TESTING ASSOCIATION BETWEEN SPATIAL PROCESSES

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ABSTRACT

This paper addresses the study of association between two spatial processes. In particular, we consider the properties of modified tests of the empirical correlation coefficient between the processes. We show that by a simple adjustment, correct level of significance can be reached and that the power under a simple linear alternative is compatible with that of a standard test in an equivalent situation. These tests can be applied both to regularly and irregularly spaced points and can be considered as a first step in an analysis of association when detailed spatial modelling is not suitable.

Application of these tests to data gives pivotal confidence interval for the regression coefficient. Furthermore, if one is prepared to model the observed covariance structure, Monte Carlo tests of association can be performed. In the examples investigated, which concern the relationship between lung cancer, smoking, and industrial factors,the results from the two types of testing procedures were close.

1. Introduction

This paper is concerned with a topic in spatial statistics, that of testing for association between two spatial processes. This question arises frequently in various fields and examples abound in geography and regional sciences (Cliff and Ord, 1981), geology (Malin and Hyde, 1982), sociology (Doreian, 1981)... In the epidemiology of chronic diseases, etiological clues are sometimes sought by studying joint geographic variations of environmental risk factors and disease rates (Doll 1980). The examples discussed in the later part of this paper will be taken from this field.

Throughout the data will consist of a set A of N locations and pairs of variables (X_α, Y_α), $\alpha \in A$, indexed by their location. These variables will be spatially autocorrelated.

In the first part a method for approximating the critical values of the product moment correlation coefficient r_{XY} will be summarized. This method has been described in detail in Clifford, Richardson, Hémon(1989).

In the following sections we give some complementary results on the performance of the tests for small domains and on their power. We also discuss the influence of the choice of the partition of the covariance structure. Finally we give some examples, comparing our results with those of Monte Carlo tests and giving pivotal confidence intervals for the regression coefficient.

2. Modified tests of associations

We have devised modified tests of association based either on s_{XY}: the empirical covariance between pairs of observations (X_α, Y_α), $\alpha \in A$, or based on r_{XY} : the corresponding empirical correlation coefficient. We shall use the notation :

$$\overline{X} = N^{-1}(\Sigma X_\alpha),\ s_{XY} = N^{-1}\Sigma(X_\alpha - \overline{X})(Y_\alpha - \overline{Y}),\ s_X^2 = N^{-1}\Sigma(X_\alpha - \overline{X})^2$$

and similarly for $\overline{Y}$ and s_Y^2.

Method

Suppose that X and Y are independent but that both X and Y are multivariate normal vectors with constant means and variance-covariance matrices Σ_X and Σ_Y respectively. A stratified structure for Σ_X and Σ_Y is imposed. Pairs in $A \times A$ are divided into strata $S_0, S_1, S_2, \ldots$ such that the covariances within strata remain constant, i.e.

$$\mathrm{cov}\,(X_\alpha, X_\beta) = C_X(k) \quad \text{if } (\alpha, \beta) \in S_k.$$

An estimate of the conditional variance of s_{XY} is then derived :

$$N^{-2} \sum_k N_k \widehat{C}_X(k) \widehat{C}_Y(k), \tag{1}$$

N_k is the number of pairs in strata S_k and $\widehat{C}_X(k)$ (respectively $\widehat{C}_Y(k)$) is the estimated autocovariance :

$$\widehat{C}_X(k) = \sum_{S_k} (X_\alpha - \overline{X})(X_\beta - \overline{X})/N_k\,.$$

Thus the estimate takes into account the autocorrelation of both X and Y.

Further it can be shown that, to the first order, the variance of r_{XY}, σ_r^2 is :

$$\sigma_r^2 = \frac{\mathrm{Var}\,(s_{XY})}{E(s_X^2)E(s_Y^2)} \tag{2}$$

which leads to the following estimate :

$$\widehat{\sigma}_r^2 = \frac{\Sigma N_k \widehat{C}_X(k)\widehat{C}_Y(k)}{N^2 s_X^2 s_Y^2}\,. \tag{3}$$

Note that an equivalent expression for $\mathrm{Var}\,(s_{XY})$ is $N^{-2}\,\mathrm{tr}\,(\Sigma_\xi \Sigma_\eta)$ where Σ_ξ and Σ_η are the variance-covariance matrices of the centered vectors $X - \overline{X}$ and $Y - \overline{Y}$.

In the classical non autocorrelated case, when either Σ_X or $\Sigma_Y = I$, it can be shown that the approximation given by (2) is exact and that r_{XY} follows a t-distribution with $N-2$ d.f. (t_{N-2}). Further : $N = 1 + (\sigma_r^2)^{-1}$

In general, an estimated effective sample size, $\widehat{M}$, is defined by the relationship

$$\widehat{M} = 1 + (\widehat{\sigma}_r^2)^{-1},$$

where $\widehat{\sigma}_r^2$ is given by (3). A modified t-test : $t_{\widehat{M}-2}$ is proposed which rejects the null hypothesis of no association when :

$$|(\widehat{M}-2)^{1/2} r(1-r^2)^{-1/2}| > t^{\alpha}_{\widehat{M}-2}$$

where $t^{\alpha}_{\widehat{M}-2}$ is critical value of the t-statistic with $\widehat{M}-2$ d.f.

Equivalently a standardized covariance can be used :

$$W = N s_{XY} (\Sigma N_k \widehat{C}_X(k)\widehat{C}_Y(k))^{-1/2}$$

and tested as a standard normal relying upon central limit theorems for spatially dependent variables.

Results on the performance of W and $t_{\widehat{M}-2}$

The performance of these tests under the null hypothesis of stochastic independence between X and Y was first assessed by Monte Carlo simulations for two models :

(a) X and Y were generated on 3 lattice sizes ($12 \times 12, 16 \times 16, 20 \times 20$) as nearest neighbor isotropic autoregressive Gaussian processes, with 1st order autocorrelation $\rho_X(1)$ and $\rho_Y(1)$ ranging from 0.2 to 0.8 ;

(b) X and Y were generated on the grid of the administrative centers of the French départments as Gaussian variables with a disc model for their autocovariance (Ripley 1981) and arbitrarily defined 1st order autocorrelation ranging from 0.2 to 0.9.

In both cases and for several levels of autocorrelation in X or Y, 500 trials were performed with a nominal rejection level of 5%.

For the two statistics, $t_{\widehat{M}-2}$ and W, the type I errors found did not vary in any systematic way with the level of autocorrelation and fluctuated around the nominal 5% level (the confidence interval excluded 5% in only 2 cases out of 45 simulated in (a)). This contrasted with the performance of the standard t-test procedure based on $N-2$ d.f. where the type I error increased systematically with the autocorrelation to reach values around 50% in the highly autocorrelated cases (Clifford, Richardson, Hémon, 1989).

3. Comparison of W and $t_{\widehat{M}-2}$ on small lattices

Based on our first results the performance of the statistics $t_{\widehat{M}-2}$ and W seemed indistinguishable. The number of sample points was reasonably large and a difference between their respective performances could be better highlighted by studying smaller samples.

Results from further simulations on smaller lattices of sizes 6×6, 8×8, 10×10 are shown in Figure 1. The type I errors of the W statistic exhibit a systematic downward trend with increasing autocorrelation which is not apparent for the modified $t_{\widehat{M}-2}$ statistic. Consequently, for small domains, the use of the modified $t_{\widehat{M}-2}$ statistic is preferable to that of W.

We also note that the convergence of W to normality is slower as the autocorrelation increases. Figure 2 shows a Q.Q. plot of W, i.e. quantiles of a standard normal distribution against the sample quantiles of W for 500 independent trials, in the case $\rho_X(1) = \rho_Y(1) = 0.8$ and a 12×12 lattice. We note that the distribution of the W statistic has short tails compared to the normal distribution. The departure from normality is confirmed by a Kolmogorov-Smirnov test which is significant at the 5% level but not at the 1% level.

4. Power of the modified tests

The power of the modified tests was assessed under a simple alternative hypothesis of a linear regression between Y and X : H_1 : $Y = aX + W$, $X \sim N(\mu_X, \Sigma_X)$, $W \sim N(\mu_W, \Sigma_W)$ and X and W independent. It is difficult to calculate theoretically the power of the modified tests because their distribution under H_1 is not precisely known. Their power can be assessed by simulations.

Two independent spatially autocorrelated processes X and W were generated on the grid of the administrative centers of French déparments as Gaussian variables with a disc model for their autocovariance. Without loss of generality $\sigma_X^2 = \sigma_W^2$ was chosen and hence the correlation r_{XY} between X and Y was only dependent on the parameter a. Five hundred trials were carried out for several levels of autocorrelation in X and W and for the values $\rho_{XY} = 0.2$ and 0.4. The grid contained $N = 82$ points. Results for higher values of ρ_{XY} are not reported because the power of the tests was very close to 1. In these simulations, the power of W and $t_{\widehat{M}-2}$, was evaluated with a 5% nominal level.

On the other hand it might be interesting to calculate the power $\pi_T(s_{XY})$ of a test on the covariance s_{XY}, similar to W but where the estimate : $N^{-2}\Sigma N_k \widehat{C}_X(k)\widehat{C}_Y(k)$, of the variance of s_{XY} is replaced by its theoretical value under H1:

$$N^{-2}\,\mathrm{tr}\,(\Sigma_\xi \Sigma_\eta) = N^{-2}[a^2\,\mathrm{tr}\,(\Sigma_\xi^2) + \mathrm{tr}\,(\Sigma_\xi \Sigma_\theta)].$$

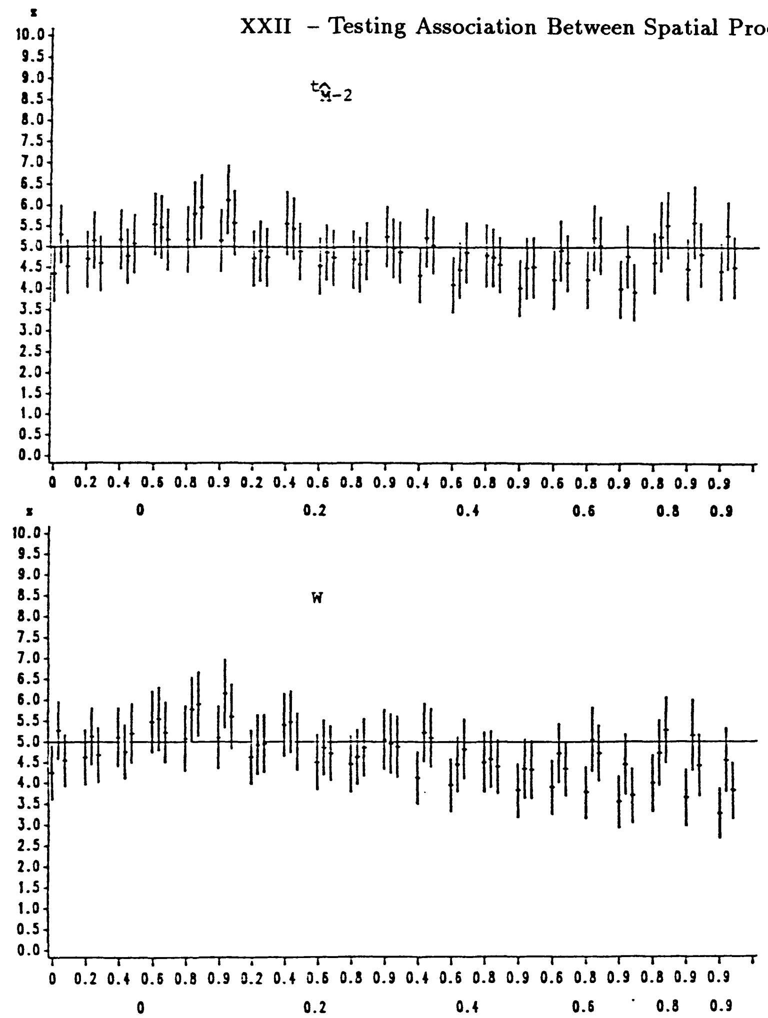

Figure 1. Comparison of the performance of $t_{\hat{M}-2}$ and W tests for small lattices: 6×6, 8×8, 10×10. 95% C.I. for the percentage of type I error for a 5% nominal test.

where Σ_θ denotes the variance-covariance matrix of the centered vector $W - \overline{W}$ and Σ_ξ and Σ_η are defined as before.

In order to carry out the calculation of $\pi_T(s_{XY})$, it is necessary to suppose that the distribution of $\sqrt{N} s_{XY}$ under H1 is approximately normal. This approximation can be justified by central limit theorems if appropriate hypotheses are placed on the rate of decrease of the autocovariances of X and W as a function of the lag.

The expectation and the variance of s_{XY} under H1 are given by:

$$
\begin{aligned}
E_{H1}(N s_{XY}) &= a \operatorname{tr}(\Sigma_\xi) \\
V_{H1}(N s_{XY}) &= 2a^2 \operatorname{tr}(\Sigma_\xi^2) + \operatorname{tr}(\Sigma_\xi \Sigma_\theta).
\end{aligned}
$$

The traces of the matrices $\Sigma_\xi \Sigma_\theta$ or their product can be expressed in terms of Σ_X and Σ_W and thus evaluated for specific models for Σ_X and Σ_W.

The power $\pi_T(s_{XY})$ of the test of the covariance using the statistic $N s_{XY} \operatorname{tr}(\Sigma_\xi \Sigma_\eta)^{-1/2}$

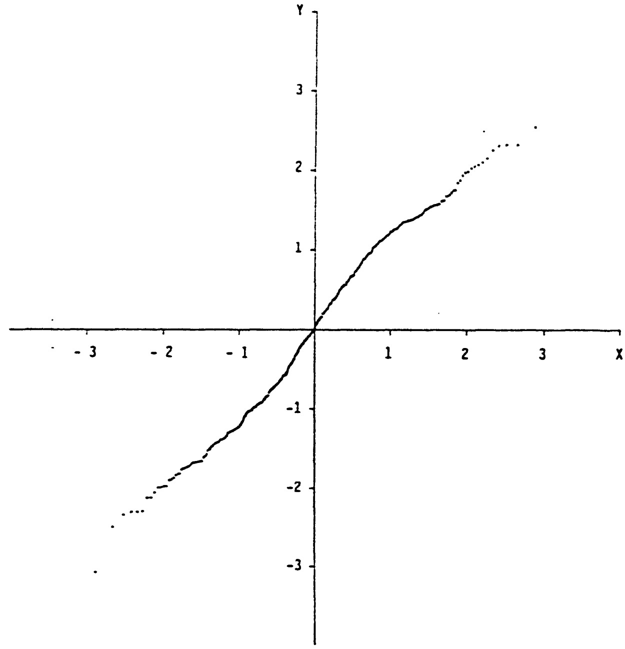

Figure 2. QQ-plot (sample quantiles (Y) against quantiles of a standard normal distribution (X)) of 500 trials for the W statistic with two mutually indepoendent simultaneous autoregressive processes (12 × 12 lattice, $\rho_X(1) = \rho_Y(1) = 0.8$).

for a bilateral test of nominal level α is thus equal to $1 - [\Phi(M_1) - \Phi(M_2)]$ with :

$$M_1 = \frac{C_\alpha[a^2 \operatorname{tr}(\Sigma_\xi^2) + \operatorname{tr}(\Sigma_\xi\Sigma_\theta)]^{1/2} - a\operatorname{tr}(\Sigma_\xi)}{[2a^2 \operatorname{tr}(\Sigma_\xi^2) + \operatorname{tr}(\Sigma_\theta\Sigma_\theta)]^{1/2}}$$

$$M_2 = \frac{-C_\alpha[a^2 \operatorname{tr}(\Sigma_\xi^2) + \operatorname{tr}(\Sigma_\xi\Sigma_\theta)]^{1/2} - a\operatorname{tr}(\Sigma_\xi)}{[2a^2 \operatorname{tr}(\Sigma_\xi^2) + \operatorname{tr}(\Sigma_\xi\Sigma_\theta)]^{1/2}}$$

Φ being the $N(0,1)$ distribution function and C_α being such that $P\{|N(0,1)| > C_\alpha\} = \alpha$.

It is also interesting to be able to compare the power observed by simulations to a reference value. Along these lines, we thus also calculated the power of the classical test of r_{XY}: $\pi_{N^*}(r)$, in a case which would be compatible with the observed empirical variance of r_{XY}, v_e, estimated by the Monte Carlo simulations. Recall that in the case of non autocorrelated variables X and Y and large samples, the variance of r_{XY} is approximately equal to $(1 - \rho_{XY}^2)^2/N - 1$ for a sample of N observations. For autocorrelated X and Y, we thus computed an approximately equivalent sample size, N^* :

$$N^* = 1 + (1 - \rho_{XY}^2)^2/v_e\,.$$

This number N^* was used to compute the reference value, $\pi_{N^*}(r)$, power of the classical test of r_{XY} based on N^* observations.

A summary of the results is given in Table 1a and 1b. In those tables, the observed power of W is only given since it is almost identical to that of $t_{\widehat{M}-2}$. Overall all the powers, whether observed or calculated, are close. The only differences are seen for a few cases of high autocorrelation for either X and W.

In summary we can say that the "theoretical power" $\pi_T(s_{XY})$ gives a good approximation of the observed power in most cases and does not require Monte-Carlo simulations. Furthermore, the modified W and $t_{\widehat{M}-2}$ tests have comparable power to that of a classical test based on an "equivalent" number of observations N^*.

5. Choice of the partition for the covariance structure of X and Y

Computations needed in order to apply the modified tests to a data set are straight forward but rely upon a choice of strata $\{S_0, S_1, S_2, \ldots\}$ of $A \times A$, on each of which the covariance of the two processes is assumed to be constant.

In Table 2 the performance of the $t_{\widehat{M}-2}$ statistic is investigated for different choices of strata in the case of the irregular grid network and Gaussian disc models. Six partitions were defined, ranging from 5 to 21 strata and corresponding to different discretizations of the distance between pairs of locations (assuming isotropy). Overall the type I error was close to 5% for most of the partitions. As the autocorrelation increased, the type I error for the 5-strata partition was inflated whereas there was a stability in the observed error rate when the number of strata increased. The results for the W statistic were similar.

This confirmed that a balance has to be reached between choosing too few classes which can bias expression (1) or too many resulting in less precise estimates of the autocovariances. Clearly in the case simulated where the autocorrelation decreases smoothly with distance, the performance of the $t_{\widehat{M}-2}$ statistic is robust to various choices of partitions.

6. Confidence interval for regression coefficient

Once we have a test for independence it is possible to construct a confidence interval for the regression coefficient b, in the model

$$Y = a1 + bX + Z$$

where Z is a process independent of X and 1 is a vector with unit elements. The confidence interval for b is the set of values of b which we would not actually reject i.e the set of b such that $Y - bX$ has no significant correlation with X.

Defining : $f_\alpha = X_\alpha - \overline{X}$ and $g_\alpha = Y_\alpha - \overline{Y}$ the standardized covariance between $Y - bX$ and X is :

$$W_b = \frac{(g^T f - b f^T f)}{\sqrt{\sum_k N_k \widehat{C}_X(k)\widehat{C}_{Y-bX}(k)}}$$

where $\widehat{C}_{Y-bX}(k) = \widehat{C}_Y(k) + b^2\widehat{C}_X(k) - 2bN_k^{-1} \sum_{S_k} f_\alpha g_\beta$.

This standardized covariance can be used as a pivot to obtain a confidence interval for b. We do not reject the null hypothesis when

$$|W_b| < C_\alpha \text{ where } : P\{|N(0,1)| > C_\alpha\} = \alpha\,.$$

Therefore the confidence interval is :

$$\{b : (g^T f = b f^T f)^2 \leq C_\alpha^2 \Sigma N_k \widehat{C}_X(k)\widehat{C}_{Y-bX}(k)\} \tag{4}$$

or

$$b - T_2 \pm (T_2^2 + T_1 - T_1T_3 - 2bT_2 + b^2T_3)^{1/2}(1 - T_3)^{-1}$$

where $b = s_{XY}/s_X^2$; $T_1 = d_X \sum_k N_k \widehat{C}_Y(k)\widehat{C}_X(k)$; $T_2 = d_X \sum_k N_k \; \widehat{C}_X(k)\widehat{C}_{XY}(k)$; $T_3 = d_X \sum_k N_k \widehat{C}_X^2(k)$ and where $\widehat{C}_{XY}(k) = \Sigma f_\alpha g_\alpha / N_k$ and $d_X = C_\alpha^2 N^{-2} s_X^{-4}$.

7. Examples

Our examples concern the relationship between lung cancer, smoking and industrial factors. We calculate W, $t_{\widehat{M}-2}$ and a confidence interval for the regression coefficient for each example. To provide an additional check on the performance of our tests, we also carried out a Monte Carlo test based on the disc model for cases in which such a model is plausible.

The data

For 82 "départements" we considered male lung cancer mortality rate (LC) over a 2 year period, 1968-1969, standardized over the age 35-74, cigarette sales per inhabitant (CS) in 1953 (a fifteen year time lag was chosen to account for the delay between exposure and the onset of the pathology) and demographic data on the percentage of employed males in the metal industry (MW) and the textile industry (TW) recorded by census in 1962.

The coordinates of the points of the network were identified with the geographical locations of the administrative centers ("préfectures") of French "départements". The spatial structure of these variables was investigated by means of a variogram. In this analysis, $N = 82$ locations were retained after grouping the "départements" around Paris into one area. The distances between the centers of "départements" were partitioned into 15 classes of 70 kilometers intervals each. This gives 15 strata $S_1, \ldots S_{15}$.

The observed variograms, i.e the plot of

$$N_k^{-1} \sum_{(\alpha,\beta)\in S_k} (X_\alpha - X_\beta)^2, k = 1, \ldots 15,$$

against the average distance, d_k, for "départements" in S_k, for the four variables considered are shown in Figure 3. Note that the last two classes contain few pairs and hence have a large variability. Three of the variograms (LC, CS, MW) exhibit clearly an upward trend with increasing distance. Up until the 10th class, the shape of this trend is fairly linear with increasing distance, thus compatible with the disc model for the covariance matrix discussed by Ripley (1981). The variogram for TW gives little indication of any spatial autocorrelation.

Results

Using the standard test base on r_{XY} there is a highly significant positive association both between LC and CS and between LC and MW. The association between LC and TW on the other hand is less strong but still significant at the 1% level (Table 3). The W and $t_{\widehat{M}-2}$ statistics for these 3 examples are shown in Table 3. One can see a substantial reduction of the degrees of freedom when the autocorrelation is taken into account. We note that this occurs also for the case LC-TW which is surprising if one

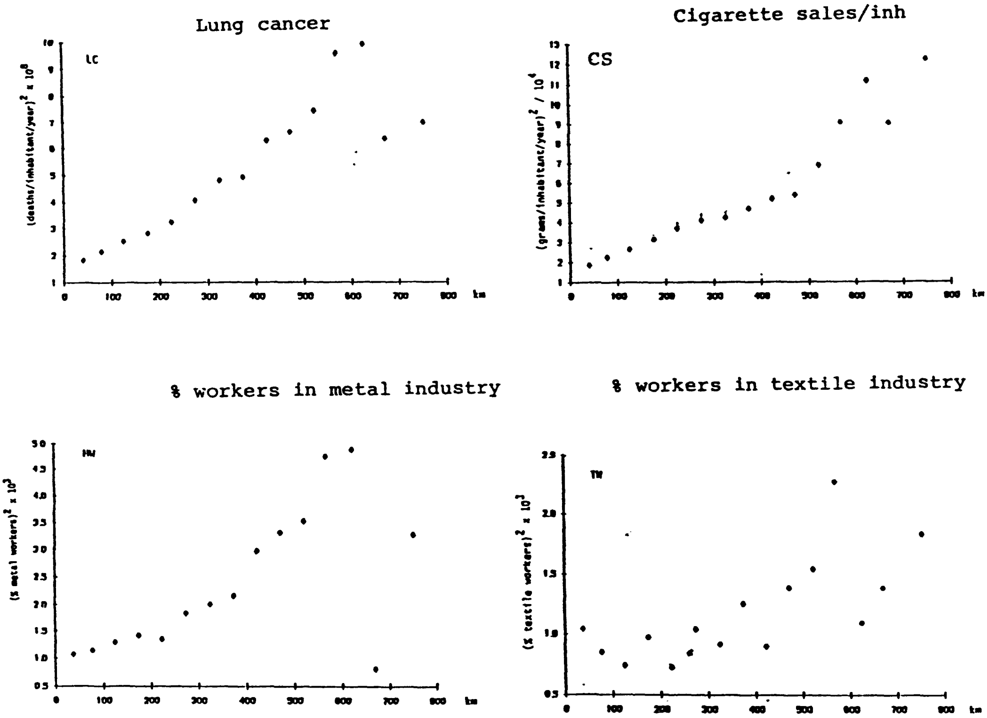

Figure 3. Variograms of the variables considered in §7. Fifteen classes of distance are considered. The numbers of pairs in each class are: 82, 400, 582, 674, 822, 812, 726, 630, 476, 304, 178, 94, 58, 40.

recalls the shape of the variogram of TW. A possible explanation for this is that the geographically small "départements", which are over-represented in the first few strata, are atypical for this particular variable (TW). For CS and MW the effective sample size is about 20% of the original sample size. Consequently the significance levels are reduced but even after this "adjustment" these two factors are statistically significantly associated with lung cancer. For TW the significance disappears after adjustment.

The first two examples were also investigated by a Monte Carlo test. A disc model for the covariance was fitted by maximum likelihood. The parameters were found by direct search and corresponded to autocorrelation ρ (1) of 0.91, 0.85 and 0.91 respectively for LC, CS and MW.

For each example, 1000 pairs of mutually independent variables, with covariance given by the estimated disc model, were generated and the observed correlation coefficient was ranked among the 1000 generated coefficients. The significance levels obtained are given in Table 3. The agreement between them and the significance levels of the W or $t_{\widehat{M}-2}$ tests is better for CS than for MW ; this is possibly due to a better fit of the disc model for the CS variable. Confidence intervals, given by (4), for the regression coefficients were also calculated. We note that they are not symmetric.

8. Discussion

In this paper we have studied the properties of modified tests of the empirical correlation coefficient between two spatial processes. We have shown that by a simple adjustment, correct level of significance can be reached and that the power under a simple linear alternative is compatible with that of a standard test in an equivalent situation. These tests can be applied both to regularly and irregularly spaced points and can be considered as a first step in an analysis of association when detailed spatial modelling is not suitable.

The performance did not vary much when different strata of equal covariance were chosen. On small lattices, the modified $t_{\widehat{M}-2}$ statistic is better than the standardized covariance.

Application of these tests to data may also give pivotal confidence interval for the regression coefficient. Furthermore, if one is prepared to model the observed covariance structure, Monte Carlo tests of association can be performed. In the examples investigated, the results from the two types of testing procedures were close.

It would be interesting to develop distribution-free tests of association based on permutations and to compare their performance to that of the proposed modified tests in the case of non Gaussian spatial distributions.

Table 1a

Power of the modified tests: results concerning the testing of the correlation between X and $Y = aX + W$ where both X and Y are spatially autocorrelated, X and W independent and of equal variance and a is chosen so that the correlation ρ_{XY} between X and Y takes the value 0.2.

		$\rho_{XY} = 0.2$				
ρ_W	ρ_X	0.0	0.2	0.4	0.6	0.8
	N^*	79	82	78	79	84
	$\pi_{N^*}(r)$	0.42	0.44	0.42	0.42	0.44
0.0	power of W	0.42	0.44	0.43	0.39	0.35
	$\pi_T(s_{XY})$	0.44	0.44	0.43	0.42	0.37
		76	74	67	71	63
0.2		0.41	0.40	0.36	0.38	0.34
		0.42	0.42	0.39	0.34	0.30
		0.44	0.41	0.39	0.36	0.32
		80	78	66	57	54
0.4		0.42	0.42	0.36	0.32	0.30
		0.44	0.36	0.37	0.34	0.21
		0.44	0.39	0.36	0.32	0.27
		78	75	61	46	35
0.6		0.41	0.40	0.33	0.25	0.20
		0.48	0.36	0.36	0.28	0.20
		0.45	0.39	0.34	0.27	0.20
		78	66	54	39	19
0.8		0.41	0.36	0.30	0.22	0.12
		0.52	0.48	0.42	0.28	0.12
		0.48	0.40	0.44	0.23	0.13

500 simulations were carried out. The observed power of W is compared with $\pi_{N^*}(r)$ and $\pi_T(s_{XY})$ (cf. §4). The observed power of $t_{\hat{M}-2}$ is almost identical to that of W. Standard deviations and confidence intervals for the observed proportions can be calculated according to binomial sampling.

Table 1b

Power of the modified tests: results concerning the testing of the correlation between X and $Y = aX + W$ where both X and Y are spatially autocorrelated, X and W independent and of equal variance and a is chosen so that the correlation ρ_{XY} between X and Y takes the value 0.4.

		$\rho_{XY} = 0.4$				
ρ_W	ρ_X	0.0	0.2	0.4	0.6	0.8
	N^*	83	87	81	70	63
	$\pi_{N^*}(r)$	0.97	0.97	0.96	0.93	0.90
0.0	power of W	0.97	0.96	0.96	0.92	0.87
	$\pi_T(s_{XY})$	0.94	0.93	0.92	0.89	0.74
		82	73	76	68	55
0.2		0.96	0.94	0.95	0.90	0.86
		0.96	0.96	0.94	0.92	0.83
		0.94	0.91	0.90	0.85	0.70
		84	67	61	63	49
0.4		0.97	0.92	0.89	0.90	0.81
		0.97	0.92	0.91	0.88	0.79
		0.94	0.91	0.87	0.81	0.65
		83	69	66	44	30
0.6		0.97	0.93	0.92	0.77	0.58
		0.96	0.95	0.93	0.81	0.56
		0.94	0.90	0.85	0.74	0.54
		68	52	51	33	20
0.8		0.92	0.83	0.82	0.63	0.39
		0.97	0.93	0.89	0.76	0.42
		0.95	0.91	0.84	0.68	0.38

500 simulations were carried out. The observed power of W is compared with $\pi_{N^*}(r)$ and $\pi_T(s_{XY})$ (cf. §4). The observed power of $t_{\hat{M}-2}$ is almost identical to that of W. Standard deviations and confidence intervals for the observed proportions can be calculated according to binomial sampling.

Table 2

Percentage of type I errors of the $t_{\hat{M}_2}$ statistic for different partitions of the covariance structure

Number of strata in each partition	5	9	13	15	17	21
$\rho_X^{(1)} = \rho_Y^{(1)} = 0$	0.056	0.052	0.05	0.056	0.054	0.054
$\rho_X^{(1)} = \rho_Y^{(1)} = 0.2$	0.04	0.04	0.036	0.032	0.034	0.03
$\rho_X^{(1)} = \rho_Y^{(1)} = 0.4$	0.062	0.046	0.05	0.05	0.052	0.05
$\rho_X^{(1)} = \rho_Y^{(1)} = 0.6$	0.066	0.058	0.054	0.056	0.052	0.052
$\rho_X^{(1)} = \rho_Y^{(1)} = 0.8$	0.068	0.058	0.048	0.04	0.046	0.046

Table 3

Comparison of the significance levels for tests of the association between lung cancer mortality rates and several risk factors given by standard test, W and $t_{\hat{M}-2}$ tests and Monte Carlo (MC$^+$) simulations, $\hat{\gamma}$ is the estimated regression coefficient.

	r	t^*_{N-2}/p	W/p	$\hat{M}$	$t_{\hat{M}-2}/p$	MC $^+$	95%CI for γ
cigarette sales per inhabitant (1953) (CS)	0.76	10.48 10^{-21}	2.94 0.0032	15	4.22 0.001	2/1000	0.78 [0.54; 0.88]
% male workers in metal industry (1962) (MW)	0.63	7.16 10^{-11}	2.48 0.0136	16	3.00 0.01	45/1000	0.29 [0.11; 0.36]
% male workers in textile industry (1962) (TW)	0.28	2.57 0.01	1.51 0.13	30	1.52 0.15	–	0.18 [−0.07; 0.37]

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