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Ivor Grattan-Guinness (editor)
Landmark Writings in Western Mathematics 1640-1940
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REVIEW

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Ivor Grattan-Guinness's *Landmark Writings in Western Mathematics 1640-1940* is the first "Great Books" type history in the field of mathematics. It covers all the major lines of development in modern mathematics from the middle of the seventeenth century until the Second World War. The work is certainly substantial: more than one thousand pages, equalling four and a half pounds of excellent expert surveys on the most important published mathematical contributions of the modern period. The book consists of an introduction by the editor and seventy-seven case-studies, covering eighty-nine path-breaking writings on geometries, algebras, calculus, number theory, functions, series, differential equations, real and complex numbers, general mechanics, astronomy, probability and statistics, dynamics, mathematical physics, topology, the history of mathematics, set theory, logic, the foundations of mathematics, and socio-economic sciences with mathematical aspects. The contributors are the world's leading experts on the subjects they discuss, including mathematicians, logicians, physicists, and historians of science.

In the editor's words, the texts discussed are all "writings that have made a major impact on the branches and aspects of mathematics to which they refer, and maybe also to other branches and even disciplines not originally within their purview" (p. ix). In a great many cases the text discussed is a book, but sometimes it is one or more journal articles. Most chapters discuss one text each. However, in quite a few cases more than one contribution are taken together; for example, Dedekind's and Peano's famous booklets in the late 1880s on the foundations of arithmetic are discussed jointly (pp. 613-626).

The editor justifies the chosen time-period by pointing out that it was really not until around the 1640s that mathematics began to "show the first signs of professional employment and diffusion of information as we know it; for example, somewhat more publications than before, the founding of the Royal Society of London and the *Académie des Sciences* in Paris in the 1660s, and the launch of scientific journals such as the *Acta Eruditorum*" (pp. x-xi). This definition, however, rules out such giants of the Renaissance as Kepler, Galileo, Stevin, and Napier. The outbreak of the Second World War is one reason for closing the time-frame at the turn of the 1940s—the other quite acceptable reason is the abundance of later works that would have had to be considered.

According to Grattan-Guinness, the leading editorial principle was "to exhibit the *range* and *variety of theories* within mathematics as it has developed over the period considered" (p. xi). Consequently, the selection of landmarks includes contributions in both pure and applied mathematics. All the selected writings have more or less global importance. They either launched new phases of work, or improved the known state of theory on a topic, or both. What is more, in order to be chosen as a landmark, the writing had to have had an international impact. Given this requirement, a number of Soviet contributions were left out.

Almost all of the writings chosen live up to these qualifications. However, there are a few border-line cases. Of course, with any such work there is a certain amount of pleasure to be had from playing with the list of selected landmarks. One can assume that every serious historian of mathematics might be inclined to modify the list here and there. However, it is regrettable that the editorial board was not able to find a single sufficiently valuable pure or applied mathematical writing composed by a female author during the three hundred years 1640-1940. Undoubtedly, there would have been a number of good candidates to be considered (cf. *e.g.* www.agnesscott.edu/lriddle/women/chronol.htm). To be fair it should be noted that the list of commentators does include several female experts.

The commentary articles are fairly uniform in structure. Each one begins with a succinct description of the significance of the text discussed, followed by bibliographical information about the publication history of the writing(s). In most cases the substantial part begins with a brief outline of the author's career, with special attention to the significance of the particular text for his career. Thereafter, the reader is provided with a careful analysis of the structure and contents of the text(s) discussed. Most contributions close with interesting information about the reception, influence, and later fame of the particular text(s). All contributions are carefully written, concise, easy to follow, and entertaining. The volume also includes a few portraits, some original diagrams, as well as reprints of beautiful title pages and parts of manuscripts. It ends with a welcome list of affiliations of the authors and their articles, followed by a carefully prepared index. All in all, Grattan-Guinness and his editorial board have done outstanding work in planning the project, choosing the authors, composing the guidelines, and, finally, in preparing the volume. Elsevier, the publisher, deserves special recognition for the high-quality of the printing.

In what follows, I will briefly comment upon a limited but representative selection of the seventy-seven articles. Unfortunately, it is not possible to discuss all of them here.

The main body of the work begins with Chapter 1 and Michel Serfati's lengthy essay on René Descartes's *La Géométrie* (pp. 1-22), which was originally published in 1637 at the end of Descartes's philosophical classic work *Discours de la Méthode*. In this essay Descartes established a relation between curves and algebraic calculation, both in the continuum of geometry and the discontinuity of number. Curves were defined by equations with integer degrees, and algebra thus provided geometry with the most natural hierarchies and principles of classification. Moreover, it is remarkable that this pioneering work of analytic geometry, "a cornerstone of our 'modern' mathematical era" (p. 1), is almost directly accessible to modern-day mathematicians, Descartes's notation being very close to our own notation. In 1649 the text was published separately in Latin, entitled *Geometria*, which is the actual landmark-text discussed.

After the bibliographical details, Serfati's article begins with a biographical introduction, a table summarizing the contents of Descartes's book, and a nice reproduction of the cover page of the Latin edition. Thereafter, Serfati carefully comments upon the revolutionary new ideas to be found in the *Geometria*. The article closes with remarks on the enthusiastic but also somewhat controversial reception of the work. Immediately after its publication, the text became "a long-lasting object of study for European mathematicians and a veritable bedside read for geometers" (p. 17), the Latin translation ensuring its international dissemination. Curiously, the *Geometria* represents the culmination of Descartes's mathematical work. According to Serfati, at the end of the 1640s Descartes felt "no need to go any further in mathematics" (p. 20).

In Chapter 4, Silvia Roero discusses Gottfried Wilhelm Leibniz's first three papers on the differential and integral calculus, published in 1684-1693 (pp. 46-58). Undoubtedly, these contributions represent one of the greatest revolutions in the history of mathematics. Roero correctly points out that although Leibniz shares the glory of the invention with Isaac Newton, priority of publication is due to Leibniz (p. 46).

Leibniz's short paper of 1684, that is, the very first public presentation of differential calculus, is rather disorganized, and appears to have been written in a hurry. Therefore, it was not easily accessible to Leibniz's contemporary mathematical audience—even Jacob Bernoulli and Christiaan Huygens had difficulties in understanding its importance. Roero explains this pragmatically: in his view, Leibniz was expecting his use of infinitely small quantities to arouse considerable criticism, and therefore hid his new ideas in a poorly organized and obscure presentation (p. 49). Another, similarly pragmatic, explanation could be that Leibniz was simply in too much of a hurry with several of his other scientific and political projects.

Roero first gives a clear account of Leibniz's presentation of his differential method, comments upon its reception, and thereafter moves on to discussing Leibniz's first article on the integral calculus, published in 1686. Furthermore, he briefly comments on the success of the Leibnizian calculus not only among mathematicians, but also among scholars of a number of applied disciplines.

Roero's article is clear, well-composed, and easy to follow. It closes with a brief description of the tragic last scene in the first act of the success story of Leibniz's calculus, namely, the infamous battle between the Leibnizians and the advocates of Newton's corresponding theory. As a consequence of this episode, "Newton's theory was practiced almost exclusively in Britain and Leibniz's on the Continent, the latter with eventual greater success" (p. 57).

Chapter 8 provides Douglas Jesseph's account of the Irish philosopher George Berkeley's *The Analyst* (1734), a work which contained a highly influential criticism of both the Newtonian calculus of fluxions and the Leibnizian *calculus differentialis* (pp. 121-130). The work had both mathematical and theological motivation. Mathematically, it voiced Berkeley's reservations about the foundations of the calculus, particularly concerning infinite quantities. Theologically, it was part of his crusade against freethinking. Berkeley's critique contains both metaphysical and logical objections: metaphysically speaking, the calculus postulates incomprehensible infinitesimal magnitudes and ratios of transient quantities. From the logical point of view, even the most elementary proofs in the calculus commit logical errors by employing inconsistent assumptions. Ultimately, the advocates of the calculus managed to fend off the attack, but not without significant concessions: undoubtedly Berkeley succeeded in showing that (1) the Newtonian calculus of fluxions was fundamentally unrigorous, and (2) that there was no practicable distinction between the Newtonian and the Leibnizian formulations of the calculus.

The Analyst immediately provoked a lively debate that monopolized British mathematical discussion for a considerable time. Indeed, because of its remarkable impact, the publication of *The Analyst* has been regarded as “the most spectacular event” in the eighteenth-century British mathematics (p. 122). Jesseph closes his excellent article with some remarks on the most important responses to Berkeley, namely, those of James Jurin, Benjamin Robins, Colin Maclaurin, and Roger Paman.

In Chapter 13, Karin Reich discusses Leonhard Euler’s two-volume *Introductio in analysin infinitorum* (1748), a truly major landmark and perhaps the greatest mathematical textbook of modern times (pp. 181-190). The first volume of the *Introductio* presents old and new results on functions and infinite series, and related notions such as continued fractions. The second one scrutinizes aspects of analytic, coordinate, and differential geometry. In other words, it contains all the background information for successfully approaching infinitesimal calculus. In Reich’s words, it is a “repository of a mass of useful information about functions, series, curves and surfaces of many kinds” (p. 189). According to Reich, the work contains only one new specialized topic, namely, the partition of numbers into additive parts (p. 189). Being written in Latin, the *Introductio* soon became a necessary working resource for every serious mathematician. Reich’s well worked-out table of contents for the *Introductio* deserves special recognition.

In Chapter 21, Pierre Crépel and Alain Coste provide a critical account of the second enlarged edition of Jean-Etienne Montucla’s *Histoire des mathématiques* (1799-1802)—the first wide-ranging history of mathematics (and mathematicians) for a general readership (pp. 292-302). It is remarkable that this huge four-volume work discusses not only pure mathematics, but also applied mathematics, theoretical and applied mechanics, astronomy, and optics. After Montucla’s death, his friends Lalande, Laxroix, and others, ensured the publication of the last two volumes. Crépel and Coste earnestly point out the strengths and weaknesses of the work. Given that both editions of the *Histoire des mathématiques* were effectively circulated, it soon gained a great reputation. According to Crépel and Coste, “the book was ignored by nobody, from Bossut through Condorcet to Lagrange, and everybody made use of it” (p. 301).

Undoubtedly Montucla’s *Histoire des mathématiques* is a classic work. However, it is somewhat unreliable, being written for the most part by a self-educated amateur. When it comes to the most influential modern works in the history of mathematics, Moritz Cantor's *Vorlesungen über die Geschichte der Mathematik* (1880-1908) is definitely another major landmark.

Chapter 22 gives Olaf Neumann's excellent, detailed, and well-organized introduction to Carl Friedrich Gauss's *Disquisitiones arithmeticae* (1801) which rigorously defined the substance and methods of number theory for the rest of the nineteenth century (pp. 303-315). In short, the *Disquisitiones* unified various considerations on number theory from a collection of bits and pieces into a rigorous and extensive mathematical theory.

Neumann discusses both the contents and the reception of the *Disquisitiones* section by section. From a didactical viewpoint, this is a very clever approach making the article easy to follow. Neumann's article ends with some remarks on the reception of the *Disquisitiones* and its later fame. Even though the work was quickly recognized by contemporary experts, it has been said that Dirichlet was the first not only to fully grasp it, but also to make it accessible to other mathematicians. Nevertheless, in Neumann's words, "Nobody who speaks of the number theory and algebra of the last 200 years can remain silent about their sources in the *Disquisitiones arithmeticae* of Gauss. Notwithstanding all the new proofs of results in detail, this work belongs to the 'eternal canon' of mathematics, and thus of human culture" (p. 314).

In Chapter 32, Albert C. Lewis discusses Hermann Günther Grassmann's *Ausdehnungslehre*, with the focus on its first edition of 1844 (pp. 431-440). The work contains the basic seeds for the development of vector analysis and linear algebra. Grassmann's project was based on his abstraction of the purely mathematical foundations of geometry from the subject of geometry as the fundamental doctrine of space. It was this foundation Grassmann attempted to represent with his calculus of extension, as presented in the *Ausdehnungslehre*, by combining the synthetic and analytic approaches to geometry. This new branch had an algebraic aspect, though it was capable of dealing with continuous geometrical entities. Unlike ordinary algebra, it could also represent geometrical dimensions without being confined to the three dimensions of physical space.

Grassmann's *Ausdehnungslehre* exercised no influence on any of his notable contemporaries. No reviews were written. It was only after a thoroughly reworked edition was published in 1861 that Grassmann's ideas slowly began to gain recognition not only in Germany but also internationally. Later it played an important role in the works of, among others, W. K. Clifford, C. S. Peirce, and A. N. Whitehead. By the turn of the twentieth century, its importance was generally acknowledged. Nevertheless, in comparison to most of Grattan-Guinness's other landmark writings, Grassmann's *Ausdehnungslehre* is of secondary importance.

Chapter 36, written by Ivor Grattan-Guinness, discusses George Boole's *An Investigation of the Laws of Thought* (1854), that is, the book which established the definitive foundation of the algebra of logic (pp. 470-479). It must be noted that the idea of an algebraic structure of logic was by no means a nineteenth-century invention. It was Leibniz who provided the initiative from which the basic idea began to grow, even though it was Boole who really started its systematic development. Boole's extensional theory was far more elaborate than the theories of his predecessors, and, thanks to his systematic algebraic notation, he could well be nominated as one of the leading candidates for the title of the initiator of modern symbolic logic.

Grattan-Guinness begins his excellent article with a brief introduction to Boole's career and his work prior to the *Laws of Thought*. In particular, he briefly describes the new ideas in Boole's first monograph *The Mathematical Analysis of Logic* (1847). Boole's mature algebra of logic, as presented in the *Laws of Thought*, built upon his previous work, especially *The Mathematical Analysis of Logic*. According to Grattan-Guinness, one of the most striking differences between these two works was the status of syllogistic logic: while preparing his *Laws of Thought*, Boole had come to understand that he had advanced his logic far beyond the Aristotelian organon (p. 474). Another notable difference was the appearance of probability theory in the *Laws of Thought*.

Though slow to develop, Boole's algebra of logic had an important influence on practically every noteworthy late nineteenth and early twentieth century logician. Gradually Boolean algebras become part of the canon of logic. Surely everyone has heard about their successful applications to electrical circuit theory, communication, and computing, for example. It should be noted that Boole himself regarded his algebra of logic a contribution to applied mathematics (p. 472).

In Chapter 41, Jean-Pierre Potier and Jan van Daal scrutinize the first edition of *The Theory of Political Economy* (1871) by William Stanley Jevons (pp. 534-543). This work was the first presentation of economics as a mathematical science based on the assumption of utility-maximizing individuals. Jevons was not a mathematician by training, and he is best remembered for his work in economics, logic, and the philosophy of science.

Jevons was inspired by Jeremy Bentham's utilitarian moral theory, according to which an action should be taken only if it augments the happiness of all those affected. Jevons, however, even went so far as to declare that "pleasure and pain are undoubtedly the ultimate objects of the Calculus of Economy" (p. 535). For his purposes, utility became a precisely defined notion relating to goods and services, and a good's utility the expression of its relation with mankind's pleasures and pain. Starting from these basic assumptions, Jevons in *The Theory of Political Economy* discusses theories of pleasure and pain, utility, exchange, labor, rent, and capital. The main contribution of the work concerns the theories of utility and exchange.

According to Potier and van Daal, Jevons's mathematics was of the level of an average English scientist of his time (p. 541). Jevons was more interested in the practical applications of mathematics than in mathematics as such. In Jevons's own words, "I do not write for mathematicians, nor as a mathematician, but as an economist wishing to convince other economists that their science can only be satisfactorily treated on an explicitly mathematical basis" (p. 541). Nevertheless, with *The Theory of Political Economy* Jevons opened an important new path in applied mathematics.

In Chapter 46, Joseph W. Dauben discusses Georg Cantor's 1883 ground-breaking work *Grundlagen einer allgemeinen Mannigfaltigkeitslehre* which presented the earliest detailed version of Cantor's transfinite set theory, including a theory of transfinite ordinal numbers and their arithmetic (pp. 600-612). Hence, it virtually created a new discipline. Also, it included another major achievement, namely, a theory of transfinite ordinal numbers and their arithmetic. Dauben's presentation is clear, well-composed, and remarkably enjoyable to read. It is one of the best contributions in the *Landmark Writings*—if not the best.

Dauben begins his article with a short description of Cantor's early work on trigonometric series. Thereafter, he gives an introduction to Cantor's theory of real numbers as presented in his 1872 paper "Über die Ausdehnung eines Satzes aus der Theorie der trigonometrischen Reihen", where Cantor disclosed his revolutionary discovery of the fact that the set of natural numbers was of a lower magnitude of infinity than the set of real numbers. The conservative Berlin authority Leopold Kronecker could not digest Cantor's new ideas and created a number of obstacles to Cantor's career, at the same time forcing Cantor to review and improve the foundations of his set theory in the process of its creation. The rest of Dauben's article discusses the foundations of set theory as presented in the *Grundlagen*, Cantor's mental disorders, his activities concerning the creation of the *Deutsche Mathematiker-Vereiningung*, and the consequences of his results for later mathematics.

Dauben points out that some of the results of Cantorian set theory are reflected in a number of later chapters in Grattan-Guinness's volume, notably those on Hilbert, Lebesgue and Baire, and Russell and Whitehead. Baire and Hilbert drew on set-theory notions to develop increasingly abstract concepts of space, whilst others extended Cantor's theory of transfinite numbers into the realm of inaccessible cardinals and a host of other theories of transfinite numbers. Moreover, Cantor's legacy has been of particular importance for the development of mathematical logic and other work on the foundations of mathematics associated with, for example, Zermelo, Fraenkel, Gödel, and Cohen.

In Chapter 50, David Singmaster surveys the long tradition of recreational mathematics (pp. 653-663). The token landmark here is the first edition of *Mathematical Recreations and Problems of Past and Present Times* (1892) by Walter William Rouse Ball. However, it is not completely clear if this work can be regarded as an outstanding landmark even in the context its own tradition. As Singmaster correctly points out, "Ball's book was one of the first substantial books devoted to recreational mathematics" (p. 653). It is indicative that no more than four pages of Singmaster's ten-page article directly discuss Ball's work.

Singmaster begins his paper by briefly introducing some of the most important landmark writings in the long history of recreational mathematics, including *The Greek Anthology*, the *Aryabhatiya*, and Fibonacci's *Liber abbaci*. However, according to Singmaster, the first work genuinely devoted to recreational mathematics was Luca Pacioli's *De viribus quantitatis*, which was published during the sixteenth century (p. 654). Furthermore, Singmaster lists a number of other, more or less significant, contributions to recreational mathematics, and in his discussion of them even provides some biographical details about their authors. The last few pages of the article are devoted to Ball's work and career. The choice of his very popular but somewhat inadequate *Mathematical Recreations* as the landmark writing of recreational mathematics appears somewhat arbitrary. The importance of the work is limited to its own period. Nevertheless, Singmaster's contribution is well-written and a pleasure to read.

Judging by the number of landmarks chosen for this volume, David Hilbert stands out as the champion of modern mathematics with his score of four writings: In Chapter 54, Norbert Schappacher discusses Hilbert's 1897 report on algebraic number fields; Chapter 55 is Michael

Toepell's account of the first edition of Hilbert's *Grundlagen der Geometrie*; in Chapter 57, Michiel Hazewinkel introduces Hilbert's 1901 paper on mathematical problems; and finally, in Chapter 77, Wilfried Sieg and Mark Ravaglia conclude the whole volume with a careful analysis of the first edition of the *Grundlagen der Mathematik* (1934/1939) by Hilbert and Paul Bernays. Also, perhaps the editorial board could have considered some of Hilbert's other works, for example, his joint-contribution with Wilhelm Ackermann, entitled *Grundzüge der theoretischen Logik* (1928).

In his famous report "Die Theorie der algebraischen Zahlkörper", better known perhaps as the *Zahlbericht*, Hilbert established algebraic number theory as a genuine domain of pure mathematics, providing at the same time what became and remained the principal reference work on the subject for several decades after its appearance. It was one of the reports on the state of mathematical disciplines commissioned by the *Deutsche Mathematiker-Vereinigung*. However, Hilbert's contribution went beyond the mere description of the state-of-the-art. One reason for its great impact was Hilbert's ability to present algebraic number theory as a major mathematical discipline in accordance with what he saw as the dominant values of the time. In the *Zahlbericht* Hilbert boldly acclaimed that "the whole modern development of pure mathematics takes place principally under the badge of number" (p. 702).

Even though later generations used the *Zahlbericht* as a standard reference, its reception was somewhat ambivalent: its advanced character made it difficult to access for the average working mathematician of the late nineteenth century. In Schappacher's words, the *Zahlbericht* was "the most lucky blend of past, present, and future [...] the perfect command and exposition of the past, the solutions of new problems, and the most refined prescience of the things to come" (p. 708).

Hilbert's essay of 1901, entitled "Mathematische Probleme", stands out as a turning point in the history of modern mathematics. Several later contributions in Grattan-Guiness's *Landmark Writings* include references to Hazewinkel's report on this famous paper, which was based on a lecture at the International Congress of Mathematicians in Paris in 1900. In this lecture, Hilbert presented a collection of ten acute, inspiring, and highly challenging mathematical problems. The full published version contains 23 problems. These well-chosen and timely problems had a remarkable influence on the development of a number of mathematical disciplines.

Hazewinkel provides the reader with short descriptions of these problems, together with possible full or partial solutions. Hence, his article is a very useful check-list on what these famous problems were all about. Also, Hazewinkel briefly discusses some later and less successful attempts to produce new stimulating lists of important mathematical problems for the twenty-first century (cf. the seven millennium problems by the Clay Institute). According to Hazewinkel, the great success of Hilbert's list was partly based on the stature of the lecture, partly on Hilbert's clever choice of the problems, partly on his unique insight into the mathematics of his time, and perhaps most of all on "the incurable optimism in it all, a flat denial of Emil Du Bois-Reymond's claim 'Ignoramus et ignoramibus'" (p. 733). Hazewinkel certainly has it right when he states that "today seems to be less a period of problem solving, nor a period of large theory building. Instead we seem to live in a period of discovery where new beautiful applications, interrelations and phenomena appear with astonishing frequency" (p. 743).

In Chapter 61, Ivor Grattan-Guinness provides a critical account of the three- volume *Principia Mathematica* (1910-1913) by Alfred North Whitehead and Bertrand Russell (pp. 784-794). In this huge work on the foundations of mathematics, Russell and Whitehead attempted to show that all mathematics could be built upon a combination of pure mathematical logic and set theory. Even though the program was not a complete success, it stimulated a great amount of later work on logic, philosophy, and the foundations of mathematics.

Grattan-Guinness's clear and well-composed essay draws an illuminating picture of the golden decades in foundational studies during the early twentieth century. Undoubtedly the greatest value of his contribution lies in its insightful discussion of the historical context of the *Principia Mathematica*, including its international reception. The story begins with Russell's admiration of the work of Cantor and Peano, moves on to Russell's stimulating discovery of his famous set-theoretical paradox, introduces his logicist program and the contents of the landmark itself—not forgetting to say a few words about the originally planned fourth volume on geometry—and ends with Gödel's decisive refutation of the program with his 1931 incompleteness results (the topic of another outstanding landmark-article by Richard Zach).

In Chapter 63, Tilman Sauer discusses Albert Einstein's revolutionary paper of 1916, entitled "Die Grundlage der allegmeinen Relativitätstheorie" (pp. 802-822). This review paper was the first comprehensive overview of the final version of Einstein's general theory of relativity.

Sauer's essay is an excellent short introduction to the early history of Einstein's theory of relativity. Sauer begins his paper with a short account of Einstein's 1905 introduction of his special theory of relativity. He correctly points out that from the mathematical point of view Einstein's contribution was not too sophisticated and relied on standard techniques of elementary calculus (p. 803). Sauer's account of the subsequent generalization of the special theory of relativity to a generally covariant theory of gravitation proceeds step-by-step: he first presents the 1907 formulation of the equivalence hypothesis; then discusses the introduction of the metric tensor as the crucial mathematical concept for a generally relativistic theory of gravitation in 1912; and moves on to comment upon the discovery of the generally covariant field equations of gravitation in 1915. Finally, this development is brought to a culmination by the 1916 landmark-paper.

Sauer concludes his article with highly interesting observations about the early reception of the final version of general relativity, the first exact solutions to the field equations, and Arthur Eddington's famous expedition to Brazil in 1919 which turned Einstein into a world celebrity. And, finally, he concludes his remarks with a few words about the most relevant later developments, such as Hermann Weyl's geometrized unification of the gravitational and electromagnetic fields.

In Chapter 75, Jan von Plato discusses Andrei Kolmogorov's *Grundbegriffe der Wahrscheinlichkeitsrechnung* (1933), a work which established the set-theoretic foundations of probability theory (pp. 960-969). It has become the symbol of modern measure-theoretic probability theory, and the year of its publication became a turning point that made earlier studies dispensable. According to von Plato, Kolmogorov's treatment of conditional probabilities and infinite fields of probability was of particular importance (p. 960).

Kolmogorov's results have an immediate connection to the sixth problem in Hilbert's 1901 list, that is, the mathematical treatment of the axioms of relevant physical disciplines such as mechanics and the calculus of probability. Kolmogorov based his solution to this problem on his strong conviction that probability theory could be formalized precisely the same abstract way as geometry or algebra. As Kolmogorov himself says, in the preface to his *Grundbegriffe*, "these new questions arose out of necessity from certain very concrete physical questions" (p. 967).

Von Plato emphasizes that Kolmogorov's epoch-making contribution lies on what he achieved by using the traditional measure-theoretic approach to probability theory. The two most important new mathematical results brought about by the *Grundbegriffe* were the theory of conditional probabilities when the condition has probability zero, and the general theory of random or stochastic processes. In von Plato's words, "one could speak of a set-theoretic foundation of the whole of probability theory only after conditional probabilities as well as distributions in infinite product spaces had been incorporated" (pp. 962-963).

Von Plato closes his insightful article with some remarks on the impact of Kolmogorov's book. Even though the *Grundbegriffe der Wahrscheinlichkeitsrechnung* was later recognized as a revolutionary advancement in the mathematics of probability, it was not immediately digested by other leading experts in mathematical probability. Its global acceptance took some time and this was at least partly due to resistance from other competing approaches by von Mises and others.

All in all, Grattan-Guinness's *Landmark Writings in Western Mathematics 1640-1940* is a great hand-book, an excellent source-book, and a very entertaining advanced introduction to the history of modern mathematics. This volume offered me the most rewarding reading experience I have had for a long time.

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