The Review of Modern Logic
Volume 11 Numbers 1 & 2 (June 2007–December 2008) [Issue 33], pp. 127–131.

Danielle Macbeth
Frege's Logic
Cambridge MA: Harvard University Press, 2005
x + 206pp. ISBN 0674017072

MACBETH'S "HITHERTO UNIMAGINABLE" FREGE

IGNACIO ANGELELLI

The first part of this essay surveys the main aspects of Macbeth's book; the second part offers some critical comments.

1. Overview of *Frege's Logic*.

After a *Preface* and an *Introduction*, five chapters follow: 1) *The starting point*, 2) *Logical generality*, 3) *A more sophisticated instrument*, 4) *The work brought to maturity*, 5) *Courses of values and Basic Law V*. The volume is completed by an *Epilogue*, *Notes*, *Abbrevations for works by Gottlob Frege*, and an *Index*[1].

In the first section (1.1) of Chapter 1 the author points out that for Frege Euclid's demonstrations are not only deductively but also "expressively" defective (p. 12). Not that Euclid leaves only the logic tacit; he also fails to express "material rules of inference", such as "If an angle is smaller than a second one, the former is not larger than the latter" (p. 12). Such material rules of inference are the main point of chapter 1, and indeed the "starting point" (cf. the title of ch. 1) for the author's analysis of Frege. In section 1.2 Macbeth claims that Frege's *Begriffsschrift* notation is intended to fill that expressive gap by means of generalized conditionals (p. 17), and in section 1.3 she emphasizes that Frege's generalized conditionals are what the just mentioned purpose demands, namely rules of inference, not narrations of facts (for example, p. 35, line 6 of the second paragraph).

In ch. 2 the generalized conditionals from the first chapter are said to be "Frege's primary concern" (p. 57, line 2), and they are more closely examined, both as conditionals and as generalizations. With regard to the first aspect, Macbeth highly praises the Fregean two-dimensional

[1]Two editorial comments: on p. 86, should be "Pünjer" instead of "Punjar". It is very inconvenient not to have all references grouped together.

notation, in which she sees much more than the mere avoidance of parentheses or of any other type of grouping indicator (p. 47, end of the first paragraph). This "much more" is that "*Begriffsschrift* conditionals can be read in various ways" (p. 47, bottom). "By contrast with sentences in our standard linear notation, each of which has one and only one main connective ...sentences in *Begriffsschrift* have a main connective only relative to an analysis" (p. 50). Consider for example the Fregean two-dimensional conditional having three antecedents: S, R, Q, and one consequent: P. In the standard notation four different sentences emerge: $S \to (R \to (Q \to P))$, $S \to ((R \,\&\, Q) \to P)$, $(S \,\&\, R) \to (Q \to P)$, $(S \,\&\, R \,\&\, Q) \to P$. Each of these "represents ... one path through Frege's two-dimensional structure, one perspective it is possible to take on it" (p 51). Macbeth adds: "The equivalence of these four formulae, though it must be proven in standard (one-dimensional) notation, is a given of Frege's two dimensional notation." (p. 51). With regard to the second aspect of the generalized conditionals, namely the generalization, the principal role is played by Frege's Latin italic letters.

Now, within the author's discussion of this generality of conditionals a new theme emerges: the Latin italic letters have also the effect of "moving everything up a level" (p. 71, last line). Thus, the conditional "If a is human, then a is a living being", with or because of the Latin letter "a" becomes a statement not about objects which are or fail to be humans or living beings, but about two concepts: the concept human and the concept living being.

Chapter 3 looks at four of Frege's mature revisions of his earlier views: 1) the distinction between concept and object is "more sharply" characterized, which is related to the distinction of levels of concepts. 2) Functions, initially described as expressions, become laws of correlation from concepts to truth-values (3.2). 3) The German letters not only help to demarcate the quantified scope but also play an "essential" role in the expression of higher level functions (3.3). 4) Finally, while logic was first conceived as a theory of maximal generality, it becomes subsequently a theory of higher-level functions or concepts (cf. 3.4).

Chapter 4 describes Frege as having started with a defective view of the semantics of concept words, namely that they are purely predicative and have no reference. Such a view, obviously, ruins the project of understanding logic as a theory of concepts. If concept words do not refer to anything, the sentence "All P are Q" is really about nothing: it expresses no thought, and the subordination of concepts supposedly expressed by generalized conditionals evaporates. The mature Frege

corrects the situation by introducing a reference (*Bedeutung*) for concept words. Thus, the generalized conditional "All P are Q" is about the concepts P and Q, just like, for example, "John is tall" is about John. Also, the mature Frege extends *Bedeutung* to sentences, an extension regarded as "natural" by the author (p. 145). Also, in ch. 4 the theme of the multiple analyzability of any given expression recurs. This theme is regarded by Macbeth as a "central insight" in Frege (first sentence of ch. 5); the multiple analyzability is an "essential" feature of Frege's logic (p. 143, last paragraph of 4.3).

Chapter 5 looks at one aspect of the mature Frege that went wrong: the introduction of the notion of class, or "course of values". Frege's error was to assume, as a law, that any two concepts that are mutually subordinated "share a common course of values in common" (introduction to ch. 5, p. 156).

In sum, what Macbeth tells us is basically the following: 1) For Frege, logic is about concepts, their properties, the properties of the properties, etc. and logical theory focuses on the study of laws that apply generally to all concepts. 2) For Frege, expressions are analyzable in multiple ways: the application of this principle extends from the various ways in which an atomic sentence like "Romeo loves Juliet" can be viewed in terms of function-argument(s) to the different groupings according to which logically compound formulae, such as the two dimensional conditionals, can be read.

2. Reviewer's remarks

If the above overview is adequate, then it is imposible, for this reviewer, to understand the author's claim that she has unveiled a Frege "hitherto unknown" (p. 1), or even unknowable: "hitherto unimaginable" (p. 179). With special reference to the last paragraph of the preceding section, the ontological interpretation of Frege's higher- order logic as the theory of properties, properties of properties, etc., would not surprise in the least, for example, H. Scholz, the first thorough Frege scholar (*cf.* References), while the multiple analyzability of expressions appears to me as a trivial matter, accessible to any reader of Frege.

Leaving aside this astonishingly superlative self-evaluation, I would like to state, in what follows, my disagreement over a number of topics in Macbeth's book.

(1) Macbeth says that Frege's logic is a "...fundamentally different kind of language from that of quantificational logic" (for example, p. 72). This is, in my view, wrong. What we see in logic

books, from treatises to textbooks, in the 20th century, and continue to see in the early years of the 21st century is essentially the impact of the work of Frege in conjunction, of course, with that of other pioneers of modern logic. The philosophical interpretation of the sentential and predicate calculi may vary, from viewing it as a platonic ontology of properties and higher properties to nominalist readings, but the Fregean *novum organon* remains the same. (The artificial conflict created by Macbeth between modern quantificational logic and Frege's logic seems to have been largely fueled by her overstating the significance of the Fregean two-dimensional notation.)

(2) The content of *Begriffsschrift* expressions is regarded by Macbeth as "essentially two-dimensional" (p. 143). Why? Macbeth explains: "for what matters to the correctness of judgment and inference is that sentences be variously analyzable" (*ibid.*). I fail to see that a two-dimensional notation (as a matter of fact, only for conditionals) essentially or necessarily follows from the multiple analyzability of expressions.

(3) Contrary to Macbeth's claim that the interchangeability of antecedents is, in Frege, "a given" (*cf.* text quoted in the preceding section), the fact is that Frege says that such interchangeability must be proved (*nachgewiesen*, in *Grundgesetze*, §12), just as in our ordinary presentations of sentential logic the convenient equivalence of the various groupings of the so-called "continued" disjunctions $(p \vee q \vee r)$ or conjunctions $(p \,\&\, q \,\&\, r)$ must be demonstrated–and is not "a given".

(4) With regard to the "Latin italic letters", Macbeth reiterates that Frege introduced such letters for the sake of expressing generality. There is however another, indeed previous purpose or use of those letters, apparently not recognized by the author. This use becomes obvious in a Fregean text quoted by Macbeth herself (p. 64), where Frege considers algebraic expressions such as "$a + (b + c) = (a + b) + c$". As a first step, Frege wants to examine this expression "quite independently of the sign of addition". To this end he writes "f" instead of "+": "$f(a, f(b, c)) = f(f(a, b), c)$". At this exploratory stage, one does not know if this is true for any interpretation of "f", and consequently the Latin letter f does not express generality. Should we regard the expression "$f(a, f(b, c)) = f(f(a, b), c)$" as senseless, just as Macbeth says that "a is greater than 2" is senseless (p. 63)? The answer is negative, perhaps *pace* Frege himself. Frege, while trying to see if the truth of "$a + (b + c) =$

$(a + b) + c$" holds "independently of the sign of addition", uses the Latin letter f not to express generality but to *formalize*, at least partially, the given expression.

(5) With regard to German letters, Macbeth writes that "higher-level concepts...are designated by expressions that make essential use of his concavity and German letters" (3.3, end of first paragraph). The word "essential" appears to be inappropriate here; there are higher level functions without any concavity or German letters, *e.g.*, in *Grundgesetze* I, §22, p. 39, right column, top.

(6) From someone who, like Macbeth, emphasizes the view of Frege's logic as a theory of higher level properties, one would expect a discussion of *what* is unsaturatedness—not the trivial unsaturatedness of symbols: a blank space in the written expression, but the alleged unsaturatedness of the *entities* designated by the unsaturated expressions. For the same reason, one would expect a study of the issues related to (im)predicativity, inevitably starting with a critique of the Fregean analysis of the ancestral and its shocking circularity.

(7) In her ch. 5, as mentioned above, the author goes into "what went wrong" in Frege's project. Here one would expect a discussion and critique of the method employed by Frege in his analysis of the concepts of number, of set (*Wertverlauf*), and perhaps of *Bedeutung* as well: a two-stages procedure moving from an equivalence relation to a semantical assignment to the expressions whose meaning one wants to make more precise (*cf.* my "The Troubled History of Abstraction").

References

[1] Angelelli, Ignacio, "The Troubled History of Abstraction," in *Logical Analysis and History of Philosophy*, Uwe Meixner and Albert Newen (eds.), Paderborn: Mentis Verlag, **8**, 2005, pp. 157-175.

[2] Frege, Gottlob, *Grundgesetze der Arithmetik, I*, Jena, 1893.

[3] Scholz, Heinrich, *Logik, Grammatik, Metaphysik*, reprinted in *Mathesis Universalis*, H. Hermes et al. (eds.), Darmstadt: Wissenschaftliche Buchgesellschaft, 1961, pp. 399-436.

[4] Scholz, Heinrich, and Hasenjäger, Gisbert, *Grundzüge der mathematischen Logik*,Berlin: Springer Verlag, 1961.

Department of Philosophy, The University of Texas at Austin, 1 University Station C3500, Austin TX 78712-0310

E-mail address: angelelli@mail.utexas.edu