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ON RANDOM COINCIDENCE POINT THEOREMS

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Abstract. Some random coincidence point theorems are proved. The results of Benavides et. el. [2], Itoh [8], Shahzad and Latif [23], Tan and Yuan [24] and Xu [25] are either extended or improved.

1. Introduction

The fundamental theory of random operators is an important branch of stochastic analysis and plays a key role in many applied areas. Random fixed point theory is the core arround which the theory of random operators has developed. The systematic study of random fixed points was initiated by the Prague school of probabilists about fifty years ago. However, it received the attention after the appearance of the survey paper by Bharucha–Reid [3] in 1976. Since then this discipline has been developed further, in which several results were established in the general framework and many applications presented. We refer the reader to Beg and Shahzad [1], Benavides, Acedo and Xu [2], Itoh [8], Lin [12], Liu [13], O'Regan [15], O'Regan and Shahzad [16], Papageorgiou [17, 18], Sehgal and Singh [20], Shahzad [21], Shahzad and Latif [23], Tan and Yuan [24] and Xu [25].

Random coincidence point theorems are stochastic generalizations of classical coincidence point theorems. Recently, Shahzad and Latif proved in [23] a random

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coincidence point theorem for a pair of commuting random operators. The aim of this note is to prove some coincidence point theorems for a new class of non-commuting random maps. We also obtain a random common fixed point theorem for a pair of R-subweakly commuting random maps. Our results improve and extend the work of Benavides, Acedo and Xu [2], Itoh [8], Shahzad and Latif [23], Tan and Yuan [24] and Xu [25].

2. Preliminaries

Let (Ω, Σ) be a measurable space and M a subset of a Banach space $X = (X, \|\cdot\|)$. Let 2^M denote the family of all nonempty subsets of M, $CB(M)$ all nonempty closed bounded subsets of M, $K(M)$ all nonempty compact subsets of M, and $WK(M)$ all nonempty weakly compact subsets of M, respectively.

A multifunction $T: \Omega \to 2^M$ is called measurable if, for any open subset C of M,

$$T^{-1}(C) = \{\omega \in \Omega : T(\omega) \cap C \neq \phi\} \in \Sigma.$$

Let $\xi: \Omega \to M$ be a mapping. Then ξ is called a *measurable selector of a multifunction* $T: \Omega \to 2^M$ if ξ is measurable and $\xi(\omega) \in T(\omega)$ for each $\omega \in \Omega$.

A mapping $f: \Omega \times M \to M$ (resp. $T: \Omega \times M \to 2^M$) is called a *random operator* if, for each $x \in M$, $f(\cdot, x)$ (resp. $T(\cdot, x)$) is measurable. A measurable mapping ξ is called a *random coincidence point of random operators* $f: \Omega \times M \to M$ and $T: \Omega \times M \to 2^M$ if for each $\omega \in \Omega$, $f(\omega, \xi(\omega)) \in T(\omega, \xi(\omega))$; a *random fixed point of a random operator* f (resp. T) if for each $\omega \in \Omega$, $f(\omega, \xi(\omega)) = \xi(\omega)$ (resp. $\xi(\omega) \in T(\omega, \xi(\omega))$).

Let $f: M \to M$ and $T: M \to CB(M)$ be any mappings. Then T is called

(1) *upper* (resp. *lower*) *semicontinuous* if for any closed (resp. open) subset V of M, $T^{-1}(V)$ is closed (resp. open);

(2) *continuous* if T is both upper and lower semicontinuous.

If $T(x) \in K(M)$ for all $x \in M$, then T is *continuous* if and only if T is continuous from M into the metric space $(K(M), H)$, where H is the Hausdorff metric on $K(M)$. The mapping T is said to be f-*nonexpansive* if

$$H(T(x), T(y)) \leq \|f(x) - f(y)\| \quad \text{for all } x, y \in M,$$

where H is the Hausdorff metric on $CB(M)$. If $f = I$, the identity map on M, then an f-nonexpansive map T is nonexpansive.

The mapping f is called *weakly continuous* if $\{x_n\}$ converges weakly to x implies $\{f(x_n)\}$ converges weakly to $f(x)$. If M is convex, then

(3) T is said to be *semiconvex* if for any $x, y \in M$, $z = kx + (1-k)y$, where $0 \leq k \leq 1$, and any $x_1 \in T(x)$, $y_1 \in T(y)$, there exists $z_1 \in T(z)$ such that

$$\|z_1\| \leq \max\{\|x_1\|, \|y_1\|\};$$

(4) f is called *affine* if

$$f(kx + (1-k)y) = kf(x) + (1-k)f(y) \quad \text{for all } x, y \in M \text{ and } 0 < k < 1.$$

The set M is said to be *starshaped with respect to* $q \in M$ if $kx + (1-k)q \in M$ for any $x \in M$ and $0 < k < 1$. The pair $\{f, T\}$ is said to be

(5) *commuting* if $fT(x) = Tf(x)$ for all $x \in M$; and

(6) *R-weakly commuting* if for all $x \in M$, $fTx \in CB(M)$ and there exists $R > 0$ such that

$$H(Tfx, fTx) \le Rd(fx, Tx).$$

Suppose M is starshaped with respect to q and $f(q) = q$. Then $\{f, T\}$ is called *R-subweakly commuting* if for all $x \in M$, $fTx \in CB(M)$ and there exists $R > 0$ such that

$$H(Tfx, fTx) \le Rd(fx, A_k x)$$

for every $k \in [0, 1]$, where $A_k x = kTx + (1-k)q$. Here $d(x, A) = \inf\{\|x - y\| : y \in A\}$ for $A \subset M$. It is clear that every commuting pair of maps is R-subweakly commuting.

The following example shows that the converse is not true in general. Consider $M = [1, \infty)$. Let T and f be defined by $Tx = [1, 4x-3]$ and $fx = 2x^2 - 1$ for all $x \in M$. Then the pair $\{f, T\}$ is R-subweakly commuting but not commuting.

A mapping $T\colon M \to CB(X)$ is called *demiclosed* at y_0 if $\{x_n\} \subset M$ and $y_n \in T(x_n)$ are sequences such that $\{x_n\}$ converges weakly to x_0 and $\{y_n\}$ converges to y_0 in X, then $y_0 \in T(x_0)$.

The space X is said to *satisfy Opial's condition* (cf. Opial [14]) if the following holds: if $\{x_n\}$ converges weakly to x_0 and $x \ne x_0$, then

$$\liminf_{n\to\infty} \|x_n - x\| > \liminf_{n\to\infty} \|x_n - x_0\|.$$

A random operator $f\colon \Omega \times M \to M$ is called *continuous* (*weakly continuous*, etc.) if for each $\omega \in \Omega$, $f(\omega, \cdot)$ is continuous (weakly continuous, etc.). Similarly, a random operator $T\colon \Omega \times M \to CB(M)$ is called continuous if for each $\omega \in \Omega$, $T(\omega, \cdot)$ is continuous. The pair $\{f, T\}$ of random operators is called *R-subweakly commuting* if for each $\omega \in \Omega$, the pair $\{f(\omega, \cdot), T(\omega, \cdot)\}$ is so.

3. Main results

We begin with the following result.

Theorem 3.1. *Let M be a nonempty separable weakly compact subset of a Banach space X which is starshaped with respect to $q \in M$, and let $f\colon \Omega \times M \to M$ be a continuous affine random operator such that $f(\omega, q) = q$ for each $\omega \in \Omega$. Let $T\colon \Omega \times M \to K(M)$ be an f-nonexpansive random operator such that*

$T(\omega, M) \subset f(\omega, M)$. Suppose that the pair $\{f, T\}$ is R-subweakly commuting and that one of the following two conditions is satisfied:

(a) *$(f - T)(\omega, \cdot)$ is demiclosed at zero for each $\omega \in \Omega$,*
(b) *f is weakly continuous and X satisfies Opial's condition.*

Then f and T have a random coincidence point.

PROOF. Suppose, first, condition (a) holds. For each n, define $T_n: \Omega \times M \to K(M)$ by

$$T_n(\omega, x) = k_n T(\omega, x) + (1 - k_n)q,$$

where $\{k_n\}$ is a sequence such that $0 < k_n < 1$ and $k_n \to 1$ as $n \to \infty$. Then each T_n satisfies $T_n(\omega, M) \subset f(\omega, M)$ and

$$H(T_n(\omega, x), T_n(\omega, y)) \leq k_n \|f(\omega, x) - f(\omega, y)\|$$

for all $x, y \in M$ and all $\omega \in \Omega$. This shows that each T_n is a random f-contraction. Since the pair $\{f, T\}$ is R-subweakly commuting, it follows that $f(\omega, T_n(\omega, x)) \in K(M)$ and

$$\begin{aligned} H(T_n(\omega, f(\omega, x)), f(\omega, T_n(\omega, x))) &= k_n H(T(\omega, f(\omega, x)), f(\omega, T(\omega, x))) \\ &\leq R k_n d(f(\omega, x), T_n(\omega, x)) \end{aligned}$$

for all $x \in M$ and all $\omega \in \Omega$. Consequently, for each n, the pair $\{f, T_n\}$ is Rk_n-weakly commuting. By Beg and Shahzad [1, Theorem 3.1], there exists a measurable mapping $\xi_n: \Omega \to M$ such that $f(\omega, \xi_n(\omega)) \in T_n(\omega, \xi_n(\omega))$ for all $\omega \in \Omega$. For each n, define $L_n: \Omega \to WK(M)$ by $L_n(\omega) = \text{w-cl}\{\xi_i(\omega) : i \geq n\}$, where w-cl denotes the weak closure. Let the multifunction L be defined by

$$L(\omega) = \text{w-ls}\, L_n(\omega) = \{x \in M : x = \text{w-lim}\, \xi_k(\omega), \xi_k(\omega) \in L_{n(k)}(\omega)\},$$

where $\{L_{n(k)}(\omega)\}$ is a subsequence of $\{L_n(\omega)\}$. Because of the separability condition, M is a compact metrizable space for the weak topology. This implies that

$$L(\omega) = \bigcap_{k \geq 1} \text{w-cl}\left(\bigcup_{n \geq k} L_n(\omega) \right).$$

Since $\omega \to \text{w-cl}(\bigcup_{n \geq k} L_n(\omega))$ is w-measurable for each k, it is measurable by Hess [7, Lemma 2.1]. Now, Hess [7, Theorem 4.2] also shows that L is measurable. Since $L_n(\omega)$ is contained in a weakly compact subset M of X, it follows that L is weakly compact valued and so it is closed valued.

An application of the Kuratowski and Ryll–Nardzewski selection theorem (see [10]) yields that L has a measurable selector ξ. We show that ξ is a random coincidence point of f and T. Indeed, fix any $\omega \in \Omega$. Then some subsequence

$\{\xi_m(\omega)\}$ of $\{\xi_n(\omega)\}$ converges weakly to $\xi(\omega)$. Further, for each m, there is some $u_m \in T(\omega, \xi_m(\omega))$ such that

$$f(\omega, \xi_m(\omega)) - u_m = (1 - k_m)(q - u_m).$$

This implies that $\{f(\omega, \xi_m(\omega)) - u_m\}$ converges to 0. Since $f(\omega, \xi_m(\omega)) - u_m \in (f - T)(\omega, \xi_m(\omega))$ and $(f - T)(\omega, \cdot\,)$ is demiclosed at zero, it follows that $f(\omega, \xi(\omega)) \in T(\omega, \xi(\omega))$.

Suppose now condition (b) holds. Then $(f - T)(\omega, \cdot\,)$ is demiclosed at zero (cf. [11]) and the result follows immediately from part (a). □

Corollary 3.2. *Let M be a nonempty separable weakly compact starshaped subset of a Banach space X. Let $T : \Omega \times M \to K(M)$ be a nonexpansive random operator. Suppose that one of the following two conditions is satisfied:*

(a) *$(I - T)(\omega, \cdot\,)$ is demiclosed at zero for each $\omega \in \Omega$,*
(b) *X satisfies Opial's condition.*

Then T has a random fixed point.

Recall that a Banach space X is almost smooth (see [9]) if $SM(B)$ is dense in X^*, where $SM(B)$ is the set of all functionals of X^* which attain their norm at a smooth point of the unit ball B. A subset M of X is called Chebyshev if to each point x of X there exists a unique point of M that is nearest to x.

Corollary 3.3. *Let M be a nonempty separable weakly compact Chebyshev subset of an almost smooth Banach space X, and let $f : \Omega \times M \to M$ be a continuous affine random operator. Let $T : \Omega \times M \to K(M)$ be an f-nonexpansive random operator such that $T(\omega, M) \subset f(\omega, M)$ for each $\omega \in \Omega$. Suppose that the pair $\{f, T\}$ is R-subweakly commuting and that one of the following two conditions is satisfied:*

(a) *$(f - T)(\omega, \cdot\,)$ is demiclosed at zero for each $\omega \in \Omega$,*
(b) *X satisfies Opial's condition.*

Then f and T have a random coincidence point.

Proof. Since every weakly compact Chebyshev subset of X is convex ([9]), the result now follows from Theorem 3.1. □

Corollary 3.4. *Let M be a nonempty separable weakly compact Chebyshev subset of a Hilbert space X, and let $f : \Omega \times M \to M$ be a continuous affine random operator. Let $T : \Omega \times M \to K(M)$ be an f-nonexpansive random operator such that $T(\omega, M) \subset f(\omega, M)$ for each $\omega \in \Omega$. Suppose that the pair $\{f, T\}$ is R-subweakly commuting. Then f and T have a random coincidence point.*

Proof. Since every weakly compact Chebyshev subset of X is convex ([9]) and $(f - T)(\omega, \cdot\,)$ is demiclosed at zero for each $\omega \in \Omega$ (cf. [11]), the result follows immediately from Theorem 3.1. □

To prove the next result, we need the following lemma.

LEMMA 3.5. *Let M be a closed convex subset of a Banach space X, and let $f: M \to M$ be an affine continuous mapping. If $T: M \to CB(M)$ is a continuous multifunction such that $f - T$ is semiconvex, then*

(a) *for any $x, y \in M$ and $z = kx + (1-k)y$, where $0 \le k \le 1$, we have*

$$d(f(z), T(z)) \le \max\{d(f(x), T(x)), d(f(y), T(y))\},$$

(b) *for any $r > 0$, the set $H_r = \mathrm{cl}(\{x \in M : d(f(x), T(x)) < r\})$ is closed and convex (or equivalently, weakly closed).*

THEOREM 3.6. *Let M be a nonempty separable weakly compact convex subset of a Banach space X, and let $f: \Omega \times M \to M$ be a continuous affine random operator. Let $T: \Omega \times M \to K(M)$ be an f-nonexpansive random operator such that $T(\omega, M) \subset f(\omega, M)$ for each $\omega \in \Omega$. Suppose that the pair $\{f, T\}$ is R-subweakly commuting and that $(f - T)(\omega, \cdot)$ is semiconvex for each $\omega \in \Omega$. Then f and T have a random coincidence point.*

PROOF. Let $\{k_n\}$ be a sequence such that $0 < k_n < 1$ and $k_n \to 1$ as $n \to \infty$. For each n, define T_n as follows:

$$T_n(\omega, x) = k_n T(\omega, x) + (1 - k_n)q,$$

where $q = f(\omega, q)$ for all $\omega \in \Omega$. Then, as in the proof of Theorem 3.1, we have $f(\omega, \xi_n(\omega)) \in T(\omega, \xi_n(\omega))$ for all $\omega \in \Omega$.

Fix $\omega \in \Omega$. For each n, there is $u_n \in T(\omega, \xi_n(\omega))$ such that

$$f(\omega, \xi_n(\omega)) - u_n = (1 - k_n)(q - u_n).$$

Since M is bounded and $k_n \to 1$, it follows that $d(f(\omega, \xi_n(\omega)), T(\omega, \xi_n(\omega))) \to 0$ as $n \to \infty$. Consequently,

$$\inf\{d(f(\omega, x), T(\omega, x)) : x \in M\} = 0$$

for all $\omega \in \Omega$. Define a mapping $h_n : \Omega \times M \to \mathbb{R}$ by

$$h_n(\omega, x) = d(f(\omega, x), T(\omega, x)) - \frac{1}{n}, \quad n \ge 1.$$

Then, by Rybinski [19, Lemmas 1 and 2], each h_n is a Caratheodory function (that is, continuous in $x \in M$ and measurable in $\omega \in \Omega$). Set

$$G_n(\omega) = \{x \in M : h_n(\omega, x) < 0\}.$$

Thus, the multifunction L_n defined by $L_n(\omega) = \mathrm{cl}(G_n(\omega))$ is measurable and is closed convex valued (see Lemma 3.5). Since M is weakly compact, by Hess

[7, Theorem 4.2], $L := \bigcap_{n\geq 1} L_n$ is measurable. The Kuratowski and Ryll–Nardzewski selection theorem [10] further implies that L has a measurable selector ξ. This ξ is the desired random coincidence point of f and T. □

COROLLARY 3.7. *Let M be a nonempty separable weakly compact convex subset of a Banach space X. Let $T:\Omega\times M\to K(M)$ be a nonexpansive random operator. Suppose that $(I-T)(\omega,\cdot)$ is semiconvex for each $\omega\in\Omega$. Then T has a random fixed point.*

THEOREM 3.8. *Let M, f, T and q have the same meanings as in Theorem* 3.1. *If for any $x\in M$ and $\omega\in\Omega$, $\lim_{n\to\infty} f^n(\omega,x)$ exists whenever $f(\omega,x)\in T(\omega,x)$, then f and T have a common random fixed point.*

PROOF. Fix $\omega\in\Omega$ and let ξ be a random coincidence point of f and T. Since f and T commute at coincidence points, it follows that

$$f^n(\omega,\xi_0(\omega)) = f^{n-1}(\omega, f(\omega,\xi_0(\omega))) \in T(\omega, f^{n-1}(\omega,\xi_0(\omega))).$$

Let $\xi(\omega)=\lim_{n\to\infty} f^n(\omega,\xi_0(\omega))$. Then, taking $n\to\infty$, we get $\xi(\omega)\in T(\omega,\xi(\omega))$. Also $\xi(\omega)=f(\omega,\xi(\omega))$. The mapping $\xi:\Omega\to M$ is the pointwise limit of measurable mappings and so it is measurable by Di Bari and Vetro [4, Lemma 3]. Hence ξ is a common random fixed point of f and T. □

REMARKS 3.9. (a) It is well-known [6] that a closed bounded convex subset M of a Frechet space (that is, a complete metrizable locally convex space) X is weakly compact if and only if for every closed convex subset N of M, each continuous affine self-map of N has a fixed point. Consequently, the existence of a fixed point q of $f(\omega,\cdot)$ for each $\omega\in\Omega$ in Corollaries 3.3 and 3.4 and Theorem 3.6 follows. We further add that an affine continuous map is weakly continuous (see [5]) and so the weak continuity of f is not required as well.

(b) In Theorem 3.1 the assumption that $f(\omega,q)=q$ for all $\omega\in\Omega$ becomes redundant when M is convex.

(c) Theorem 3.1 improves [23, Theorem 3.1] in the following ways:

(i) for each $\omega\in\Omega$, the range of $f(\omega,\cdot)$ need not be M; and
(ii) the pair $\{f,T\}$ may be non-commuting (more precisely, R-subweakly commuting).

Theorem 3.1 applies when $f=I$, so it generalizes [2, Corollaries 3.1 and 3.2], [8, Theorem 3.4] and [24, Theorem 3.4]. It also improves [25, Theorem 1(ii)], where X is strictly convex and M is convex and has the fixed point property.

(d) Theorem 3.8 extends [23, Theorem 3.3] to a class of non-commuting maps.

(e) The proof of Theorem 3.8 suggests the following general result.

"Let M be a nonempty separable complete subset of a metric space X, and let $f:\Omega\times M\to M$ and $T:\Omega\times M\to CB(M)$ be continuous random operators. Suppose that f and T commute at coincidence points and that for any $x\in M$

and $\omega \in \Omega$, $\lim_{n\to\infty} f^n(\omega, x)$ exists whenever $f(\omega, x) \in T(\omega, x)$. If f and T have a random coincidence point, then they have a common random fixed point."

We further remark that the existence of a random coincidence point may be replaced by the existence of a deterministic coincidence point.

Now, we suppose that every closed convex subset of M has the fixed point property for continuous affine mappings. Recall that the metric d is said to be μ-monotone if there exists $0 \leq \mu \leq 1$ such that $d(\lambda x, 0) \leq \mu d(x, 0)$ for every $x \neq 0$ and $0 \leq \lambda \leq 1$.

Theorem 3.10. *Let M be a nonempty closed bounded convex subset of a separable Frechet space X with μ-monotone metric, and let $f: \Omega \times M \to M$ be a continuous affine random operator. Let $T: \Omega \times M \to K(M)$ be an f-nonexpansive random operator such that $T(\omega, M) \subset f(\omega, M)$ for each $\omega \in \Omega$. Suppose that the pair $\{f, T\}$ is R-subweakly commuting and that one of the following two conditions is satisfied:*

(a) *$(f - T)(\omega, \cdot)$ is demiclosed at zero for each $\omega \in \Omega$,*
(b) *$(f - T)(\omega, \cdot)$ is semiconvex for each $\omega \in \Omega$.*

Then f and T have a random coincidence point.

Proof. As in the proof of Theorems 3.1 and 3.6, it can be shown that there exists a measurable mapping ξ_n such that $f(\omega, \xi_n(\omega)) \in T(\omega, \xi_n(\omega))$ for all $\omega \in \Omega$. Clearly, M is weakly compact.

For each n, define $L_n: \Omega \to WK(M)$ by $L_n(\omega) = \text{w-cl}\{\xi_i(\omega) : i \geq n\}$ when (a) holds or by $L_n(\omega) = \text{cl}(G_n(\omega))$ when (b) holds, where $G_n(\omega) = \{x \in M : d(f(\omega, x), T(\omega, x)) < 1/n\}$. Then, as in Shahzad and Khan [22], $L := \bigcap_{n\geq 1} L_n$ is measurable and L has a measurable selector ξ. This ξ is the desired random coincidence point of f and T. $\square$

Corollary 3.11. *Let M be a nonempty closed bounded convex subset of a separable Frechet space X with μ-monotone metric. Let $T: \Omega \times M \to K(M)$ be a nonexpansive random operator. Suppose that that one of the following two conditions is satisfied:*

(a) *$(I - T)(\omega, \cdot)$ is demiclosed at zero for each $\omega \in \Omega$,*
(b) *$(I - T)(\omega, \cdot)$ is semiconvex for each $\omega \in \Omega$.*

Then T has a random fixed point.

Remark 3.12. Theorem 3.10 (in particular, Corollary 3.11) generalizes [25, Theorem 1(ii)].

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