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A FEW PROPERTIES OF THE KOBAYASHI DISTANCE AND THEIR APPLICATIONS

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Dedicated to the memory of Juliusz P. Schauder

Abstract. Let N be a norming set in a Banach space X. In this paper we prove the lower semicontinuity with respect to the topology $\sigma(X,N)$ of the Kobayashi distance in a bounded, relatively compact in $\sigma(X,N)$, convex and open subset of a Banach space. We apply this result to the Denjoy–Wolff type theorem.

1. Introduction

In [23] the following property of the Kobayashi distance was proved: in a bounded, convex and open subset D of a reflexive Banach space $(X, \|\cdot\|)$ the Kobayashi distance is lower semicontinuous with respect to the weak topology, i.e., if $\{x_\beta\}_{\beta\in I}$ and $\{y_\beta\}_{\beta\in I}$ are nets in D which are weakly convergent to x and y, respectively, and $x, y \in D$, then

$$k_D(x,y) \leq \liminf_{\beta\in I} k_D(x_\beta, y_\beta).$$

Hence we get that the convergence in the Denjoy–Wolff theorem for condensing mappings is locally uniform in the open unit ball in a strictly convex reflexive Banach space with the Kadec–Klee property (see [20], [22] and [23]). In this paper we generalize the result of lower semicontinuity of the Kobayashi distance

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replacing the weak topology by a weaker one and apply this result to the Denjoy–Wolff type theorem.

2. Preliminaries

Throughout this paper all Banach spaces will be complex and the Kobayashi distance in a bounded domain D in a Banach space $(X, \|\cdot\|)$ will be denoted by k_D. The Kobayashi distance k_D is locally equivalent to the norm $\|\cdot\|$ ([14], [16]–[18], [41]). It is well known that all distances assigned to a convex bounded domain D by Schwarz-Pick systems of pseudometrics ([14], [16]–[18]) coincide ([11], [18], [29], [30]).

A subset C of a bounded domain D is said to lie strictly inside D if $\operatorname{dist}(C, \partial D) > 0$. Let us observe that we have the following property of a bounded convex domain.

Theorem 2.1 ([17]). *Let D be a bounded convex domain in a Banach space $(X, \|\cdot\|)$. A subset C of D is k_D-bounded if and only if C lies strictly inside D.*

The convexity of D implies the next important property of the distance k_D.

Lemma 2.1 ([20], [26]). *Let D be a convex bounded domain in a Banach space $(X, \|\cdot\|)$.*

(a) *If $x, y, w, z \in D$ and $s \in [0,1]$, then*

$$k_D(sx + (1-s)y, sw + (1-s)z) \le \max[k_D(x,w), k_D(y,z)].$$

(b) *If $x, y \in D$ and $s, t \in [0,1]$, then*

$$k_D(sx + (1-s)y, tx + (1-t)y) \le k_D(x,y).$$

A mapping $f : D \to D$ is said to be k_D-nonexpansive if

$$k_D(f(x), f(y)) \le k_D(x,y)$$

for all $x, y \in D$. Each holomorphic $f : D \to D$ is k_D-nonexpansive (see [14], [16]–[18]).

If D is a bounded convex domain in X, then the family of all holomorphic self-mappings of D is denoted by $H(D,D)$. In $H(D,D)$ we consider two topologies: the compact open topology on $H(D,D)$ generated by pseudodistances

$$p_K(f,g) = \sup\{\|f(x) - g(x)\| : x \in K\},$$

where $f, g \in H(D,D)$ and K ranges over the compact subsets of D, and the topology of locally uniform convergence on $H(D,D)$ induced by pseudodistances

$$q_{B(a,r)}(f,g) = \sup\{\|f(x) - g(x)\| : x \in B(a,r)\},$$

where $f, g \in H(D, D)$ and $B(a, r) \subset D$ ranges over the open balls in X satisfying $\operatorname{dist}(B(a, r), \partial D) > 0$ and ∂D denotes the boundary of D. It is known that the pointwise convergence (in norm) of a net $\{f_\beta\}_{\beta \in I}$ of holomorphic self-mappings of D is equivalent to its convergence in the compact open topology on D, and if X is a finite-dimensional Banach space, then the topology of locally uniform convergence on $H(D, D)$ is equivalent to the compact open topology. The last observation is no longer true in the infinite dimensional case. Finally, let us mention here that the locally uniform convergence of iterates is a necessary claim in many applications ([24], [31]–[38]).

Let $(X, \|\cdot\|)$ be a Banach space and let $\emptyset \neq N \subset X^*$ satisfy the following condition: there exist constants $c, C > 0$ such that

$$\sup\{|u(x)| : u \in F,\ \|u\| \leq C\} \geq c\|x\|$$

for each $x \in X$. We say that N is a norming set in X. It is obvious that the norming set generates a linear Hausdorff topology $\sigma(X, N)$ in X which is weaker than the weak topology $\sigma(X, X^*)$.

We have the following characterization of holomorphic mappings.

Theorem 2.2 ([5], [8], [12], [25]). *Let $(X_1, \|\cdot\|_1)$ and $(X_2, \|\cdot\|_2)$ be Banach spaces, $\emptyset \neq C \subset X_1$ be a domain in X_1 and N be a norming set in $(X_2, \|\cdot\|_2)$. A mapping $f : D \to X_2$ is a holomorphic mapping if and only if f is a locally bounded mapping and for each $u \in N$ a composition $u \circ f : \to \mathbb{C}$ is a holomorphic mapping.*

As a corollary we get the next property of holomorphic mappings.

Corollary 2.1. *Let D_1, D_2 be bounded, convex and open sets in Banach spaces $(X_1, \|\cdot\|_1)$ and $(X_2, \|\cdot\|_2)$, respectively. Let N be a norming set in $(X_2, \|\cdot\|_2)$. If $\{f_\lambda\}_{\lambda \in J}$ is a net of holomorphic mappings $f_\lambda : D_1 \to D_2$ which is pointwise convergent in $\sigma(X_2, N)$ to a function $f : D_1 \to D_2$, then f is also holomorphic.*

Let (Y, d) be a metric space and let $\emptyset \neq D \subset Y$. We say that a mapping $f : D \to D$ is α_d-condensing with respect to Kuratowski's measure of noncompactness α_d (see [28]) if

$$\alpha_d(f(A)) < \alpha_d(A)$$

for each bounded $A \subset D$ with $\alpha_d(A) > 0$. More information on condensing mappings and their applications can be found, for example, in [3], [6] and [15].

In [20] the following version of the Denjoy–Wolff theorem was proved (see also [1], [2], [7], [9], [19], [22], [23], [26], [27], [36], [39], [40] and [42]).

Theorem 2.3 ([20]). *If B is the open unit ball of a strictly convex Banach space $(X, \|\cdot\|)$ and $f : B \to B$ is holomorphic, condensing with respect to $\alpha_{\|\cdot\|}$,* *and fixed-point-free, then there exists $\xi \in \partial B$ such that the sequence $\{f^n\}$ of iterates of f converges in the compact-open topology to the constant map taking the value ξ.*

Remark. Theorem 2.3 is also valid if in place of a holomorphic mapping $f : B \to B$ we consider a k_B-nonexpansive mapping $f : B \to B$.

3. The lower semicontinuity of Kobayashi distance

We now extend results due to the second author ([23]) to our setting beginning with a reformulation of Lemma 3.1 from [23]. The proofs of the given below results are slight modifications of the ones of [23].

Lemma 3.1. *Let D_1, D_2 be bounded, convex and open sets in Banach spaces $(X_1, \|\cdot\|_1)$ and $(X_2, \|\cdot\|_2)$, respectively, and let N be a norming set in $(X_2, \|\cdot\|_2)$. If $\{f_\lambda\}_{\lambda \in J}$ is a net of holomorphic mappings $f_\lambda : D_1 \to D_2$ which is pointwise convergent in the topology $\sigma(X_2, N)$ to a function $f : D_1 \to \overline{D_2}$ and there exists a point $z_0 \in D_1$ such that $w_0 = f(z_0) \in D_2$, then f maps holomorphically D_1 into D_2.*

Proof. Choose $z \in D_1$. By Corollary 2.1 f maps holomorphically D_1 into $\overline{D_2}$, where $\overline{D_2}$ denotes a closure of D_2 in $(X_2, \|\cdot\|_2)$. Next we follow step by step as in [23]. We observe that a mapping

$$g_n = \frac{1}{n} w_0 + \left(1 - \frac{1}{n}\right) f$$

$(n \in \mathbb{N})$ transforms D_1 into $A_n = (1/n)w_0 + (1 - 1/n)\overline{D_2}$, the set A_n lies strictly inside D_2 and finally,

$$k_{D_2}(w_0, g_n(z)) = k_{D_2}(g_n(z_0), g_n(z)) \le k_{D_1}(z_0, z).$$

Hence, by Theorem 2.1, the sequence $\{g_n(z)\}$ lies strictly inside D_2 and therefore

$$f(z) = \lim_n g_n(z)$$

is an element of D_2 and this completes the proof. □

We conclude this section by stating the main result of this paper.

Theorem 3.1. *Let X be a Banach space, N a norming set in X and let $\emptyset \neq D \subset X$ be a bounded, convex and open set such that $\overline{D}$ is compact in $\sigma(X, N)$, where $\overline{D}$ denotes a closure of D in X. If $\{x_\beta\}_{\beta \in I}$ and $\{y_\beta\}_{\beta \in I}$ are nets in D which are convergent in $\sigma(X, N)$ to x and y, respectively, and $x, y \in D$, then*

$$k_D(x, y) \le \liminf_{\beta \in I} k_D(x_\beta, y_\beta).$$

If in place of nets $\{x_\beta\}_{\beta\in I}$ and $\{y_\beta\}_{\beta\in I}$ we have sequences $\{x_n\}_{n\in\mathbb{N}}$ and $\{y_n\}_{n\in\mathbb{N}}$, then the compactness of $\overline{D}$ in $\sigma(X,N)$ can be replaced by a sequentional compactness of $\overline{D}$ in $\sigma(X,N)$.

PROOF. Without loss of generality we may assume that

$$\sup_{\beta\in I} k_D(x_\beta, y_\beta) < \infty.$$

Let $\{\beta_\lambda\}_{\lambda\in J}$ be an ultranet in I ([4], [13], [21]) such that

$$\lim_{\lambda\in J} k_D(x_{\beta_\lambda}, y_{\beta_\lambda}) = \liminf_{\beta\in I} k_D(x_\beta, y_\beta).$$

and let $\varepsilon > 0$. Then there exist holomorphic functions $f_{\beta_\lambda} : \Delta \to D$ and points γ_{β_λ} in the open unit disc Δ such that

$$x_{\beta_\lambda} = f_{\beta_\lambda}(0) \quad \text{and} \quad y_{\beta_\lambda} = f_{\beta_\lambda}(\gamma_{\beta_\lambda})$$

and

$$k_\Delta(0, \gamma_{\beta_\lambda}) \le k_D(x_{\beta_\lambda}, y_{\beta_\lambda}) + \varepsilon$$

for each $\lambda \in J$ ([11]). Hence

$$\sup_{\lambda\in J} k_\Delta(0, \gamma_{\beta_\lambda}) \le [\sup_{\lambda\in J} k_D(x_{\beta_\lambda}, y_{\beta_\lambda})] + \varepsilon < \infty.$$

Since $\{\beta_\lambda\}_{\lambda\in J}$ is an ultranet in I and $\overline{D}$ is compact in the topology $\sigma(X,N)$ and

$$x = \sigma(X,N) - \lim_{\lambda\in J} x_{\beta_\lambda} = \sigma(X,N) - \lim_{\lambda\in J} f_{\beta_\lambda}(0),$$

Lemma 3.1 implies existence of a holomorphic function $f : \Delta \to D$ and a point $\gamma \in \Delta$ with

$$\sigma(X,N) - \lim_{\lambda\in J} f_{\beta_\lambda}(z) = f(z)$$

for each $z \in \Delta$ and $\gamma_{\beta_\lambda} \to {}_\lambda \gamma$. These observations give

$$f(0) = x,$$
$$k_\Delta(0,\gamma) = \lim_{\lambda\in J} k_\Delta(0, \gamma_{\beta_\lambda}) \le \lim_{\lambda\in J} k_D(x_{\beta_\lambda}, y_{\beta_\lambda}) + \varepsilon = \liminf_{\beta\in I} k_D(x_\beta, y_\beta) + \varepsilon < \infty,$$
$$\lim_{\lambda\in J} k_\Delta(\gamma, \gamma_{\beta_\lambda}) = 0,$$
$$\limsup_{\lambda\in J}\left[\operatorname{arg\,tanh}\left(\frac{\|f_{\beta_\lambda}(\gamma) - f_{\beta_\lambda}(\gamma_{\beta_\lambda})\|}{\operatorname{diam} D}\right)\right]$$
$$\le \limsup_{\lambda\in J} k_D(f_{\beta_\lambda}(\gamma), f_{\beta_\lambda}(\gamma_{\beta_\lambda})) \le \lim_{\lambda\in J} k_\Delta(\gamma, \gamma_{\beta_\lambda}) = 0$$

(see [17]). Consequently,

$$f(\gamma) - y = \sigma(X,N) - \lim_{\lambda\in J}[f_{\beta_\lambda}(\gamma) - y_{\beta_\lambda}] = \sigma(X,N) - \lim_{\lambda\in J}[f_{\beta_\lambda}(\gamma) - f_{\beta_\lambda}(\gamma_{\beta_\lambda})] = 0.$$

This means that

$$k_D(x,y) = k_D(f(0), f(\gamma)) \le k_\Delta(0,\gamma)$$

and

$$k_D(x,y) \le \liminf_{\beta\in I} k_D(x_\beta, y_\beta) + \varepsilon$$

for each $\varepsilon > 0$. Therefore

$$k_D(x,y) \le \liminf_{\beta\in I} k_D(x_\beta, y_\beta),$$

which completes the first part of the proof. If in place of nets $\{x_\beta\}_{\beta\in I}$ and $\{y_\beta\}_{\beta\in I}$ we have sequences $\{x_n\}_{n\in\mathbb{N}}$ and $\{y_n\}_{n\in\mathbb{N}}$ and in place of compactness of $\overline{D}$ in $\sigma(X,N)$ we have sequentional compactness of $\overline{D}$ in $\sigma(X,N)$, then we can repeat our earlier considerations taking into account the following facts: (Δ, k_Δ) is a separable metric space, the distance k_Δ is locally equivalent to the euclidean metric and holomorphic mappings are nonexpansive with respect to the Kobayashi distances. Finally, we replace the ultranet technique by the usual diagonal procedure. □

Before our next result let us recall the definition of the Kadec-Klee property with respect to $\sigma(X,N)$.

Definition 3.1 ([6], [10], [15]). Let N be a norming set in a Banach space $(X,\|\cdot\|)$. A Banach space $(X,\|\cdot\|)$ is said to have the Kadec–Klee property with respect to $\sigma(X,N)$, if for every sequence $\{x_n\}$ in X the following implication holds:

$$\left.\begin{array}{l} \|x_n\| \le 1, \\ \operatorname{sep}\{x_n\} = \inf\{\|x_n - x_m\| : n \ne m\} > 0, \\ \sigma(X,F) - \lim\limits_{n\to\infty} x_n = x, \end{array}\right\} \Rightarrow \|x\| < 1.$$

Now we are ready to state the following lemma.

Lemma 3.2. *Let N be a norming set in a strictly convex Banach space $(X,\|\cdot\|)$ and B the unit open ball in $(X,\|\cdot\|)$. Assume that $\overline{B}$ ($\overline{B}$ denotes the closure of B in $(X,\|\cdot\|)$) is sequentionally compact in the topology $\sigma(X,N)$ and $(X,\|\cdot\|)$ has the Kadec–Klee property with respect to $\sigma(X,N)$. If $\{x_n\}$ and $\{y_n\}$ are sequences in B, $\{x_n\}$ is strongly convergent to $\xi \in \partial B$ and*

$$\sup\{k_B(x_n, y_n) : n = 1, 2, \dots\} = c < \infty,$$

then $\{y_n\}$ is strongly convergent to ξ.

Proof. We can assume that the sequence $\{y_n\}$ is $\sigma(X,N)$-convergent to η. If $\xi = \eta$, then $\{y_n\}$ is strongly convergent to ξ by the Kadec–Klee property of X. Assume $\xi \ne \eta$. In this case by the strict convexity of B, each point of the open segment

$$(\xi,\eta) = \{z \in X : z = s\xi + (1-s)\eta \text{ for some } 0 < s < 1\}$$

lies in B. Next, for $0 < s, t < 1$, the sequences $\{sx_n + (1-s)y_n\}$ and $\{tx_n + (1-t)y_n\}$ tend in $\sigma(X, N)$ to $s\xi + (1-s)\eta$ and $t\xi + (1-t)\eta$, respectively. Now, applying Lemma 2.1 and Theorem 3.1, we get

$$\begin{aligned} k_B(s\xi + (1-s)\eta, t\xi + (1-t)\eta) &\leq \liminf_{n\to\infty} k_B(sx_n + (1-s)y_n, tx_n + (1-t)y_n) \\ &\leq \limsup_{n\to\infty} k_B(x_n, y_n) \leq c < \infty \end{aligned}$$

for all $s, t \in (0, 1)$. By Theorem 2.1 this means that (ξ, η) lies strictly inside B, which is impossible since $\xi \in \partial B$. □

4. The Denjoy–Wolff theorem

In this section we show a locally uniform convergence of the sequence of iterates in the Denjoy-Wolff type theorem (see Theorem 2.3).

Theorem 4.1. *Let N be a norming set in a strictly convex Banach space $(X, \|\cdot\|)$, B the open unit ball in $(X, \|\cdot\|)$, such that $\overline{B}$ is sequentionally compact in the topology $\sigma(X, N)$. If $(X, \|\cdot\|)$ has the Kadec–Klee property with respect to $\sigma(X, N)$ and $f : B \to B$ is k_B-nonexpansive, condensing with respect to $\alpha_{\|\cdot\|}$ and fixed-point-free, then there exists $\xi \in \partial B$ such that the sequence $\{f^n\}$ of iterates of f converges locally uniformly to the constant map taking the value ξ.*

Proof. By Theorem 2.3 $\{f^n(0)\}$ converges to $\xi \in \partial B$. Let us choose an arbitrary sequence $\{f^{n_i}(x_i)\}$ with $\sup_i \|x_i\| = c_1 < 1$. We can assume that $\{f^{n_i}(x_i)\}$ is $\sigma(X, N)$-convergent to η. By the inequality

$$\begin{aligned} \sup_i k_B(f^{n_i}(0), f^{n_i}(x_i)) &\leq \sup_i k_B(0, x_i) \\ &= \sup_i [\operatorname{arg\,tanh} \|x_i\|] = \operatorname{arg\,tanh} c_1 = c < \infty \end{aligned}$$

and Lemma 3.2 we obtain the strong convergence of $\{f^{n_i}(x_i)\}$ to ξ. □

Corollary 4.1. *Since each holomorphic $f : B \to B$ is k_B-nonexpansive, then Theorem 4.1 is valid for holomorphic mappings.*

Theorem 4.1 is strictly related with Theorem 4.1 in [23] but as the example given below shows our theorem is stronger.

Example 4.1. Let l^1 be furnished with a new norm

$$\|x\| = \left(\|x\|_1^2 + \sum_{i=1}^{\infty} \frac{x_i^2}{2^i} \right)^2,$$

where $\|\cdot\|_1$ is a usual norm in l^1 and $x = \{x_i\} \in l^1$. If we take $N = c_0 \subset (l^1)^*$, i.e.

$$u(x) = \sum_{i=1}^{\infty} u_i x_i$$

for $u = \{u_i\} \in c_0$ and $x = \{x_i\} \in l^1$, then we can observe that N is a norming set in a strictly convex Banach space $(l^1, \|\cdot\|)$, the closed unit ball $\overline{B}$ in $(l^1, \|\cdot\|)$ is sequentionally compact in the topology $\sigma(l^1, N)$ and $(l^1, \|\cdot\|)$ has the Kadec–Klee property with respect to $\sigma(l^1, N)$.

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