal{H}^{s-1}_{0,1}$ is continuous.

From now on we will also assume that the Levi form of ∂Y has at least one negative eigenvalue in every point of ∂Y. Then we have

Proposition 3.3. *There are constants C_s such that for $u \in \mathcal{H}^s$*

$$\|u\|_s \le C_s(\|\bar{D}u\|_{s-1} + \|u\|_0). \tag{3.1}$$

Proof. (3.1) is proved in [17, Theorem 3.1] for $u \in C^\infty(L)$; since $C^\infty(L)$ is dense in $\mathcal{H}^s$ it is true for $u \in \mathcal{H}^s$ as well.

Now assume that $(Y, \mathcal{J}_0)$ contains a nonsingular compact complex hypersurface Z without boundary, and that $j(t)$ is a smooth family of complex structures on $\bar{Y}$ parametrized by some open neighborhood $\mathcal{T}'$ of 0 in a Fréchet space T. We will also assume that $\mathcal{J}(0) = \mathcal{J}_0$, and that the tangent bundle TZ is invariant under all $\mathcal{J}(t)$; thus Z is a complex submanifold of all $(Y, \mathcal{J}(t))$. Let $L(t) \to (\bar{Y}, \mathcal{J}(t))$ denote the line bundles determined by the divisor Z, $L(0) = L$, and $\bar{D}(t) : C^1(L(t)) \to C_{0,1}(L(t))$ the corresponding Cauchy-Riemann operators, $\bar{D}(0) = \bar{D}$.

Proposition 3.4. *There are a neighborhood $\mathcal{T}'' \subset \mathcal{T}'$ of 0, a family of smooth bundle isomorphisms $\Phi_t : L \to L(t)$, and a smooth family of first order linear partial differential operators $\Lambda_t : C^\infty(L) \to C^\infty_{0,1}(L)$, $t \in \mathcal{T}''$, such that*

(i) $\Lambda_0 = 0$,

(ii) for every $s = 1, 2, \dots$, Λ_t extends to a smooth family of operators $\mathcal{H}^s \to \mathcal{H}^{s-1}_{0,1}$;

(iii) for $u \in C^\infty(L)$ the Cauchy-Riemann equation $\bar{D}(t)\Phi_t \circ u = 0$ is equivalent to $\bar{D}u + \Lambda_t u = 0$.

The proof of this parallels the proof of [17, Lemma 4.2], so we omit it. Here we record that according to the proof in [17], Φ_t, Λ_t satisfy

$$\bar{D}(t)\Phi_t \circ u = \Phi_t \circ \bar{D}u + \Phi_t \circ \Lambda_t u. \tag{3.2}$$

It follows that for $t \in \mathcal{T}''$, $s = 1, 2, \dots$ there are constants $C_{s,t}$ such that for $u \in \mathcal{H}^s$

$$\|u\|_s \le C_{s,t}(\|\bar{D}u + \Lambda_t u\|_{s-1} + \|u\|_0). \tag{3.3}$$

Indeed, this is just (3.1) applied to $\bar{D}(t)$, $L(t)$ instead $\bar{D}$, L (here we assume $\mathcal{T}''$ is so small that the Levi form of $\partial(Y, \mathcal{J}(t))$ has a negative eigenvalue for $t \in \mathcal{T}''$).

Proposition 3.5. *Given s, the constant $C_{s,t}$ in (3.3) is locally uniform in $t \in \mathcal{T}''$.*

Proof. Fix $\tau \in \mathcal{T}''$. With $t \in \mathcal{T}''$ we have

$$\begin{aligned}\|u\|_s &\leq C_{s,\tau}(\|(\bar{D} + \Lambda_\tau)u\|_{s-1} + \|u\|_0)\\ &\leq C_{s,\tau}(\|(\bar{D} + \Lambda_t)u\|_{s-1} + \|u\|_0) + C_{s,\tau}\|(\Lambda_\tau - \Lambda_t)u\|_{s-1}.\end{aligned}$$

If t is sufficiently close to τ, Proposition 3.4 (ii) implies that the last term here is $\leq \|u\|_s/2$. It then follows that $C_{s,t} = 2C_{s,\tau}$ can be chosen in (3.3).

Proposition 3.6. *There is a neighborhood $\mathcal{T}''' \subset \mathcal{T}''$ of 0 such that given $s > 2n + 4$ we have with constants $\bar{C}_{s,t}$ ($t \in \mathcal{T}'''$)*

$$\|u\|_s \leq \bar{C}_{s,t}\|\bar{D}u + \Lambda_t u\|_{s-1},$$

whenever $u \in \mathcal{H}^s$ is orthogonal to $H^0(L)$ with respect to $(\ ,\ )_0$. Here $\bar{C}_{s,t}$ can be chosen locally uniform in $t \in \mathcal{T}'''$.

Proof. Let $\tau \in \mathcal{T}''$. If we can not find uniform constants $\bar{C}_{s,t}$ in any neighborhood of τ then in view of Proposition 3.5 there exist a sequence $t_i \to \tau$ and $u_i \in \mathcal{H}^s$ orthogonal to $H^0(L)$ with $\|u_i\|_0 = 1$, $\|\bar{D}u_i + \Lambda_{t_i}u_i\|_{s-1} \to 0$. By Proposition 3.5, $\|u_i\|_s$ is bounded, hence by Proposition 3.2 and the Arzelà-Ascoli theorem a subsequence u_{ν_i} converges to $u \in C^1(L)$ in C^1-norm. Then $\|u\|_0 = 1$. Also $\bar{D}u + \Lambda_\tau u = 0$, so that $\phi_\tau \circ u \in H^0(L(\tau))$, and u is orthogonal to $H^0(L)$. But by [17, Proposition 5.2] this implies $u = 0$ if τ is in a sufficiently small neighborhood $\mathcal{T}'''$ of 0, which is a contradiction.

Corollary 3.7. *The operator*

$$\bar{D} + \Lambda_t : \mathcal{H}^s \to \mathcal{H}^{s-1}_{0,1}$$

has closed range if $s > 2n + 4$, $t \in \mathcal{T}'''$.

Proof of Theorem 3.1. We can assume that M is a hypersurface in $\mathbb{C}^2$ and its CR structure J_0 is inherited from $\mathbb{C}^2$; further, that f_0 is the inclusion $M \subset \mathbb{C}^2$. M divides $\mathbb{P}_2 \supset \mathbb{C}^2$ in two parts, let Y denote the pseudoconcave one, and $\bar{Y} = Y \cup M$. Denote the complex structure on $\bar{Y}$ inherited from $\mathbb{P}_2$ by $\mathcal{J}_0$. Also, let $Z \subset Y$ be the line at infinity. In

[16] we pointed out that Kiremidjian's theorem in [11] implies that for $t \in \mathcal{T}_1$ near 0 $\bar{Y}$ admits a complex structure $\mathcal{J}(t)$ that induces the CR structure $j(t)$ on (M, HM), and which agrees with $\mathcal{J}_0$ in points of Z. In fact Kiremidjian's proof constructs a unique such $\mathcal{J}(t)$, and one checks that $\mathcal{J}(t)$ depends smoothly on t. Also $\mathcal{J}(0) = \mathcal{J}_0$.

Let now $L(t)$, $\bar{D}(t)$, $\mathcal{H}^s$, $\mathcal{H}^{s-1}_{0,1}$ be as above, and Φ_t, Λ_t as in Proposition 3.4. The homogeneous coordinates on $\mathbb{P}_2$ give rise to three sections $u^i_0 \in H^0(L)$, $i = 0, 1, 2$ that span $H^0(L)$. In [17, Section 5] we proved that with a sufficiently small neighborhood $\mathcal{T}_2$ of 0 there are unique sections $u^i_t \in C^\infty(L)$, $t \in \mathcal{T}_2$, such that $\Phi_t \circ u^i_t \in H^0(L(t))$ and $u^i_t - u^i_0$ is orthogonal to $H^0(L)$ with respect to $(\ ,\ )_0$, $i = 0, 1, 2$. We will assume $\mathcal{T}_2 \subset \mathcal{T}'''$ from Proposition 3.6, and proceed to show that $u^i_t \in C^\infty(L)$ depends smoothly on t. It will suffice to show that for every $s > 8$ every $\tau \in \mathcal{T}_2$ has a neighborhood on which the mapping $t \mapsto u^i_t \in \mathcal{H}^s$ is smooth.

Observe that $\Phi_t \circ u^i_t \in H^0(L(t))$ implies $(\bar{D} + \Lambda_t)u^i_t = 0$, hence also

$$(\bar{D} + \Lambda_\tau)(u^i_t - u^i_\tau) + (\Lambda_t - \Lambda_\tau)(u^i_t - u^i_\tau) = (\Lambda_t - \Lambda_\tau)u^i_\tau. \tag{3.4}$$

Introduce a left inverse Q of $\bar{D} + \Lambda_\tau$ as follows. Given $s > 8$ and $\alpha \in \mathcal{H}^{s-1}_{0,1}$, write $\alpha = \beta + \gamma$ with β in the range of $\bar{D} + \Lambda_\tau : \mathcal{H}^s \to \mathcal{H}^s_{0,1}$ and γ orthogonal to this range *with respect to* $(\ ,\ )_{s-1}$. By virtue of Corollary 3.7 this can be done. Put $Q\alpha = u \in \mathcal{H}^s$ if $(\bar{D} + \Lambda_\tau)u = \alpha$ and u is orthogonal to $H^0(L)$ with respect to $(\ ,\ )_0$. Such a u can be found because dim $H^0(L) =$ dim Ker$(\bar{D} + \Lambda_\tau) = 3$, see [17, Proposition 5.1, also Proposition 5.2]. Also note that $Q : \mathcal{H}^{s-1}_{0,1} \to \mathcal{H}^s$ is a bounded operator.

Putting $(\bar{D} + \Lambda_t)(u^i_t - u^i_\tau) = \alpha_t$ we can write (3.4) as

$$\alpha_t + (\Lambda_t - \Lambda_\tau)Q\alpha_t = (\Lambda_t - \Lambda_\tau)u^i_\tau.$$

If t is sufficiently close to τ, the norm of the operator $(\Lambda_t - \Lambda_\tau)Q : \mathcal{H}^{s-1}_{0,1} \to H^{s-1}_{0,1}$ is less than one, whence it follows that

$$\alpha_t = (I + (\Lambda_t - \Lambda_\tau)Q)^{-1}(\Lambda_t - \Lambda_\tau)u^i_\tau \in \mathcal{H}^{s-1}_{0,1}$$

depends smoothly on t. Therefore the same holds for $u^i_t = u^i_\tau + Q\alpha_t \in \mathcal{H}^s$, and this proves that $t \mapsto u^i_t \in C^\infty(L)$ is smooth.

Assuming $\mathcal{T}_2$ is sufficiently small we now obtain the smooth family $f(t)$ of the Theorem in the form

$$f(t) = \left(\frac{u^1_t}{u^0_t}, \frac{u^2_t}{u^0_t}\right)\bigg|_M = \left(\frac{\Phi_t \circ u^1_t}{\Phi_t \circ u^0_t}, \frac{\Phi_t \circ u^2_t}{\Phi_t \circ u^0_t}\right)\bigg|_M.$$

§4. Moduli spaces of convex domains

Our approach to Theorem 1.3 will be through moduli spaces of convex domains. In this section we will review the relevant facts.

We will denote by $\mathcal{X}_0$ the space of strongly convex smoothly bounded domains $D \subset \mathbb{C}^2$ that contain the origin. As explained in [15] (where the notation was $\mathcal{X}$) this is a Fréchet manifold, in fact an open convex cone in some Fréchet space. The set of those $D \in \mathcal{X}_0$ that are invariant under the circle action

$$(4.1) \qquad \gamma_t : z \mapsto e^{it} z \qquad (t \in \mathbb{R}),$$

i.e. the circular domains, will be denoted $\mathcal{C}_0$. We will work with marked domains, too. A marking for $D \in \mathcal{X}_0$ consists of a pair η of linearly independent vectors $\eta_1, \eta_2 \in T_0^{1,0}D$. The space of marked domains (D, η) with $D \in \mathcal{X}_0$ will be denoted $\mathcal{X}$; this is again a Fréchet manifold. We will say that two marked domains (D, η), $(D', \eta') \in \mathcal{X}$ are equivalent, $(D, \eta) \sim (D', \eta')$, if there is a biholomorphism $\Phi : D \to D'$ that fixes 0 and maps the marking $\eta = (\eta_1, \eta_2)$ to $\eta' = (\eta_1', \eta_2')$. The moduli space $\mathcal{X}/\sim$ of strongly convex smooth domains was described in [2] and [15] in terms of invariants associated with the Kobayashi metric. We will briefly recall how this can be done, mostly following [2], though not its notation.

Given any strongly convex domain D, a point $a \in D$ and a vector $v \in T_a^{1,0}D$, consider holomorphic mappings f of the unit disc

$$\Delta = \{\zeta \in \mathbb{C} : |\zeta| < 1\}$$

into D such that $f(0) = a$ and $f_*(0)\, \partial/\partial\zeta = \lambda v$ with some $\lambda \geq 0$. There is a unique mapping f that maximizes the value of λ, called extremal map (determined by a and v); this map is smooth on $\bar{\Delta}$ and maps the circle $\partial\Delta$ into ∂D, see [13]. If, for fixed a we let v vary, the vectors $f_*(0)\partial/\partial\zeta$ for the corresponding extremal maps f will trace the smooth boundary of a strongly convex circular domain in $T_a^{1,0}D$, called the Kobayashi indicatrix.

Now the first invariant of $(D, \eta) \in \mathcal{X}$ is obtained by looking at the Kobayashi indicatrix $I^* \subset T_0^{1,0}D$ of D at 0. There is a unique linear map $A : T_0^{1,0}D \to \mathbb{C}^2$ that sends the marking (η_1, η_2) to the standard basis $(1,0)$, $(0,1)$ of $\mathbb{C}^2$; the image of I^* under A will be denoted $I = I(D, \eta)$.

An exponential-like mapping $r : \partial I \to \partial D$, called the circular representation, can be defined as follows. Given $v \in \partial I$, let $f : \Delta \to D$ be the extremal map determined by $0 \in D$, $A^{-1}v \in T_0^{1,0}D$. Then putting

$r(v) = f(1)$ a diffeomorphism $r : \partial I \to \partial D$ is obtained. This diffeomorphism has a natural extension to a homeomorphism $\bar{I} \to \bar{D}$, smooth off 0, which is also called circular representation, but we will not deal with this extension.

The hypersurfaces ∂D, $\partial I \subset \mathbb{C}^2$ inherit a CR structure from $\mathbb{C}^2$, denoted $(\partial D, H(\partial D), J_D)$ resp. $(\partial I, H(\partial I), J_I)$, and it turns out that the circular representation r maps $H(\partial I)$ to $H(\partial D)$, i.e. r is a contact diffeomorphism. However, in general it does not intertwine J_D, J_I - in other words, it is not a CR isomorphism - and one can measure the extent to which it distorts the CR structure by looking at the complex line bundles over ∂D resp. ∂I

$$\begin{aligned} H^{0,1}(\partial D) &= \{\xi + iJ_D\xi : \xi \in H(\partial D)\} \subset \mathbb{C} \otimes H(\partial D), \\ H^{0,1}(\partial I) &= \{\xi + iJ_I\xi : \xi \in H(\partial I)\} \subset \mathbb{C} \otimes H(\partial I), \\ H^{1,0}(\partial I) &= \overline{H^{0,1}(\partial I)}, \end{aligned}$$

and the CR deformation tensor which is a bundle map

$$\Phi_{D,\eta} : H^{0,1}(\partial I) \to H^{1,0}(\partial I)$$

with the property that the pull back

$$r_*^{-1} H^{0,1}(\partial D) \subset \mathbb{C} \otimes H(\partial I) = H^{0,1}(\partial I) \oplus H^{1,0}(\partial I)$$

is the graph of $\Phi_{D,\eta}$. By [2], the pair $(I(D,\eta), \Phi_{D,\eta})$ depends only the equivalence class of $(D,\eta) \in \mathcal{X}$, and conversely, the knowledge of $(I(D,\eta), \Phi_{D,\eta})$ allows one to reconstruct the equivalence class of (D,η). Furthermore, the range of the invariants $I(D,\eta)$, $\Phi_{D,\eta}$ can also be described to some extent.

To this end notice that the circle action (4.1) decomposes any tensor $\Phi : H^{0,1}(\partial I) \to H^{1,0}(\partial I)$ into homogeneous tensors: $\Phi = \sum_{-\infty}^{\infty} \Phi_\nu$; here $\gamma_t^* \Phi_\nu = e^{i\nu t}\Phi_\nu$. Denote by $\mathcal{D}_I$ the Fréchet space of smooth tensors $\Phi : H^{0,1}(\partial I) \to H^{1,0}(\partial I)$ whose decomposition contains homogeneous terms Φ_ν with $\nu > 0$ only. These spaces patch together to form a smooth Fréchet bundle $\mathcal{D} = \bigcup_{I \in \mathcal{C}_0} \mathcal{D}_I \to \mathcal{C}_0$. By [2] for any $(D,\eta) \in \mathcal{X}$ the CR deformation tensor $\Phi_{D,\eta}$ is in $\mathcal{D}_{I(D,\eta)}$, whence we obtain a (smooth) mapping $\widetilde{h} : \mathcal{X} \to \mathcal{D}$ that associates with $(D,\eta) \in \mathcal{X}$ the pair of invariants $(I(D,\eta), \Phi_{D,\eta})$. By the above discussion $\widetilde{h}$ descends to a mapping $h : \mathcal{X}/\sim \to \mathcal{D}$, and it turns out that h is a homeomorphism onto an open neighborhood of the zero section in the bundle $\mathcal{D} \to \mathcal{C}_0$. This is essentially contained in [2], although Bland and Duchamp consider only domains with fixed indicatrix I. The proof, however, carries over for variable indicatrices; see also çquivalent result. As in [15, Theorem 10.2] one can prove that $\tilde{h}$ has smooth local right inverses. That is, given $(D,\eta) \in \mathcal{X}$, there are a neighborhood $\mathcal{V} \subset \mathcal{D}$ of $\tilde{h}(D,\eta)$ and a smooth mapping $k : \mathcal{V} \to \mathcal{X}$ with $\tilde{h} \circ k = \mathrm{id}_{\mathcal{V}}$ and $k(\tilde{h}(D,\eta)) = (D,\eta)$.

In the sequel we will endow $\mathcal{X}/\sim$ with the smooth structure that is induced from $\mathcal{D}$ by the homeomorphism h. Thus we have

Proposition 4.1. *The canonical projection $\mathcal{X} \to \mathcal{X}/\sim$ is smooth and has smooth local right inverses.*

In addition to the circular representation $r : \partial I \to \partial D$ discussed above, later we will also need a canonical contact diffeomorphism between $(\partial D, H(\partial D))$ and (S^3, H_0). We end this section by describing how such a diffeomorphism can be constructed. This construction is not holomorphically invariant (unlike the circular representation); it could be made invariant for marked domains (D,η), but invariance will not be the issue in our discussion.

Our first observation is the following. Let H_t ($t \in [0,1]$) be a smooth family of contact structures on a manifold M. Then one can canonically associate with this family a contact diffeomorphism $g : (M, H_0) \to (M, H_1)$.

We will justify this observation under the assumption that H_t are orientable, hence given by a smooth family α_t of one forms on $M : H_t = \operatorname{Ker} \alpha_t$. The forms $d\alpha_t$ restricted to H_t are nondegenerate, whence there is a unique smooth family V_t of vector fields tangent to H_t such that

$$V_t \lrcorner d\alpha_t \Big|_{H_t} = \frac{d\alpha_t}{dt}\Big|_{H_t}.$$

Denoting Lie derivative by $\mathcal{L}$, these vector fields therefore satisfy

$$\mathcal{L}_{V_t}\alpha_t = d(V_t \lrcorner \alpha_t) + V_t \lrcorner d\alpha_t = V_t \lrcorner\, d\alpha_t \equiv \frac{d\alpha_t}{dt} \mod \alpha_t.$$

This implies that the flow g_t of the time dependent field V_t pulls back α_t to some multiple of α_0, in particular $g = g_1$ is a contact diffeomorphism between (M, H_0) and (M, H_1).

If now $D \in \mathcal{X}_0$, the mapping $\rho(z) = z/\|z\|$ defines a diffeomorphism of ∂D to S^3, although this is not in general a contact diffeomorphism between $(\partial D, H(\partial D))$ and (S^3, H_0). To remedy this, denoting the unit ball of $\mathbb{C}^2$ by B, and putting $D_t = tD + (1-t)B$, $\rho|_{\partial D_t}$ will push forward the contact structures $H(\partial D_t)$ to a smooth family of contact structures H_t on S^3. Our observation above now supplies a canonical contact diffeomorphism $g : (S^3, H_0) \to (S^3, H_1)$, whence also a canonical contact diffeomorphism

$$g^{-1} \circ \rho : (\partial D, H(\partial D)) \to (S^3, H_0). \tag{4.2}$$

§5. Proof of Theorem 1.3

The previous section described the spaces $\mathcal{X}$, $\mathcal{X}/\sim$ of convex domains; we shall now connect those spaces with the spaces $\mathcal{B}(S^3, H_0)$, $\mathcal{M}(S^3, H_0)$ of CR manifolds. Thus, let $(D, \eta) \in \mathcal{X}$ be a marked domain. Its boundary ∂D inherits a CR structure from $\mathbb{C}^2$, $(\partial D, H(\partial D), J_D)$. The marking η of D determines a marking μ' of this CR manifold as follows. For $i = 1, 2$, consider the extremal mapping $e^i : \Delta \to D$ determined by η_i, and put

$$p_i = e^i(1), \qquad v_i = -\frac{1}{\lambda}\, e^i_*(1) \mathrm{Im}\, \frac{\partial}{\partial \zeta}.$$

The marking $\mu' = (p_i, v_i)$ defines a marked CR manifold $(\partial D, H(\partial D), J_D, \mu')$. Via the contact diffeomorphism (4.2) constructed in Section 4 this marked CR manifold can be identified with a marked CR manifold $(J, \mu) \in \mathcal{B}(S^3, H_0)$, which we will also denote $\Theta(D, \eta)$. Thus Θ is a mapping from $\mathcal{X}$ to $\mathcal{B}$, and indeed a smooth mapping that descends to a mapping $\theta : \mathcal{X}/\sim \to \mathcal{M}$.

Conversely, consider a marked CR manifold $(J, \mu) \in \mathcal{B}$, where J is sufficiently close to the standard CR structure J_0 of the sphere. By [17] this implies there is a smooth CR embedding

$$f : (S^3, H_0, J) \to \mathbb{C}^2 \tag{5.1}$$

with image a strongly convex hypersurface. Denote the domain bounded by this hypersurface by D; the marking $\mu = (p_i, v_i)$ then defines points $p'_i = f(p_i) \in \partial D$ and vectors $v'_i = f_* v_i \in T'_{p_i} \partial D$.

By [5] there are extremal mappings $e^i : \bar{\Delta} \to \bar{D}$ with

$$e^i(1) = p'_i, \qquad e^i_*(1) \mathrm{Im}\, \partial/\partial\zeta = -\lambda_i v'_i \qquad (\lambda_i > 0).$$

These extremal mappings are unique up to composition by holomorphic automorphisms of Δ that fix 1. When $J = J_0$ and $\mu = \mu_0$ is an elliptic marking, the extremal discs $e^1(\Delta)$, $e^2(\Delta)$ intersect in one point inside

D. More generally, if a CR manifold (S^3, H_0, J) admits a strongly convex embedding into $\mathbb{C}^2$, we will say that a marking μ is elliptic if the two extremal discs $e^1(\Delta)$, $e^2(\Delta)$ determined by the marking as above, intersect (in which case they intersect in exactly one point). The set of elliptically marked CR manifolds will be denoted $\mathcal{E} \subset \mathcal{B}$.

Below we will need the fact that the extremal discs $e^i(\Delta)$ depend smoothly on D and p_i', v_i'. This does not seem to have been published anywhere, but a proof can be easily obtained by a small modification of the arguments in [14]. A slightly weaker theorem is proved in [5], where D is, however, kept fixed. In any case, this shows that the subset $\mathcal{E}$ of elliptically marked CR manifolds is open in $\mathcal{B}$; in particular, if the marking μ above was close to an elliptic marking μ_0 of (S^3, H_0, J_0) then itself is elliptic, that is, the extremal discs $e^1(\Delta)$, $e^2(\Delta)$ above intersect in some point. By modifying the embedding of (S^3, H_0, J) into $\mathbb{C}^2$ we can assume that the point of intersection is 0; and also that $e^1(0) = e^2(0) = 0$. With this normalization the λ_i in (4.2) are uniquely determined, and we can define a marking η of $D \in \mathcal{X}_0$ by

$$\eta_i = \frac{1}{\lambda_i}\, e^i(0)\frac{\partial}{\partial\zeta} \in T_0^{1,0}D \qquad (i = 1, 2).$$

We have thus associated a marked domain $(D, \eta) \in \mathcal{X}$ with an elliptically marked CR structure $(J, \mu) \in \mathcal{E}$, in particular, with (J, μ) close to (J_0, μ_0). This association is not unique, for it depends on the CR embedding (5.1) we choose. However, Bland's theorem (Theorem 1.2) implies that (J_0, μ_0) has a neighborhood $\mathcal{N} \subset \mathcal{S}$ such that $\mathcal{U} = \mathcal{N} \cap \mathcal{B}$ is a submanifold of $\mathcal{N}$, and then Theorem 3.1 implies that (after a possible shrinking) the mapping f in (5.1) can be chosen to depend smoothly on (J, μ). This then makes the passage from $(J, \mu) \in \mathcal{U}$ to (D, η) a smooth mapping $\Psi : \mathcal{U} \to \mathcal{X}$. The construction was such that for $u \in \mathcal{U}$ $\Theta(\Psi(u))$ is on the Cont orbit of u, and for $x \in \mathcal{X}$ the marked domains $\Psi(\Theta(x))$ and x are equivalent. By replacing $\mathcal{N}$ by its Cont orbit we can assume that $\mathcal{N}$, hence $\mathcal{U}$ are Cont invariant; then Ψ descends to a continuous open mapping $\psi : \mathcal{U}/\text{Cont} \to \mathcal{X}/\sim$ and $\theta \circ \psi = \text{id}_{\mathcal{U}/\text{Cont}}$.

Ideas like the ones employed in the construction of the mappings Θ, Ψ also let one understand the action of Cont on the open set $\mathcal{E} \subset \mathcal{B}$.

Proposition 5.1. *If two elliptically marked CR manifolds are CR diffeomorphic then there is a unique CR diffeomorphism between them. Moreover, if $\mathcal{T}_1$ is an open set in some Fréchet space and $F, G : \mathcal{T}_1 \to \mathcal{E} \subset \mathcal{S}$ are smooth mappings (as mappings into $\mathcal{S}$) such that for every $t \in \mathcal{T}_1$, $F(t)$ and $G(t)$ are CR diffeomorphic, then the CR diffeomorphism between them (an element of Cont) depends smoothly on t.*

Proof. If the elliptically marked CR manifolds (S^3, H_0, J, μ) and $(S^3, H_0, \hat{J}, \hat{\mu})$ are CR diffeomorphic, the unique CR diffeomorphism between them can be constructed as follows. Taking suitable convex embeddings of these CR manifolds into $\mathbb{C}^2$ the image hypersurfaces will bound strongly convex domains D, $\hat{D}$ and the markings μ, $\hat{\mu}$ will induce markings η, $\hat{\eta}$ on them, as explained above. It will suffice to show that there is a unique biholomorphism H between (D, η) and $(\hat{D}, \hat{\eta})$, this latter being a consequence of the biholomorphic invariance of extremal discs. Indeed, let the linear map $L : T_0^{1,0}D \to T_0^{1,0}\hat{D}$ map η to $\hat{\eta}$, then for any $z \in \bar{D} \setminus \{0\}$ $H(z) \in \bar{\hat{D}}$ can be obtained as follows. Let $e : \Delta \to \Delta$ be the unique extremal mapping such that $e(0) = 0$, $e(\alpha) = z$ with some α, $0 < \alpha \leq 1$, and let $\hat{e} : \Delta \to \hat{D}$ be the unique extremal mapping such that $\hat{e}_*(0)\partial/\partial\zeta = \lambda L e_*(0)\partial/\partial\zeta$, with some $\lambda > 0$. Then $H(z)$ is given by $\hat{e}(\alpha)$; in particular H is unique.

The second half of the Proposition is proved using the above passage from $\mathcal{E}$ to $\mathcal{X}$, and in addition Theorem 3.1 and the smooth dependence of extremal maps on the data (the target domain, base points, resp. tangent vector).

At this point we are ready to prove Theorem 1.3. Let π, ω denote the canonical projections $\mathcal{B} \to \mathcal{B}/\text{Cont}$ resp. $\mathcal{X} \to \mathcal{X}/\sim$, see the diagrams:

$$\begin{array}{ccc} \mathcal{B} & \xleftarrow{\;\Theta\;} & \mathcal{X} \\ \pi\big\downarrow & & \big\downarrow\omega \\ \mathcal{B}/\text{Cont} & \xleftarrow{\;\theta\;} & \mathcal{X}/\sim \end{array} \qquad\qquad \begin{array}{ccc} \mathcal{B} \supset \mathcal{U} & \xrightarrow{\;\Psi\;} & \mathcal{X} \\ \pi\big\downarrow & & \big\downarrow\omega \\ \mathcal{U}/\text{Cont} & \xrightarrow{\;\psi\;} & \mathcal{X}/\sim . \end{array}$$

The pullback of the smooth structure of $\mathcal{X}/\sim$ by ψ defines a smooth structure on $\mathcal{U}/\text{Cont}$. Thus ψ and its inverse $\theta|_{\psi(\mathcal{U}/\text{Cont})}$ are diffeomorphisms. We need to show that $\pi : \mathcal{U} \to \mathcal{U}/\text{Cont}$ is a trivial smooth principal bundle with structure group Cont. First, $\pi = \theta \circ \omega \circ \Psi$ is smooth. Second, assuming $\mathcal{U}$ is sufficiently small, a section of π can be gotten by looking at a smooth local right inverse σ of ω defined near $\omega(\Psi(J_0, \mu_0))$ (cf. Proposition 4.1); then $\kappa = \Theta \circ \sigma \circ \psi$ is a smooth section of π. Finally, denoting the action of a contact diffeomorphism $\gamma \in \text{Cont}$ on $\mathcal{U}$ by a superscript, we define a smooth Cont equivariant mapping

$$\text{Cont} \times \mathcal{U}/\text{Cont} \ni (\gamma, \bar{u}) \mapsto (\kappa(\bar{u}))^\gamma \in \mathcal{U}.$$

This has a smooth inverse

$$\mathcal{U} \ni u \mapsto (\Gamma(\kappa(\pi(u)), u), \pi(u)),$$

where $\Gamma(v, u)$ denotes the unique CR diffeomorphism between $v \in \mathcal{U}$ and $u \in \mathcal{U}$, cf. Proposition 5.1. Hence the Cont bundles $\mathcal{U} \to \mathcal{U}/$ Cont and Cont $\times \mathcal{U}/$Cont $\to \mathcal{U}/$Cont are isomorphic, whence the theorem follows.

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