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A MICROLOCAL VERSION OF THE RIEMANN-HILBERT CORRESPONDENCE

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Dedicated to Jean Leray

1. Introduction

Let X be a complex n-dimensional manifold. Recall that the "Riemann-Hilbert correspondence" consists of the following diagram, together with the assertion that the arrows are equivalences of categories quasi-inverse to each other:

$$\mathrm{Perv}(X)^{\circ} \underset{\mathrm{Sol}}{\overset{RH}{\rightleftarrows}} \mathrm{Reghol}(\mathcal{D}_X). \tag{1}$$

We make use of the following notations:

- $D^b_{\mathbb{C}-c}(X)$ is the derived category of bounded complexes of sheaves of $\mathbb{C}$-vector spaces on X with $\mathbb{C}$-constructible cohomology,
- $\mathrm{Reghol}\,(\mathcal{D}_X)$ is the abelian category of regular holonomic (left) $\mathcal{D}_X$-modules,
- $\mathcal{H}\mathrm{ol}(\mathcal{D}^\infty_X)$ is the category of modules of the form $\mathcal{D}^\infty_X \underset{\mathcal{D}_X}{\otimes} \mathcal{M}$ where $\mathcal{M}$ is a holonomic $\mathcal{D}$-module,
- $D^b_{r-h}(\mathcal{D}_X)$ is the derived category of bounded complexes of $\mathcal{D}_X$-modules with regular holonomic cohomology,
- $D^b_h(\mathcal{D}^\infty_X)$ is the derived category of bounded complexes of admissible $\mathcal{D}^\infty_X$-modules (in the sense of [11]) with cohomology in $\mathcal{H}\mathrm{ol}(\mathcal{D}^\infty_X)$,
- $\mathrm{Perv}(X)$ is the full abelian subcategory of $D^b_{\mathbb{C}-c}(X)$ whose objects are "perverse sheaves", where we adopt for our purpose a definition shifted

by $n = \dim_{\mathbb{C}} X$ from the usual one, i.e. given $F \in Ob(D^b_{\mathbb{C}-c}(X))$, we say F is an object of $\mathrm{Perv}(X)$ if and only if $F[n]$ is perverse in the usual sense of [2] (e.g. if $Y \subset X$ is a purely d-codimensional complex set then we say that $\mathbb{C}_Y[-d]$ is perverse; see §4.

Recall that one sets $\mathrm{Sol}(\mathcal{M}) = R\mathcal{H}om_{\mathcal{D}}(\mathcal{M}, \mathcal{O})$ or $R\mathcal{H}om_{\mathcal{D}^\infty}(\mathcal{M}, \mathcal{O})$ accordingly, and that the arrow bearing that name in (1) was constructed in [4]. The construction of the temperate $R\mathcal{H}om(\cdot, \mathcal{O})$-functor RH and proof that RH is an equivalence was performed in [4], [5]. Also recall that the equivalence between $\mathrm{Reghol}(\mathcal{D}_X)$ and $\mathcal{H}\mathrm{ol}(\mathcal{D}_X^\infty)$ under $\mathcal{D}_X^\infty \overset{\mathbf{L}}{\underset{\mathcal{D}_X}{\otimes}} (\cdot)$ was proven in [7].

An independent proof that Sol is an equivalence is performed in [10], but it could not be used for microlocalization.

The point of interest here is to give a microlocal version of (1). Namely, if $\pi : T^*X \to X$ is the cotangent bundle of X, and $p \in \overset{\circ}{T}{}^*X \underset{\mathrm{def}}{=} T^*X \backslash T^*_X X$, one has the abelian category $\mathrm{Reghol}(\mathcal{E}_{X,p})$ of germs of regular holonomic modules over the ring of microdifferential operators $\mathcal{E}_{X,p}$ of [11] which should be equivalent to a category defined by a suitable microlocalization of $\mathrm{Perv}(X)$. The precise statement goes as follows.

We set $\mathbb{C}^\times := \mathbb{C}\backslash\{0\}$ and $\gamma : T^*X \to T^*X/\mathbb{C}^\times$.

THEOREM 1.1. *One has the following commutative diagram* (2) *and all the horizontal arrows are equivalences of categories.*

$$(2)\qquad \begin{array}{ccc} \mathrm{Perv}(X;\mathbb{C}^\times p)^\circ & \xrightarrow{\quad \mu RH \quad} & \mathrm{Reghol}(\mathcal{E}_{X,p}) \\ \Big\downarrow & & \Big\downarrow {\scriptstyle \mathcal{E}^{\mathbb{R},f}_{X,p} \underset{\mathcal{E}_{X,p}}{\otimes} (\cdot)} \\ \mathrm{Perv}(X;p)^\circ & \underset{\mathrm{Sol}_p}{\overset{T\text{-}\mu\mathrm{hom}(\cdot,\mathcal{O}_X)}{\rightleftarrows}} & \mathrm{Reghol}(\mathcal{E}^{\mathbb{R},f}_{X,p}). \end{array}$$

We make use of the following notations:

- $\mathcal{E}_X^\infty$ is the sheaf of infinite order microdifferential operators of [11],
- $\mathcal{E}_X^{\mathbb{R}}$ is the sheaf of holomorphic microlocal operators of [11],
- $\mathcal{E}_X^{\mathbb{R},f}$ is the temperate analogue of $\mathcal{E}_X^{\mathbb{R}}$ introduced in [1],
- an object of $\mathrm{Reghol}(\mathcal{E}^{\mathbb{R},f}_{X,p})$ is by definition of the form $\mathcal{E}^{\mathbb{R},f}_{X,p} \underset{\mathcal{E}_X}{\otimes} \mathcal{M}$ with $\mathcal{M} \in Ob(\mathrm{Reghol}(\mathcal{E}_{X,p}))$, with a similar definition for $\mathcal{H}ol(\mathcal{E}^\infty_{X,p})$ and $\mathcal{H}ol(\mathcal{E}^{\mathbb{R}}_{X,p})$,
- the categories $\mathrm{Perv}(X;\mathbb{C}^\times p)$ and $\mathrm{Perv}(X;p)$ are defined below,
- $\mu\mathrm{hom}(\cdot,\cdot)$ is Kashiwara and Schapira's functor of [9],
- T-$\mu\mathrm{hom}(\cdot, \mathcal{O}_X)$ is the temperate version of $\mu\mathrm{hom}(\cdot, \mathcal{O}_X)$ of [1], while $\mu RH := \gamma^{-1} R\gamma_* \, T\text{-}\mu\mathrm{hom}(\cdot, \mathcal{O}_X)$.

Assuming the definition of $\mathrm{Perv}(X;p)$, the construction of Sol_p is implicit in [8], and explicit in [12].

The various microlocalizations of $\mathrm{Perv}(X)$ are performed by essential use of the microlocal theory of sheaves of Kashiwara and Schapira [9] and by using the microlocal characterization of perverse sheaves of loc.cit.

We stress the point that these microlocalizations rely on necessary *real* (subanalytic) geometry.

The main tool in the proof is the invariance by canonical transformations which allows one to make use of the generic position theorem of [7] which reduces the situation to that of (regular holonomic) $\mathcal{D}$-modules.

The proof of the main theorem goes here as follows. In Sections **2–4** we explain the first vertical arrow in (2). In Section 5 we prove that the morphism $\mu RH : \mathrm{Perv}(X;\mathbb{C}^\times p)^\circ \to \mathrm{Reghol}(\mathcal{E}_{X,p})$ is an equivalence of categories. Finally, the fact that the arrows T-$\mu\mathrm{hom}(\cdot,\mathcal{O}_X)$ and Sol_p appearing in (2) are quasi-inverse to each other, is easily deduced from [12].

2. The category $D^b_{\mathbb{R}-c}(X;\Omega)$

Let X be a real analytic manifold, $D^b(X)$ the derived category of the category of bounded complexes of sheaves on X and $D^b_{\mathbb{R}-c}(X)$ its full triangulated subcategory of complexes with $\mathbb{R}$-constructible cohomology. The following is detailed in [1, Appendix].

If $\Omega \subset T^*X$ is any subset of the cotangent bundle of X the fundamental category occurring in [9] is

$$D^b(X;\Omega) \underset{\mathrm{def}}{=} D^b(X)/\mathcal{N}_\Omega,$$

where $\mathcal{N}_\Omega$ is the null-system of objects F whose micro-support $SS(F)$ does not meet Ω (cf. loc.cit.).

We set here

$$D^b_{\mathbb{R}-c}(X;\Omega) = D^b_{\mathbb{R}-c}(X)/\mathcal{N}_\Omega \cap Ob(D^b_{\mathbb{R}-c}(X)).$$

Note that if $\Omega' \subset \Omega$ there is a canonical functor $\mathcal{D}^b_{\mathbb{R}-c}(X;\Omega) \longrightarrow D^b_{\mathbb{R}-c}(X;\Omega')$.

If $\Omega = \{p\}$ is a point we write $D^b(X;p)$ instead of $D^b(X;\{p\})$ and so forth.

By the results of [9] it is easy to see that

LEMMA 2.1. *$D^b_{\mathbb{R}-c}(X;p)$ is a full triangulated subcategory of $D^b(X;p)$.*

An adaptation of the microlocal kernel operations of [9] yields also the invariance under "extended canonical transformations" of loc.cit.

More precisely, let Y be another copy of X and denote by q_j the j-th projection of $X \times Y$ and by $(\cdot)^a$ the antipodal map of T^*Y.

Let $p_X \in T^*X$, $p_Y \in T^*Y$ and $K \in Ob(D^b_{\mathbb{R}-c}(X \times Y))$ satisfy the following condition:

(3) $SS(K) \cap (\{p_X\} \times T^*Y) \subset \{(p_X, p_Y^a)\}$ in the neighborhood of that point.

For $F \in Ob(D^b_{\mathbb{R}-c}(Y))$ one defines a pro-object of $D^b_{\mathbb{R}-c}(X; p_X)$ by setting

$$\Phi^{\mu}_K(F) = \text{“}\varprojlim\text{”}\, Rq_{1!}(K_{X\times V} \otimes q_2^{-1}F) \tag{4}$$

where V runs over the set of relatively compact open subanalytic neighborhoods of $y = \pi(p_Y)$. Actually one has

LEMMA 2.2. *For $K \in Ob(D^b_{\mathbb{R}-c}(X \times Y))$ satisfying* (3), *this pro-object is an object of $D^b_{\mathbb{R}-c}(X; p_X)$ and the functor $\Phi^{\mu}_K : D^b_{\mathbb{R}-c}(Y; p_Y) \to D^b_{\mathbb{R}-c}(X; p_X)$ is well defined.*

Note that the functor $\Phi_K(\cdot) = Rq_{1!}(K \otimes q_2^{-1}(\cdot))$ would not be defined here in general.

PROPOSITION 2.3. *Let $\varphi : (T^*Y)_{p_Y} \to (T^*X)_{p_X}$ be a germ of canonical transformation and Λ its associated germ of Lagrangian manifold in $T^*(X \times Y)$. One may find $K \in Ob(D^b_{\mathbb{R}-c}(X \times Y))$ with $SS(K) \subset \Lambda$ in the neighborhood of (p_X, p_Y^a), such that $\Phi^{\mu}_K : D^b_{\mathbb{R}-c}(Y; p_Y) \to D^b_{\mathbb{R}-c}(X; p_X)$ is an equivalence of categories.*

3. The category $D^b_{\mathbb{C}-c}(X; \Omega)$

Let now X be a complex n-dimensional manifold, and $X_{\mathbb{R}}$ the underlying real manifold. Recall that for $F \in Ob(D^b_{\mathbb{R}-c}(X))$ one has the following characterization (cf. [9]):

$$(F \in Ob(D^b_{\mathbb{C}-c}(X))) \Leftrightarrow (SS(F) \text{ is } \mathbb{C}^{\times}\text{-conical}) \Leftrightarrow (SS(F) \text{ is } \mathbb{C}\text{-Lagrangian}), \tag{5}$$

thus we may define for any subset $\Omega \subset T^*X$ a full triangulated subcategory of $D^b_{\mathbb{R}-c}(X; \Omega)$ by setting

$$D^b_{\mathbb{C}-c}(X; \Omega) \underset{\text{def}}{=} \text{the full subcategory of } D^b_{\mathbb{R}-c}(X; \Omega) \text{ of the objects } F \in Ob(D^b_{\mathbb{R}-c}(X)) \text{ such that } SS(F) \text{ is } \mathbb{C}^{\times}\text{-conical in a neighborhood of } \Omega. \tag{6}$$

PROPOSITION 3.1 (See [1, Appendix]). *Let Y be another copy of X, $\varphi : (T^*Y)_{p_Y} \to (T^*X)_{p_X}$ be a germ of complex canonical transformation and $\Lambda \subset T^*(X \times Y)$ its associated complex Lagrangian submanifold. Then*

(i) *there exists $K \in Ob(D^b_{\mathbb{C}-c}(X \times Y; (p_X, p_Y^a)))$ with $SS(K) \subset \Lambda$ in a neighborhood of (p_X, p_Y^a) such that the functor of Proposition 2.3 induces an equivalence of categories*

$$\Phi^\mu_K : D^b_{\mathbb{C}-c}(Y; p_Y) \to D^b_{\mathbb{C}-c}(X; p_X),$$

(ii) *if moreover φ is globally defined on the orbit $\mathbb{C}^\times p_Y$ then there is $K \in Ob(D^b_{\mathbb{C}-c}(X \times Y; \mathbb{C}^\times(p_X, p_Y^a)))$, with $SS(K) \subset \Lambda = \mathbb{C}^\times \Lambda$ in a neighborhood of $\mathbb{C}^\times(p_X, p_Y^a)$ such that Φ^μ_K induces an equivalence of categories*

$$\Phi^\mu_K : D^b_{\mathbb{C}-c}(Y; \mathbb{C}^\times p_Y) \to D^b_{\mathbb{C}-c}(X, \mathbb{C}^\times p_X).$$

Point (i) follows easily from Proposition 2.3 by (5), because Φ^μ_K preserves local $\mathbb{C}^\times$-conicity; then (ii) stems from (i) and formula (4) that shows that Φ^μ_K is defined at any point in the fiber of π over $\pi(p)$.

For example one has $D^b_{\mathbb{C}-c}(X; T^*X) = D^b_{\mathbb{C}-c}(X)$ and if $x \in X \cong T^*_X X$ one has the equivalence $(F \in Ob(D^b_{\mathbb{C}-c}(X; x))) \Leftrightarrow (F \in Ob(D^b_{\mathbb{R}-c}(X))$ and $F|_V \in Ob(D^b_{\mathbb{C}-c}(V))$ for some open neighborhood V of x).

Note that, in general, the objects of $D^b_{\mathbb{C}-c}(X; p)$ do not have $\mathbb{C}$-constructible cohomologies and the natural functor $D^b_{\mathbb{C}-c}(X)/\mathcal{N}_p \cap D^b_{\mathbb{C}-c}(X) \to D^b_{\mathbb{C}-c}(X; p)$ is not an equivalence.

On the other hand, one has the following geometrical version of the generic position theorem. Recall (cf. [7]) that a complex Lagrangian subset $\Lambda \subset T^*X$ is said to have a *generic position* at $p \in \overset{\circ}{T^*} X$ if and only if

(7) $$\Lambda \cap \pi^{-1}\pi(p) = \mathbb{C}^\times p \quad \text{in a neighborhood of } p.$$

PROPOSITION 3.2. *Let $F \in Ob(D^b_{\mathbb{C}-c}(X; p))$ such that $SS(F)$ is in a generic position at p. Then there exists $F' \in Ob(D^b_{\mathbb{C}-c}(X; \pi(p)))$ such that $F' \simeq F$ in $D^b(X; p)$.*

The proof goes by showing that one may "cut off" the non-$\mathbb{C}$-Lagrangian part of $SS(F)$ in $\pi^{-1}\pi(p)$, i.e. one finds kernels K, K^* in $D^b_{\mathbb{C}-c}(X \times X; (p, p^a))$ and an open subanalytic neighborhood U of x in X such that K, K^* satisfy the conditions of Proposition 3.1(i), $\Phi^\mu_{K^*}$ is a quasi-inverse of Φ^μ_K and $F' := \Phi^\mu_{K^*}((\Phi^\mu_K F)_U)$ is such that $SS(F')$ is $\mathbb{C}^\times$-invariant in $\pi^{-1}(U)$. Thus $F' \in Ob(D^b_{\mathbb{C}-c}(X; \pi(p)))$ by (5) and $F' \simeq F$ in $D^b(X; p)$ by Proposition 3.1.

Alternatively, the proof is obtained by using the refined version of [3] of a microlocal cut-off lemma of [9] where non-convex sets are allowed.

4. Microlocalization of perverse sheaves

In [9] one finds the following microlocal characterization of perverse sheaves: An object $F \in Ob(D^b_{\mathbb{C}-c}(X))$ is a perverse sheaf if and only if it satisfies the following condition (cf. [9, (10.3.7)])

(8) For any non-singular point $p \in SS(F)$ such that $\pi : SS(F) \to X$ has constant rank in a neighborhood of p, there exists a complex d-codimensional submanifold $Y \subset X$ such that $F \simeq \mathbb{C}^m_Y[-d]$ in $D^b(X;p)$ for some m.

Thus for any subset $\Omega \subset T^*X$ we may define a full subcategory $\mathrm{Perv}(X;\Omega)$ of $D^b_{\mathbb{C}-c}(X;\Omega)$ in the following manner.

DEFINITION 4.1. $Ob(\mathrm{Perv}(X;\Omega)) \underset{\mathrm{def}}{=} \{F \in Ob(D^b_{\mathbb{C}-c}(X;\Omega));\ F$ satisfies condition (8) at any p in a neighborhood of $\Omega\}$.

Then the following results from §3 and the characterization (8).

PROPOSITION 4.2. *Let $\Omega = \{p\}$ (resp. $\Omega = \mathbb{C}^\times p$).*

(i) $\mathrm{Perv}(X;\Omega)$ *is invariant by extended canonical transformation in the sense of Proposition* 3.1 (i) (*resp. Proposition* 3.1 (ii)).

(ii) *Let $F \in \mathrm{Perv}(X;p)$ (resp. $\mathrm{Perv}(X;\mathbb{C}^\times p)$) such that $SS(F)$ is in a generic position at p. Then there is $F' \in \mathrm{Perv}(X;\pi(p))$ such that $F \simeq F'$ in $D^b(X;p)$.*

(iii) $\mathrm{Perv}(X;\Omega)$ *is a full abelian subcategory of* $D^b_{\mathbb{C}-c}(X;\Omega)$.

5. The equivalence $\mu RH : \mathrm{Perv}(X;\mathbb{C}^\times p)^\circ \to \mathrm{Reghol}(\mathcal{E}_{X,p})$

Recall that Kashiwara's functor RH of cohomology with bounds of [5], [6] is defined on $\mathbb{R}$-constructible complexes, more precisely

$$RH : D^b_{\mathbb{R}-c}(X)^\circ \to D^b(\mathcal{D}_X)$$

(where $D^b(\mathcal{D}_X)$ stands for $D^b(\mathrm{Mod}\,\mathcal{D}_X)$), and it is microlocalized in [1] as a functor

$$T\text{-}\mu\mathrm{hom}(\cdot,\mathcal{O}_X) : D^b_{\mathbb{R}-c}(X)^\circ \to D^b_{\mathbb{R}_{>0}}(\pi^{-1}\mathcal{D}_X)\,,$$

where the latter category is the full subcategory subcategory of the complexes of $D^b(\pi^{-1}\mathcal{D}_X) := D^b(\mathrm{Mod}(\pi^{-1}\mathcal{D}_X))$ with $\mathbb{R}_{>0}$-homogeneous cohomology. Since one has

$$\mathrm{supp}(T\text{-}\mu\mathrm{hom}(F,\mathcal{O}_X)) \subset SS(F),$$

it follows that for any subset $\Omega \subset T^*X$, the functor of triangulated categories

$$T\text{-}\mu\mathrm{hom}(\cdot,\mathcal{O}_X) : D^b_{\mathbb{R}-c}(X;\Omega)^\circ \to D^b_{\mathbb{R}_{>0}}(\pi_\Omega^{-1}\mathcal{D}_X)$$

is well-defined, where $\pi_\Omega := \pi|_\Omega : \Omega \to X$. If moreover $\Omega = \mathbb{C}^\times\Omega$ is a $\mathbb{C}^\times$-invariant subset we set for $F \in Ob(D^b_{\mathbb{R}-c}(X))$,

$$(9) \qquad \mu RH(F) \underset{\mathrm{def}}{=} \gamma^{-1} R\gamma_* \, T\text{-}\mu\mathrm{hom}(F, \mathcal{O}_X) \in Ob(D^b_{\mathbb{R}_{>0}}(\pi_\Omega^{-1}\mathcal{D}_X)).$$

Recall also the following facts:

- For any $F \in Ob(D^b_{\mathbb{R}-c}(X))$ and any $j \in \mathbb{Z}$, $H^j T\text{-}\mu\mathrm{hom}(F, \mathcal{O}_X)$ is an $\mathcal{E}_X^{\mathbb{R},f}$-module,
- $\mathcal{E}_X^{\mathbb{R},f}$ is faithfully flat on $\mathcal{E}_X$ and $\gamma^{-1} R\gamma_* \mathcal{E}_X^{\mathbb{R},f} \cong \mathcal{E}_X$,

and we have invariance by canonical transformations, that is, with the hypotheses of Proposition 3.1(i), one may find a section

$$s \in H^\circ(T\text{-}\mu\mathrm{hom}(K, \Omega_{X\times Y/X}))_{(p_X, p_Y^a)}$$

(where $\Omega_{X\times Y/X}$ means the sheaf of maximum degree forms relative to $X \times Y \to X$) such that the correspondence $P \in \mathcal{E}^{\mathbb{R},f}_{X,p_X} \mapsto Q \in \mathcal{E}^{\mathbb{R},f}_{Y,p_Y}$ such that $Ps = sQ$ is a ring isomorphism compatible with a natural isomorphism $T\text{-}\mu\mathrm{hom}(F, \mathcal{O}_Y)_{p_Y} \xrightarrow{\sim} T\text{-}\mu\mathrm{hom}(\Phi^\mu_{K[n]} F, \mathcal{O}_X)_{p_X}$.

Finally, we have a basic formula:

$$T\text{-}\mu\mathrm{hom}(F, \mathcal{O}_X) \simeq \mathcal{E}_X^{\mathbb{R},f} \underset{\pi^{-1}\mathcal{D}_X}{\otimes} \pi^{-1} RH(F) \qquad \text{for} \quad F \in Ob(D^b_{\mathbb{C}-c}(X)),$$

from which we get

$$(10) \qquad \mu RH(F) = \mathcal{E}_X \underset{\pi^{-1}\mathcal{D}_X}{\otimes} \pi^{-1} RH(F) \qquad \text{for} \quad F \in Ob(D^b_{\mathbb{C}-c}(X)).$$

The key point is then

LEMMA 5.1. *Formula* (9) *actually defines a functor*

$$\mu RH : \mathrm{Perv}(X; \mathbb{C}^\times p)^\circ \to \mathrm{Reghol}(\mathcal{E}_{X,p}).$$

PROOF. Let $F \in Ob(\mathrm{Perv}(X; \mathbb{C}^\times p))$. By the invariance by extended (resp. quantized) canonical transformations, we may assume that $SS(F)$ has a generic position at p, thus, by Proposition 4.2(iii) we may find $F' \in \mathrm{Perv}(X; \pi(p))$ such that $F \simeq F'$ in $D^b(X; p)$, thus

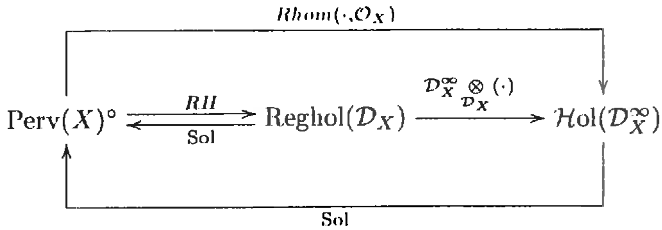
$$\mu RH(F)_p \simeq \mu RH(F')_p \simeq (\mathcal{E}_X \underset{\pi^{-1}\mathcal{D}_X}{\otimes} \pi^{-1} RH(F'))_p,$$

by (10), and the latter is an object concentrated in degree zero, which coincides with the germ at p of a regular holonomic $\mathcal{E}_X$-module. □

That $\mu RH : \mathrm{Perv}(X; \mathbb{C}^\times p)^\circ \to \mathrm{Reghol}(\mathcal{E}_{X,p})$ is an equivalence is then readily deduced, by using again invariance by canonical transformations, from Kashiwara and Kawai's generic position theorem of [7].

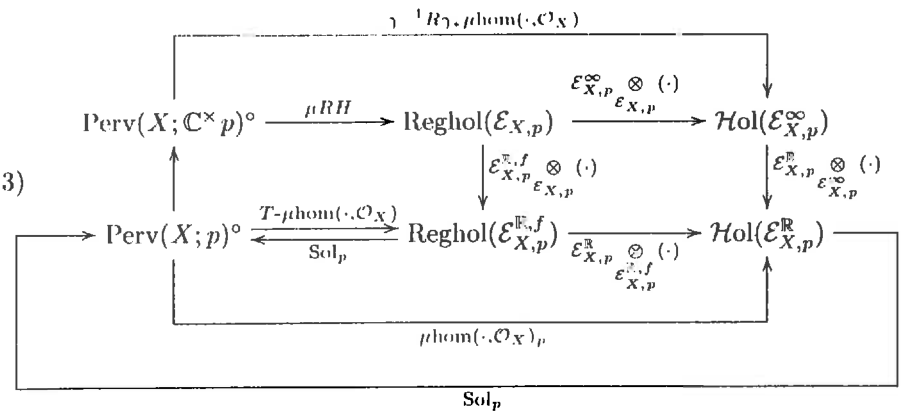
6. Final remarks

As is well known one has the diagram



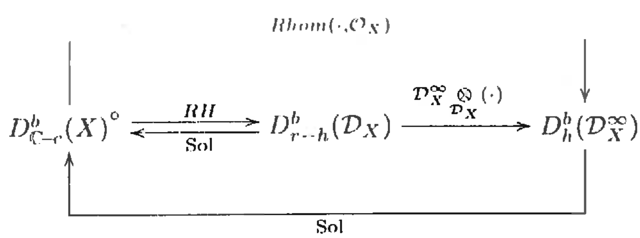
The corresponding microlocal diagram is given by

(1.3)



where the horizontal arrows are equivalences of categories.

Notice also that, up to now, we were not able to get the microlocal version of the diagram



concerning derived categories.

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