

INTRODUCTION

In 1926, seven years after accepting a post of statistician at Rothamstead Experimental Station in England, Fisher suggested the use of blocking and within-block randomization to increase the precision, and provide a valid estimate of the error, of field experiments in agriculture. The error being that ‘which cannot be ascribed to carelessness in measuring the land or weighing the produce, and which is consequently described as due to *soil heterogeneity*’. That arrangement of agricultural field experiments partially allows for the statistical dependency between yields of nearby plots of land, which arises from fertility trends across the field, competition between plants in adjacent plots, and interference between treatments.

The evidence presented by Fairfield Smith in 1938, in the *Journal of Agricultural Science*, about the relation between variance in yield and the size and shape of plots, was the earliest indication that it might not always be possible satisfactorily to remove the effects of spatial structure at all the relevant scales. Thence the need for methods to deal directly with statistically dependent quantities, observed at different locations. Spatial statistics arose to meet this call.

In the 1950’s, Peter Whittle launched a program of study of spatial processes emphasizing model formulation focusing on second-order properties, and drew attention to the potential relevance of the tools employed in Statistical Mechanics.

The methods developed by Gibbs, around the turn of the century, to study mechanical systems comprising large numbers of particles, had been brought to the attention of probabilists and statisticians mainly via Khinchin’s 1949 monograph. In *Elementary Principles in Statistical Mechanics*, published in 1902, Gibbs introduced explicit models for probability distributions of the “states” of such systems, and proceeded to show that all the corresponding thermodynamic properties can be derived from such distributions.

The model suggested by Wilhelm Lenz in 1920, and then developed and described by his student Ernst Ising in 1925, for the probability distribution of the energy states of a magnetic system, defined a paradigm whose brilliant career contradicted Ising’s own grim assessment of the long-term prospects for his model. Indeed, it paved the way for major advances in the theory of order-disorder phenomena in metallic alloys, and, many years later, would prove seminal in the development of contextual models for digital images.

The well-deserved notoriety that Lars Onsager derived from the exact solution, announced in 1942, of a special case of the two-dimensional Lenz-Ising model, would serve as an early warning that, in the field of study of systems of interacting components,, analytic solutions would be the exception, rather than the rule. Through the ensuing years, techniques for approximate computation relevant to the Gibbs-Ising program, from series expansions to Monte Carlo methods, would be independently reinvented several times, in different disciplines studying spatial phenomena.

Statistical mechanics has been a continuing source of models, methods, and metaphors for the theory and practice of statistics of spatial phenomena, and, generally, for the statistics of systems with many interacting components, be they sites in space, or nodes in an abstract network. The breadth, depth, and promise of the inspiration that springs from this source can best be appraised in Ulf Grenander’s seminal *Lectures in Pattern Theory*.

The needs expressed, during the war years and early 1950's, by geographers and silviculturists, were addressed by Bertil Matérn in a thesis, on *Spatial Variation*, presented in 1960 to the University of Stockholm. This was the first meticulous study of sampling methods for spatial processes, taking into account spatial covariances.

In 1951, Daniel Krige, concerned with optimal sampling and exploitation of ore deposits in the Witwatersrand gold province, initiated developments in all respects parallel to those prompted by the geographers and silviculturists, but here undertaken by geologists and mining engineers — George Matheron's *Traité de Géostatistique Appliquée* describes the accomplishments in the first decade of Geostatistics.

The ideas put forward by Paul Lévy, in the second half of the 1940's, for research on processes comprising random variables indexed by points of multidimensional spaces, came to fruition only about two decades later, when Dobrushin rigorously established the usage of *local conditional distributions* to construct those processes, and further elucidated the connections between the question of uniqueness, the role of boundary conditions, and the nature of spatial dependency.

In 1970-71, Averintsev in the Soviet Union, and Spitzer in the United States, independently proved first versions of the equivalence between Markov and Gibbs random fields. Presently this was generalized by Hammersley and Clifford, and provided the necessary foundation whereupon Julian Besag erected an innovative program of inference for random fields.

The approach to point processes inaugurated by Doob and Dobrushin in the 1950's set the stage for the development, by Prohorov, Mecke, and others, of the conceptual apparatus that supports the modern theory of point processes. These can model a wide variety of spatial processes, including patterns of points or lines, mosaic processes, and random aggregates of *grains*.

Perhaps nowhere else has there emerged a larger panoply of *ad hoc* techniques than in the analysis of spatial point processes: geographers keen on 'nearest-neighbor' methods, and plant ecologists relying on Greig-Smith's quadrat methods. Bartlett and Ripley defined a coherent program of inference for spatial point processes based on second-order methods, including spectral analysis.

Today, data about phenomena unfolding in space and evolving over time are collected in large amounts, often continuously by automated systems, in many branches of science and technology.

For example, the multispectral scanners aboard satellites of NASA's Landsat program have gathered data at rates of about 5×10^9 bits per hour. The data rates expected from NASA's planned Earth Observing System are much higher. The amount of data that may have to be analyzed to solve a real problem can also be very large: the order of magnitude of a typical data set, used for the study of climate at a global scale, and comprising multispectral images gathered by the Advanced Very High Resolution Radiometer, aloft a NOAA satellite that monitors cloud cover and sea-surface temperature, is 10^{12} bits.

The superabundance of data about spatio-temporal processes not only defines new challenges in data representation, storage, retrieval and transmission, but also calls for novel techniques for data summarization driven by automated methods for data understanding.

In digital image analysis, in particular, there are notable developments in opposite, yet complementary directions. On the one hand, the demand continues to grow

for increasingly sharp images, which reveal otherwise imperceptible detail, produced by techniques that resolve features at scales smaller than the support of the point-spread function of the imaging system, and even, occasionally, at sub-pixel scales. These developments have been driven mainly by requirements in astronomical and medical applications, and certainly have acquired special cogency in relation with the Hubble Space Telescope program. On the other hand, there is a pressing need for methods that can perform image smoothing intelligently, to suppress irrelevant detail, and image compression, possibly subject to constraints on the type and amount of information that can be discarded or withheld in the process — and a sense is forming that wavelet representations are bound to play a major role in this technology.

In all cases, the ever refining techniques of data acquisition are showing, ever more clearly, the pervasiveness of pattern at all scales: that the existence of interactions between entities occupying different points of space and evolving over time, is the rule rather than the exception. There has also developed a clear perception of the importance of components of structure that lie beyond the reach of second-order methods.

The National Research Council has recently published a report on *Spatial Statistics and Digital Image Analysis*, organized by a panel co-chaired by Julian Besag and James Simpson, that provides an overview designed to enhance the awareness of researchers and graduate students, in the mathematical and physical sciences, about this interdisciplinary field, and to stimulate its further growth.

The purpose of spatial statistics, including statistical digital image analysis, is to uncover and describe structure in outcomes of stochastic processes whose elements are indexed by points of multidimensional spaces, and also of developing conceptual tools to comprehend the system of interactions that govern such structure.

The realization of this purpose, at such level of generality, has opened new grounds for unexpected applications of methods that had, until very recently, been developed or used exclusively within either computational physics or spatial statistics. This speaks to the intellectual depth of the field, and bodes well for its future growth.

One of these breakthroughs was the application, by Nuala Sheehan, in 1989, of methods used in image restoration, to the estimation of genetic pedigrees. It had, until then, proven next to impossible to compute maximum likelihood estimates in many cases of practical interest, especially for pedigrees comprising large numbers of individuals, and displaying a complex network of family relations. Judea Pearl's vision of a probabilistic paradigm for the propagation of evidence in artificial reasoning systems, developed along similar lines, and led to the same methods.

Another outgrowth from spatial statistics has considerably broadened the range of practical applicability of Bayes procedures. Intimated by Grenander in his 1980 *Lectures in Pattern Theory*, and later developed by Donald and Stuart Geman into technology for image restoration, the *Gibbs Sampler*, and other Monte Carlo methods to simulate outcomes of stochastic processes on general graphs, have proven a panacea for computations involving posterior distributions. David Clayton, Adrian Smith, and co-workers recently developed striking illustrations of this technology.

Even the most venerable of the statistical old-timers, the analysis of agricultural experiments, has, very much owing to Julian Besag's continuing impetus, come full circle to benefit from the latest advances in the statistical analysis of random fields.

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